# Trigonometry

1. Find the maximum area of a sector of a circle of total perimeter $P$.

2. Sketch on the same diagrams $y = \cos 2\theta$ and $y = 6\cos\theta - 5$. Find the coordinates of their points of intersection, for $0 < \theta < 2\pi$.

3. A triangle $ABC$ has sides of length $a$, $b$, $c$ with opposite angles $A$, $B$, $C$ in the usual way.

   (a) Use the cosine rule to prove that

   $$\sin^2\left(\frac{A}{2}\right) = \frac{(a+b-c)(a-b+c)}{4bc}.$$

   (b) $M$ is the midpoint of the side $BC$. Given that the angle $CAM$ is $\theta$, use the sine rule to show that

   $$\tan\theta = \frac{c\sin A}{b + c\cos A}$$

   and show that if $b = c$, this reduces to $\theta = \frac{1}{2}A$.

4. Solve the equations $\sin\theta = 2\cos(\theta - \frac{\pi}{3})$, giving all solutions for $0 < \theta < 2\pi$.

5. Prove that

   $$\tan\theta + \cot\theta = 2\operatorname{cosec} 2\theta.$$

   Hence find all solutions to the equation $\tan\theta + \cot\theta = 4\sin 2\theta$ for $0 < \theta < 2\pi$.

6. Given that $\phi$ is a reflex angle and $\sec\phi = \frac{\sqrt{5}}{2}$, find the exact value of

   $$\frac{\sec\phi + \tan\phi}{\sec\phi - \tan\phi}.$$

7. Find $R$ and $\alpha$ such that $2\sin\theta + \cos\theta = R\cos(\theta - \alpha)$. Hence

   (a) sketch the graph of $y = 2\sin\theta + \cos\theta$ for $0 < \theta < 2\pi$;

   (b) solve the equation $2\sin\theta + \cos\theta = -2$, giving all solutions in this range.

8. Sketch the graphs of $y = \sin^{-1} x$ and $y = \cos^{-1} x$. For what values of $x$ does $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

9. Solve the equation $\tan 2x = 3\tan x$, giving all solutions for $-\pi \leqslant x \leqslant \pi$.

10. Sketch the graph of $y = \sin^{-1}(\sin x)$ for $0 \leqslant x \leqslant 2\pi$.

11. Solve the equations, giving all solutions in the range $0 \leqslant x \leqslant 2\pi$.

    (a) $|\sin x + \sqrt{3}\cos x| - 1 = 0$,

    (b) $|\sin x| + \sqrt{3}\cos x - 1 = 0$.

12. Prove the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Hence solve the following equations, giving all solutions in the range $0 \leqslant \theta \leqslant 2\pi$;

    (a) $\sin 3\theta = 2\sin\theta$,

    (b) $\sin 3\theta = \sin 2\theta$.

13. (a) By writing $\sin x + \cos x$ in the form $R\sin(x + \alpha)$, find the maximum and minimum values of $\sin x + \cos x$.

(b) By writing $\sin x(\sin x + \cos x)$ in the form $a + b\sin(2x - \alpha)$, find the maximum and minimum values of $\sin x(\sin x + \cos x)$.

14. (a) Prove that $\sin A + \sin B = 2\sin\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$.

(b) Show that for a standard triangle

$$\frac{a + b}{a - b} = \frac{\tan\frac{1}{2}(A + B)}{\tan\frac{1}{2}(A - B)}.$$

15. Given that $y = \tan^{-1} x$, find $\cos y$ in terms of $x$. Hence solve the equation

$$\tan^{-1} x = \cos^{-1} x.$$