FP2 Differential Equation Substitutions

1. (a) By using the substitution $y^3 = z$, find the general solution of the differential equation

$$3y^2\frac{dy}{dx} + 2xy^3 = e^{-x^2},$$

giving y in terms of x in your answer.

(b) Describe the behaviour of y as $x \to \infty$. [OCR]

2. The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{x^2 - y^2}{xy}. \qquad \text{(A)}$$

(a) Use the substitution $y = xz$, where z is a function of x, to obtain the differential equation

$$x\frac{dz}{dx} = \frac{1 - 2z^2}{z}.$$

(b) Hence show by integration that the general solution of the differential equation (A) may be expressed in the form $x^2(x^2 - 2y^2) = k$, where k is a constant. [OCR]

3. (a) Use the substitution $z = x + y$ to show that the differential equation

$$\frac{dy}{dx} = \frac{x + y + 3}{x + y - 1} \qquad \text{(A)}$$

may be written in the form $\frac{dz}{dx} = \frac{2(z + 1)}{z - 1}$.

(b) Hence find the general solution of the differential equation (A). [OCR]

4. The variables x and y are related by the differential equation

$$x^3\frac{dy}{dx} = xy + x + 1. \qquad \text{(A)}$$

(a) Use the substitution $y = u - \frac{1}{x}$, where u is a function of x, to show that the differential equation ma be written as

$$x^2\frac{du}{dx} = u.$$

(b) Hence find the general solution of the differential equation (A), giving your answer in the form $y = f(x)$. [OCR]

5. (a) Use the substitution $y = xz$ to find the general solution of the differential equation

$$x\frac{dy}{dx} - y = x\cos\left(\frac{y}{x}\right),$$

giving your answer in a form without logarithms. (You may quote an appropriate result from the formula booklet.)

(b) Find the solution of the differential equation for which $y = \pi$ when $x = 4$. [OCR]

6. The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{2x^2 + y^2}{xy}. \qquad \text{(A)}$$

(a) Use the substitution $y = ux$, where u is a function of x, to obtain the differential equation

$$x\frac{du}{dx} = \frac{2}{u}.$$

(b) Hence find the general solution of the differential equation (A), giving your answer in the form $y^2 = f(x)$. [OCR]

7. The substitution $y = u^k$, where k is an integer, is to be used to solve the differential equation

$$x\frac{dy}{dx} + 3y = x^2y^2 \qquad \text{(A)}$$

by changing it into an equation (B) in the variables u and x.

(a) Show that the equation (B) may be written in the form

$$\frac{du}{dx} + \frac{3}{kx}u = \frac{1}{k}xu^{k+1}.$$

(b) Write down the value of k for which the integrating factor method may be used to solve equation (B).

(c) Using this value of k, solve equation (B) and hence find the general solution of equation (A), giving your answer in the form $y = f(x)$. [OCR]