Sequences Worksheet

Don't forget that $T = 3n - 7$ is a *formula* and $3n - 7$ is an *expression*.

1. Generate the first five terms of the following sequences:
   (a) $T = n + 2$. $\boxed{3, 4, 5, 6, 7}$
   (b) $T = 5n - 1$. $\boxed{4, 9, 14, 19, 24}$
   (c) $T = n^2 - n + 5$. $\boxed{5, 7, 11, 17, 25}$
   (d) $T = 2^n$. $\boxed{2, 4, 8, 16, 32}$
   (e) $T = (-1)^n$. $\boxed{-1, 1, -1, 1, -1}$
   (f) $T = \frac{1}{6}(n^3 - 6n^2 + 11n - 6)$. $\boxed{0, 0, 0, 1, 4}$

2. Find the 100th term of the following sequences by finding an expression for the nth term.
   (a) $7, 10, 13, 16, 19, \ldots$. $\boxed{304}$
   (b) $6, 4, 2, 0, -2, \ldots$. $\boxed{-192}$
   (c) $2.7, 2.9, 3.1, 3.3, 3.5, \ldots$. $\boxed{22.5}$
   (d) $-10, -14, -18, -22, -26, \ldots$. $\boxed{-406}$

3. The number 289 is a term in the sequence $9, 16, 23, 30, \ldots$. Which term is it? $\boxed{41st}$

4. Do the sequences given by $2n+5$ and $8n+6$ ever have a common term? $\boxed{\text{No. Odd/even considerations}}$

5. Find a formula for the nth term of the following two sequences:
   (a) $2, 3, 4, 5, 6, \ldots$. $\boxed{T = n + 1}$
   (b) $3, 5, 7, 9, 11, \ldots$. $\boxed{T = 2n + 1}$

   Using your answers to (a) and (b) find a formula for the nth term of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \ldots$.
   $\boxed{T = \frac{n+1}{2n+1}}$

6. Using a similar argument to the one above, find a formula for the nth term of $\frac{3}{1}, \frac{5}{4}, \frac{7}{9}, \frac{9}{16}, \frac{11}{25}, \ldots$.
   $\boxed{T = \frac{2n+1}{n^2}}$

7. Generate the first 5 terms of the following *inductively* defined sequences:
   (a) $a_1 = 7$ and $a_{n+1} = a_n + 3$. $\boxed{7, 10, 13, 16, 19}$
   (b) $a_1 = 1$ and $a_{n+1} = 2a_n$. $\boxed{1, 2, 4, 8, 16}$
   (c) $a_1 = -3$ and $a_{n+1} = 4 - 2a_n$. $\boxed{-3, 10, -16, 36, -68}$
   (d) $a_1 = 1$ and $a_{n+1} = \frac{1}{1+a_n}$. $\boxed{1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}}$
   (e) $a_1 = 1$, $a_2 = 4$ and $a_{n+2} = 3a_{n+1} - 2a_n$. $\boxed{1, 4, 10, 22, 46}$
   (f) $a_1 = 100$ and $a_{n+1} = -a_n$. $\boxed{100, -100, 100, -100, 100}$

8. The Fibonacci sequence is defined $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = a_{n+1} + a_n$. Generate the first 20 terms of the sequence. What do you notice about the quantity a_{n+1}/a_n as n gets larger? This quantity is called the golden ratio and is talked about on one of the posters at the back of the classroom. Both the Fibonacci sequence and the golden ratio occur a lot in nature. $\boxed{\frac{a_{n+1}}{a_n} \to \text{golden ratio} = \frac{1+\sqrt{5}}{2} \approx 1.618033\ldots}$

9. Find an expression for the following *quadratic* sequences:

(a) $2, 9, 18, 29, 42, 57, \ldots$. $\boxed{n^2 + 4n - 3}$

(b) $-2, 1, 8, 19, 34, 53, \ldots$. $\boxed{2n^2 - 3n - 1}$

(c) $7, 21, 45, 79, 123, 177, \ldots$. $\boxed{5n^2 - n + 3}$

10. Find a formula for the nth term of the following cubic sequence. [Don't worry if you can't do this one.]

    (a) $0, 4, 16, 42, 88, 160, \ldots$. $\boxed{T = n^3 - 2n^2 + 3n - 2}$

11. (a) Generate the first 7 terms of $T = 2^n$. $\boxed{2, 4, 8, 16, 32, 64, 128}$

    (b) Draw a difference table. What do you notice? Using this knowledge, find a formula for the nth term of:

    i. $3, 9, 27, 81, \ldots$. $\boxed{T = 3^n}$

    ii. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$. $\boxed{T = \left(\frac{1}{2}\right)^{n-1} = 2^{1-n}}$

    iii. $2.5, 5, 10, 20, 30, \ldots$ (quite hard). $\boxed{T = 5 \times 2^{n-2}}$

    These are called *exponential* sequences. They crop up in nature a lot, such as cell growth in a petri dish (number of cells doubles every five hours), disease outbreaks (each person infects two others) and nuclear reactors (each neutron hitting an atom releases three new neutrons).