Differentiation Test I

1. Differentiate with respect to x the following:
   (a) $x^{100} - 7x^4 + 2$.
   (b) $x^2 + \frac{1}{x}$.
   (c) $3\sqrt{x} + 7x$.
   (d) $\frac{1}{2x^2}$.

2. Find the coordinates of where $y = x^4 - 9x^2$ crosses the x-axis.

3. Find the equation of the tangent to $y = x^3 + x^2 - 2x + 1$ when $x = -1$.

4. (a) Find the points of intersection of $y = 4 - x^2$ and $y = 3x$. [Draw a very rough sketch of the curve and the line to verify your answer.]
   (b) Find the equations of the tangents to $y = 4 - x^2$ at the points of intersection.
   (c) The tangents intersect at P. Find the coordinates of P.

5. Find the coordinates of where the normal to $y = x^2 + 2x - 3$ when $x = \frac{1}{2}$ crosses the x-axis.

6. Find the coordinate of the point on $y = x^2 - 2x + 3$ where the tangent is parallel to $y + 2x = 3$.

7. An *open-topped* box is to be made from a rectangular sheet of card measuring 16 cm by 10 cm. Four equal squares (each of side length x cm) are to be cut from each corner and the flaps folded up.
   (a) Find an formula for the volume V of the box in terms of x.
   (b) Find the values of x for which V is stationary.
   (c) Show that, for the value of x that is applicable in this problem only, that this stationary point is a maximum.

8. A piece of wire of length l is cut into two parts of lengths x and $l - x$. The former is bent into the shape of a square, and the latter into a rectangle of which the base is double the height.
   (a) Find an expression for the sum of the areas of the two shapes.
   (b) Prove that the only value of x for which this sum is stationary is $x = \frac{8l}{17}$.
   (c) Determine the nature of this point (maximum or minumum).