

Subtle Factorising & Simplifying

Patrons are reminded that when they see $9x(x-1)^2 - 6x^2(x-1)$ they should blur your eyes somewhat to see $9xz^2 - 6x^2z$. This clearly factories to $3xz(3z-2x)$ so the original factorises thus:

$$9x(x-1)^2 - 6x^2(x-1) = 3x(x-1)[3(x-1) - 2x] = 3x(x-1)(x-3).$$

Now factorise the following expressions fully.

1. $x(x+4) - 3(x+4)$. $(x+4)(x-3)$
2. $2x(x-3) + (x-3)$. $(x-3)(2x+1)$
3. $4(2x-1) + 3x(2x-1)$. $(2x-1)(3x+4)$
4. $x(x+2) + 2(x+2)$. $(x+2)^2$
5. $3x(y-1) + 2(y-1)$. $(y-1)(3x+2)$
6. $p(x+y) + q(x+y)$. $(x+y)(p+q)$
7. $4q(2x-y) - 3p(2x-y)$. $(2x-y)(4q-3p)$
8. $4x^2(x+2)^2 - 2x(x+2)$. $2x(x+2)(2x^2+4x-1)$
9. $10x^5(2x+1) + 5x^4(2x+1)^2$. $5x^4(2x+1)(4x+1)$
10. $49y(y-4)^{10} - 35y^2(y-4)^9$. $14y(y-4)^9(y-14)$
11. $(x+1)^4(x+2) - 2(x+1)^3(x+2)^2$. $-(x+2)(x+1)^3(x+3)$
12. $20(2x+1)^4(x-2)^5 - 25(2x+1)^5(x-2)^4$. $-5(6x+13)(2x+1)^4(x-2)^4$
13. $16x^2(x-1) - 20x(x-1)^2$. $-4x(x-1)^2$

For the next few patrons are reminded that $\frac{3x(x+1)}{(x+1)^2} = \frac{3x}{x+1}$ and that factorising the top and bottom of a fraction is a wise way to proceed. Simplify the following:

14. $\frac{4x+8}{4}$. $x+2$
15. $\frac{7}{14x-21}$. $\frac{1}{2x-3}$
16. $\frac{3x+6}{5x+10}$. $\frac{3}{5}$
17. $\frac{x^2-4x}{2x-8}$. $\frac{x}{2}$
18. $\frac{4x+2}{6x^2+3x}$. $\frac{2}{3x}$
19. $\frac{2x(x+1)^2}{x^3+x^2}$. $\frac{2(x+1)}{x}$