Introduction to Vectors

1. Calculate the following as a single vector:

   (a) $\binom{2}{3} + \binom{4}{7}$. $\boxed{\binom{6}{10}}$

   (b) $\binom{2}{1} + \binom{3}{-2}$. $\boxed{\binom{5}{-1}}$

   (c) $\binom{4}{3} - \binom{1}{0}$. $\boxed{\binom{3}{3}}$

   (d) $\binom{2}{-3} - \binom{-4}{10}$. $\boxed{\binom{6}{-13}}$

   (e) $5\binom{2}{3}$. $\boxed{\binom{10}{15}}$

   (f) $3\binom{2}{3} + \binom{2}{5}$. $\boxed{\binom{8}{14}}$

   (g) $5\binom{1}{0} - 2\binom{1}{3}$. $\boxed{\binom{3}{-6}}$

   (h) $6\binom{0}{4} + \frac{1}{2}\binom{4}{-7}$. $\boxed{\binom{2}{20.5}}$

   (i) $\binom{2}{3} + 2\binom{-3}{1} - \binom{1}{-2}$. $\boxed{\binom{-5}{7}}$

   (j) $k\binom{a}{b}$. $\boxed{\binom{ka}{kb}}$

   (k) $\binom{x}{y} + \binom{a}{b}$. $\boxed{\binom{x+a}{y+b}}$

   (l) $\binom{2x}{y-1} - \binom{x+3}{2y}$. $\boxed{\binom{x-3}{-y-1}}$

2. Find the scalars required to solve the following equations:

   (a) $\binom{3}{a} + \binom{b}{4} = \binom{7}{10}$. $\boxed{a = 6, b = 4}$

   (b) $2\binom{1}{a} + \binom{b}{-1} = \binom{12}{11}$. $\boxed{a = 6, b = 10}$

   (c) $\binom{x+2}{y} + \binom{x}{4} = \binom{6}{2y}$. $\boxed{x = 2, y = 4}$

   (d) $2\binom{2x+1}{-1} - \binom{x-1}{y} = \binom{12}{-8}$. [Signs!] $\boxed{x = 3, y = 6}$

   (e) $p\binom{1}{2} + q\binom{3}{1} = \binom{9}{8}$. $\boxed{p = 3, q = 2}$

   (f) $3\binom{p}{2p} + \binom{q}{q+1} = \binom{6}{10}$. $\boxed{p = 1, q = 3}$

   (g) $p\binom{2}{3} - q\binom{1}{5} = \binom{-2}{-10}$. $\boxed{p = 0, q = 2}$

   (h) $\binom{2x^2}{x+1} + \binom{y^2}{2} = \binom{18}{y}$. $\boxed{x = 1, y = 4 \text{ or } x = -3, y = 0}$

3. If $\mathbf{a} = \binom{2}{3}$ and $\mathbf{b} = \binom{4}{-1}$, illustrate the following vectors in a sketch:

   (a) $\mathbf{a}$.

   (b) $\mathbf{b}$.

   (c) $2\mathbf{a}$.

   (d) $3\mathbf{b}$.

   (e) $\frac{1}{2}\mathbf{a}$.

   (f) $\mathbf{a} + \mathbf{b}$.

   (g) $\mathbf{b} + \mathbf{a}$. [What do you notice about the answer to this question and the previous one?]

(h) $2\mathbf{a} + \mathbf{b}$.

(i) $3\mathbf{a} + 2\mathbf{b}$.

(j) $-\mathbf{a}$.

(k) $-3\mathbf{b}$.

(l) $-\mathbf{a} - \mathbf{b}$.

(m) $2\mathbf{a} - \mathbf{b}$.