

## Conditional Probability Worksheet

Most of the questions here were stolen from a textbook, not written by me.

- The two events  $A$  and  $B$  are such that  $\mathbb{P}(A) = 0.6$ ,  $\mathbb{P}(B) = 0.2$ ,  $\mathbb{P}(A | B) = 0.1$ . Calculate the probabilities that
  - both of the events occur;
  - at least one of the events occur;
  - exactly one of the events occurs;
  - $B$  occurs given that  $A$  occurs.
- I travel to work by route  $A$  or route  $B$ . The probability that I chose route  $A$  is  $\frac{1}{4}$ . The probability that I am late for work if I go via route  $A$  is  $\frac{2}{3}$  and the corresponding probability for route  $B$  is  $\frac{1}{3}$ .
  - What is the probability that I am late for work on Monday?
  - Given that I am late for work, what is the probability that I went via route  $B$ ?
- Susan takes exams in Maths, French and History. The probabilities that she passes are 0.7, 0.8 and 0.6 respectively. Given that her performances in each subject are independent, draw a tree diagram to show the possible outcomes. Find the probabilities that Susan
  - fails all three examinations
  - fails just one examination
  - Given that Susan fails just one examination, find the probability that it is History that she fails.
- Four coins are biased so that the probabilities of getting a head are 0.4, 0.48, 0.52 and 0.6. One coin is chosen at random and, on being tossed three times, gives three heads. What is the probability that the coin chosen was the coin with the bias 0.52?
- On a certain Pacific island, a particular disease is caught by one person in a thousand. 98% of those who have the disease respond positively to a diagnostic test, but 3% of those who do not have the disease also respond positively to the same test (a 'false positive'). If a person selected at random responds positively to the test, what is the probability that he actually has the disease?
- If  $\mathbb{P}(A) = 0.5$ ,  $\mathbb{P}(B) = 0.7$  and  $\mathbb{P}(A \cup B) = 0.8$ , find  $\mathbb{P}(A | B)$ .
- $A$  and  $B$  are independent events, and  $\mathbb{P}(A) = 0.7$  and  $\mathbb{P}(B) = 0.2$ . What are
  - $\mathbb{P}(B | A)$
  - $\mathbb{P}(A \cap B)$
  - $\mathbb{P}(A \cup B)$ ?
- Two dice are rolled. Given that there is at least one six, what is the probability that both are sixes?
- If there are 23 people in a room, find the probability that at least two share the same birthday.
- A parent at an all boys school was heard to say at a parents' meeting that she had two children. What is the probability that they are both boys?

11. A red and a blue die are rolled. Given that there is at least one six, what is the probability that the red die is showing six?  $\boxed{\frac{6}{11}}$
12. Ten cards lie face down on a table. They contain the digits  $0, 1, 2, \dots, 9$ . Two cards are chosen at random from the table. Find the probability that
- the sum of the numbers is greater than 9, given that the first number is 3,  $\boxed{\frac{1}{3}}$
  - the second number is 2, given that the sum of the two numbers is greater than 7,  $\boxed{\frac{4}{63}}$
  - the first number is 4, given that the difference between the two numbers is 4.  $\boxed{\frac{1}{6}}$
13. Two tetrahedral die are thrown; one is red and the other blue. The number on which each lands is noted, the faces being marked 1,2,3 and 4. Find the probability that
- the sum of the numbers on which the dice land is 6, given that the red die lands on an odd number,  $\boxed{\frac{1}{8}}$
  - the blue die lands on a 2 or a 3, given that the red die lands on a 2.  $\boxed{\frac{1}{2}}$
14. Two cards are drawn successively from an ordinary pack of cards and kept out of the pack. Find the probability that
- both cards are hearts,  $\boxed{\frac{1}{17}}$
  - the first card is a heart and the second card is a spade,  $\boxed{\frac{13}{204}}$
  - the second card is a diamond, given that the first card is a club.  $\boxed{\frac{13}{51}}$
15. 90% of Scandinavians are blue eyed, whereas only 25% of English are. In our establishment, 10% of the pupils are Scandinavian, and the rest English. I bump into a blue eyed person in the corridor: what is the probability that he is Scandinavian?  $\boxed{\frac{2}{7}}$
16. A bag contains 4 red balls, 3 yellow balls and 5 green balls. I take two balls from the bag, without replacement. Given that the balls are the same colour, find the probability that they are both green.  $\boxed{\frac{10}{19}}$
17. A bag contains 4 red balls, 3 yellow balls and 5 green balls. I take three balls from the bag, without replacement. Given that the balls are all the same colour, find the probability that they are all green.  $\boxed{\frac{2}{3}}$
18. A bag contains  $r$  red balls,  $y$  yellow balls and  $g$  green balls. I take two balls from the bag, without replacement. Given that the balls are different colours, find the probability that they contain a yellow.  $\boxed{\frac{ry+gy}{ry+gy+rg}}$
19. In a class of 24 girls, 7 have blonde hair, 5 have red hair and the rest have black hair. I select a committee of 5 from the class completely at random. Given that exactly two are red heads, find the probability that my committee contains exactly one blonde.  $\boxed{\frac{154}{323}}$
20. A bag contains 5 white and 7 black counters. I take 12 balls from the bag *with* replacement. Given that the first 3 balls were black, what is the probability my hand contains equal numbers of black and white?  $\boxed{0.08725}$
21. Two events A and B are such that  $\mathbb{P}(A) = 0.6$ ,  $\mathbb{P}(B) = 0.2$ , and  $\mathbb{P}(A | B) = 0.1$ . Find  $\mathbb{P}(B | A)$ .  $\boxed{\frac{1}{30}}$
22. We receive 10,000 transistors from company A, and there is a 1% failure rate on their components. We also receive 4000 transistors from company B, which have a 1.5% failure rate. A transistor fails: what is the probability that it came from company A?  $\boxed{\frac{5}{8}}$

23. All the diamonds are removed from a standard pack of cards, and a card is taken at random from this reduced pack. If  $K$  is the event the card is a King, and  $S$  is the event the card is a Spade, are  $K$  and  $S$  independent?  $\text{Yes, } P(S) \times P(K) = P(S \cap K) = \frac{1}{39}$
24. If  $A$  is independent of  $B$ , (i.e. the occurrence of  $B$  doesn't change the probability of  $A$  happening), does it also imply that  $B$  is independent of  $A$ ? Prove it! Yes
25. The random events  $A, B$  and  $C$  are defined in a finite sample space  $S$ .  $A$  and  $B$  are mutually exclusive and  $A$  and  $C$  are independent.  $\mathbb{P}(A) = \frac{1}{5}$ ,  $\mathbb{P}(B) = \frac{1}{10}$ ,  $\mathbb{P}(A \cup C) = \frac{7}{15}$ , and  $\mathbb{P}(B \cup C) = \frac{23}{60}$ . Evaluate  $\mathbb{P}(A \cup B)$ ,  $\mathbb{P}(A \cap B)$ ,  $\mathbb{P}(A \cap C)$ , and  $\mathbb{P}(B \cap C)$ , and state whether  $B$  and  $C$  are independent.  $\frac{3}{10}, 0, \frac{1}{15}, \frac{1}{20}, \text{no}$