Binomial Hypothesis Testing

Binomial Hypothesis testing was a topic I found very hard at school. It took me a long time to get my head round what was going on, but once it had sunk in it was an easy set of marks to get in the exam; you just have to have your wits about you! I am going to explain it by means of an example with commentary on the right.

QUESTION: "Vinita threw a coin 15 times and it came up heads 13 times. Mr. Kench remarked that it looked like the coin was biased in favour of heads. Test Mark's Hypothesis at the 1% significance level."

ANSWER	COMMENTARY
Let p be probability of a head from a toss of the coin $\mathrm{H}_0 : p = \frac{1}{2}$ ("The coin is not biased towards heads") $\mathrm{H}_1 : p > \frac{1}{2}$ ("The coin is more likely to give a head") Significance level : $\alpha = 1\%$ 1-tail test	We first need to set up your null hypothesis (H_0) and alternative hypothesis (H_1). The null hypothesis is always the boring option. The one where nothing is biased, nothing changes or nothing works. The alternative hypothesis is the exciting one, where something is biased, a class cheated or someone has psychic abilities(!)
Let X = "The number of heads from 15 tosses." $X \sim B(15, \frac{1}{2})$. $\mathbb{P}(X \geqslant 13) = \mathbb{P}(X = 13) + \mathbb{P}(X = 14) + \mathbb{P}(X = 15)$ $= \left({}^{15}C_{13} \left(\frac{1}{2}\right)^{13} \left(\frac{1}{2}\right)^{2}\right) + \left({}^{15}C_{14} \left(\frac{1}{2}\right)^{14} \left(\frac{1}{2}\right)^{1}\right) + \left(\left(\frac{1}{2}\right)^{15}\right)$ $= 0.00369 = 0.369\%$	We now proceed ***on the basis that H_0 is true***, and calculate the probability that, if $p = \frac{1}{2}$, then what is the probability of what we have seen ***and anything more extreme***. So in this case we need to calculate the probability that the number of heads from 15 tosses is 13, 14 or 15. i.e. $\mathbb{P}(X \geqslant 13)$.
$0.369\% < 1\%$ so we ***reject*** H_0 and conclude that at the 1% significance level the coin is biased.	We then compare our probability of 13 or more heads with our significance level. Because the likelihood of 13 or more heads is only 0.369% (i.e. very unlikely) compared with the 1% level we set, we conclude that the coin is probably biased and so we reject H_0. [If the probability is less than our significance level, then we reject H_0. If the probability is greater than our significance level, then we accept H_0.]

It should be noted for the future that $\mathbb{P}(X \geqslant 13) = 1 - \mathbb{P}(X \leqslant 12)$. This is important for using statistical tables in the exam.

This is most of the theory, but we will now need to go on to consider two-tail tests (the above example is one-tail) and how to use statistical tables to help save us from endless binomial calculations.