Intersection of Lines in 3D

1. Find the vector equation of the line (in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$) through the following points:

   (a) $(3,2,-7)$ and $(5,-13,-4)$.

   (b) $(-8,-13,-9)$ and $(12,7,1)$.

   (c) $(3,0,0)$ and $(0,0,5)$.

   (d) $(0,0,0)$ and $(-10,4,-6)$.

   (e) $(a,3a,2a)$ and $(2a,a,-a)$.

   (f) (a,a^2,a^2+b^2) and $(b,b^2,2ab)$.

2. Find whether the following lines intersect, are parallel or are skew for the following pairs of lines. Obviously if they *do* intersect find the point of intersection.

   (a) $\mathbf{r} = \begin{pmatrix}-7\\-3\\-12\end{pmatrix} + \lambda\begin{pmatrix}5\\-2\\8\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}17\\-1\\-4\end{pmatrix} + \mu\begin{pmatrix}7\\3\\-4\end{pmatrix}$. Intersect at (3, −7, 4)

   (b) $\mathbf{r} = \begin{pmatrix}-5\\-30\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\7\\1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}8\\6\\-5\end{pmatrix} + \mu\begin{pmatrix}4\\-2\\3\end{pmatrix}$. Skew

   (c) $\mathbf{r} = \begin{pmatrix}11\\-7\\2\end{pmatrix} + \lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}1\\1\\-1\end{pmatrix} + \mu\begin{pmatrix}-10\\-8\\6\end{pmatrix}$. Parallel

   (d) $\mathbf{r} = \begin{pmatrix}-1\\5\\-4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and the line that passes through $(3,5,6)$ and $(-1,1,-2)$.

   Intersect at (1, 3, 2)

   (e) $\mathbf{r} = \begin{pmatrix}0\\2\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ and the line that passes through $(-3,5,-3)$ and $(-2,3,0)$. Skew

   (f) The line that passes through $(3,2,4)$ and $(-3,-7,-8)$ and the line that passes through $(0,1,3)$ and $(-2,5,9)$. Intersect at (1, −1, 0)

   (g) The line that passes through $(3,1,0)$ and $(-3,1,3)$ and the line that passes through $(5,0,-1)$ and $(1,0,1)$. Parallel

   (h) The line that passes through $(-5,-4,-3)$ and $(5,1,2)$ and the line that passes through $(-1,-3,0)$ and $(8,0,6)$. Intersect at (−7, −5, −4)

   (i) The line that passes through $(2,0,3)$ and $(-1,2,1)$ and the line that passes through $(4,-1,5)$ and $(10,-5,1)$. Skew

   (j) The line that passes through $(0,0,1)$ and $(2,2,5)$ and the line that passes through $(1,-1,2)$ and $(-1,5,2)$. Intersect at $(\frac{1}{2}, \frac{1}{2}, 2)$

3. Find the value of the constant that makes the following lines intersect.

(a) $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$. $\boxed{a = -3}$

(b) $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ b \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix}$. $\boxed{b = 1}$

(c) $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ c \\ 4 \end{pmatrix}$. $\boxed{c = \frac{91}{6}}$

(d) $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} d \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$. $\boxed{d = \frac{1}{6}}$