

Equating Coefficients

When asked to express one expression in the form of another, then a good way of doing it is putting the two expressions equal, comparing the LHS to the RHS and ‘reading off’ the answers. For example; “Write $2x^2 + 8x + 4$ in the form $a(x+b)^2 + c$ ”. (Of course we can see that we need only complete the square, but let us assume a catastrophic loss of knowing how to do this.)

Put them equal:

$$2x^2 + 8x + 4 = a(x + b)^2 + c.$$

Multiply out the RHS to get

$$\begin{aligned} 2x^2 + 8x + 4 &= ax^2 + ab^2 + 2abx + c \\ &= (a)x^2 + (2ab)x + (ab^2 + c). \end{aligned}$$

We can then read off that $a = 2$. Then we see that $2ab = 8 \Rightarrow b = 2$ and finally $ab^2 + c = 8 + c = 4 \Rightarrow c = -4$. Therefore

$$2x^2 + 8x + 4 = 2(x + 2)^2 - 4.$$

Questions

1. Write $x^2 + 6x - 2$ in the form $a(b - x)^2 - c$.
2. Write $\frac{x + 3}{x + 5}$ in the form $a + \frac{b}{x + 5}$.
3. Write $3x^2 + 18x - 5$ in the form $\frac{(x + a)^2 + c}{b}$.
4. Write $x^2 + 4x + y^2 - 2y$ in the form $(x + a)^2 + (y + b)^2 + c$.
5. (This is a Jan12 C1 question. Need to adapt further.)
If $5x^2 + px - 8 \equiv q(x - 1)^2 + r$ find p , q and r .