

# Introduction to Vectors

1. Calculate the following as a single vector:

- |   |   |
|---|---|
| (a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ .   | $\begin{pmatrix} 6 \\ 10 \end{pmatrix}$     |
| (b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ .  | $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$     |
| (c) $\begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .   | $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$      |
| (d) $\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 10 \end{pmatrix}$ .  | $\begin{pmatrix} 6 \\ -13 \end{pmatrix}$    |
| (e) $5\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ .   | $\begin{pmatrix} 10 \\ 15 \end{pmatrix}$    |
| (f) $3\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ .  | $\begin{pmatrix} 8 \\ 14 \end{pmatrix}$     |
| (g) $5\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ .   | $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$     |
| (h) $6\begin{pmatrix} 0 \\ 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ .                              | $\begin{pmatrix} 2 \\ 20.5 \end{pmatrix}$   |
| (i) $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ . | $\begin{pmatrix} -5 \\ 7 \end{pmatrix}$     |
| (j) $k\begin{pmatrix} a \\ b \end{pmatrix}$ .   | $\begin{pmatrix} ka \\ kb \end{pmatrix}$    |
| (k) $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ .   | $\begin{pmatrix} x+a \\ y+b \end{pmatrix}$  |
| (l) $\begin{pmatrix} 2x \\ y-1 \end{pmatrix} - \begin{pmatrix} x+3 \\ 2y \end{pmatrix}$ .                                     | $\begin{pmatrix} x-3 \\ -y-1 \end{pmatrix}$ |

2. Find the scalars required to solve the following equations:

- |  |                                   |
|--|-----------------------------------|
| (a) $\begin{pmatrix} 3 \\ a \end{pmatrix} + \begin{pmatrix} b \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \end{pmatrix}$ .                  | $a = 6, b = 4$                    |
| (b) $2\begin{pmatrix} 1 \\ a \end{pmatrix} + \begin{pmatrix} b \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 11 \end{pmatrix}$ .               | $a = 6, b = 10$                   |
| (c) $\begin{pmatrix} x+2 \\ y \end{pmatrix} + \begin{pmatrix} x \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 2y \end{pmatrix}$ .                | $x = 2, y = 4$                    |
| (d) $2\begin{pmatrix} 2x+1 \\ -1 \end{pmatrix} - \begin{pmatrix} x-1 \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$ . [Signs!] | $x = 3, y = 6$                    |
| (e) $p\begin{pmatrix} 1 \\ 2 \end{pmatrix} + q\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \end{pmatrix}$ .                 | $p = 3, q = 2$                    |
| (f) $3\begin{pmatrix} p \\ 2p \end{pmatrix} + \begin{pmatrix} q \\ q+1 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$ .              | $p = 1, q = 3$                    |
| (g) $p\begin{pmatrix} 2 \\ 3 \end{pmatrix} - q\begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \end{pmatrix}$ .              | $p = 0, q = 2$                    |
| (h) $\begin{pmatrix} 2x^2 \\ x+1 \end{pmatrix} + \begin{pmatrix} y^2 \\ 2 \end{pmatrix} = \begin{pmatrix} 18 \\ y \end{pmatrix}$ .           | $x = 1, y = 4$ or $x = -3, y = 0$ |

3. If  $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ , illustrate the following vectors in a sketch:

- $\mathbf{a}$ .
- $\mathbf{b}$ .
- $2\mathbf{a}$ .
- $3\mathbf{b}$ .
- $\frac{1}{2}\mathbf{a}$ .
- $\mathbf{a} + \mathbf{b}$ .
- $\mathbf{b} + \mathbf{a}$ . [What do you notice about the answer to this question and the previous one?]

(h)  $2\mathbf{a} + \mathbf{b}$ .

(i)  $3\mathbf{a} + 2\mathbf{b}$ .

(j)  $-\mathbf{a}$ .

(k)  $-3\mathbf{b}$ .

(l)  $-\mathbf{a} - \mathbf{b}$ .

(m)  $2\mathbf{a} - \mathbf{b}$ .