

Partial Fractions: The Return!

In C4 we learnt how to split two types of expressions into partial fractions; namely:

$$\frac{px + q}{(ax + b)(cx + d)} \equiv \frac{A}{ax + b} + \frac{B}{cx + d},$$

and

$$\frac{px^2 + qx + r}{(ax + b)(cx + d)^2} \equiv \frac{A}{ax + b} + \frac{B}{cx + d} + \frac{C}{(cx + d)^2}.$$

In FP2 we add

$$\frac{px^2 + qx + r}{(ax + b)(cx^2 + d)} \equiv \frac{A}{ax + b} + \frac{Bx + C}{cx^2 + d}.$$

As before this becomes important in integrating certain rational functions in the form $p(x)/q(x)$.

1. Split the following into partial fractions:

(a) $\frac{x + 1}{(x^2 + 1)(x - 1)}$ $\frac{1}{x-1} - \frac{x}{x^2+1}$

(b) $\frac{3x + 4}{(x^2 + 4)(x + 2)}$ $\frac{x+10}{4(x^2+4)} - \frac{1}{4(x+2)}$

(c) $\frac{1 - 3x}{(1 + 2x)(1 + x^2)}$ $\frac{2}{2x+1} - \frac{x+1}{x^2+1}$

(d) $\frac{16 + 2x + 15x^2}{(1 + x^2)(2 - x)}$ $\frac{x}{x^2+1} - \frac{16}{x-2}$

(e) $\frac{3x^2 + 4x - 28}{(2x - 1)(x^2 + 25)}$ $\frac{2x+3}{x^2+25} - \frac{1}{2x-1}$

(f) $\frac{2x^2}{(x + 2)(x^2 + 4)}$ $\frac{x-2}{x^2+4} + \frac{1}{x+2}$

2. Find the values of the following integrals:

(a) $\int_1^2 \frac{2 + 3x^2}{x(x^2 + 2)} dx$ $2 \ln 2$

(b) $\int_0^1 \frac{1 - 2x - x^2}{(1 + x)(1 + x^2)} dx$ 0

(c) $\int_2^3 \frac{8 + 3x - x^2}{(x - 1)(x^2 + 4)} dx$ $\frac{13}{2} \ln 2 - \frac{3}{2} \ln 13$