OCR
Oxford Cambridge and RSA

Wednesday 24 June 2015 – Morning

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A turntable is rotating at $3\,\text{rad}\,\text{s}^{-1}$. The turntable is then accelerated so that after 4 revolutions it is rotating at $12.4\,\text{rad}\,\text{s}^{-1}$. Assuming that the angular acceleration of the turntable is constant,

(i) find the angular acceleration, **[3]**

(ii) find the time taken to increase its angular speed from $3\,\text{rad}\,\text{s}^{-1}$ to $12.4\,\text{rad}\,\text{s}^{-1}$. **[2]**

2 The region bounded by the x-axis, the lines $x = 1$ and $x = 2$, and the curve $y = kx^2$, where k is a positive constant, is occupied by a uniform lamina.

(i) Find the exact x-coordinate of the centre of mass of the lamina. **[6]**

(ii) Given that the x- and y-coordinates of the centre of mass of the lamina are equal, find the exact value of k. **[4]**

3 Two planes, A and B, flying at the same altitude, are participating in an air show. Initially the planes are 400 m apart and plane B is on a bearing of 130° from plane A. Plane A is moving due south with a constant speed of $75\,\text{m}\,\text{s}^{-1}$. Plane B is moving at a constant speed of $40\,\text{m}\,\text{s}^{-1}$ and has set a course to get as close as possible to A.

(i) Find the bearing of the course set by B and the shortest distance between the two planes in the subsequent motion. **[5]**

(ii) Find the total distance travelled by A and B from the instant when they are initially 400 m apart to the point of their closest approach. **[6]**

4 **(i)** Write down the moment of inertia of a uniform circular disc of mass m and radius $2a$ about a diameter. **[1]**

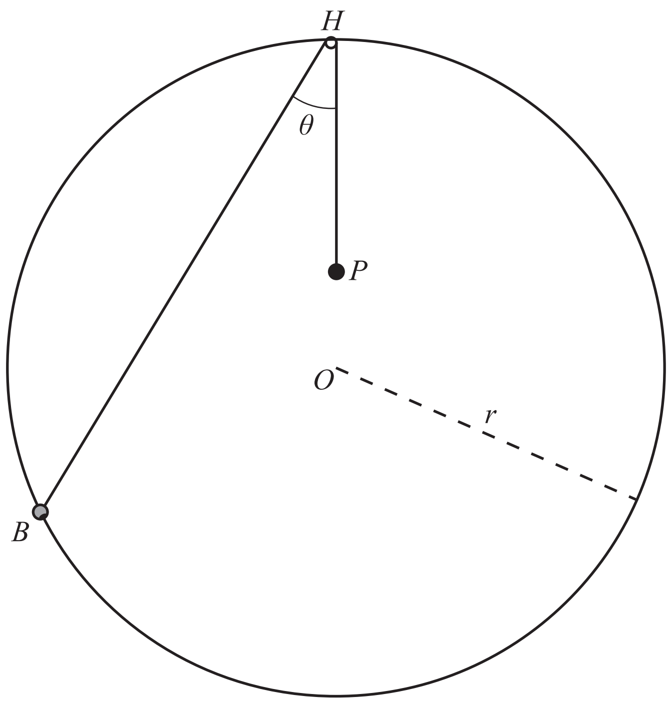
A uniform solid cylinder has mass M, radius $2r$ and height h.

(ii) Show by integration, and using the result from part **(i)**, that the moment of inertia of the cylinder about a diameter of an end face is

$$M\left(r^2 + \tfrac{1}{3}h^2\right)$$

and hence find the moment of inertia of the cylinder about a diameter through the centre of the cylinder. **[8]**

5



A smooth circular wire hoop, with centre O and radius r, is fixed in a vertical plane. The highest point on the wire is H. A small bead B of mass m is free to move along the wire. A light inextensible string of length a, where $a > 2r$, has one end attached to the bead. The other end of the string passes over a small smooth pulley at H and carries at its end a particle P of mass λm, where λ is a positive constant. The part of the string HP is vertical and the part of the string BH makes an angle θ radians with the downward vertical where $0 \leqslant \theta \leqslant \frac{1}{3}\pi$ (see diagram). You may assume that P remains above the lowest point of the wire.

(i) Taking H as the reference level for gravitational potential energy, show that the total potential energy V of the system is given by

$$V = mg(2\lambda r\cos\theta - 2r\cos^2\theta - \lambda a).$$ **[5]**

(ii) Find the set of possible values of λ so that there is more than one position of equilibrium. **[4]**

(iii) For the case $\lambda = \frac{3}{2}$, determine whether each equilibrium position is stable or unstable. **[6]**

6 A pendulum consists of a uniform rod AB of length $2a$ and mass $2m$ and a particle of mass m that is attached to the end B. The pendulum can rotate in a vertical plane about a smooth fixed horizontal axis passing through A.

(i) Show that the moment of inertia of this pendulum about the axis of rotation is $\frac{20}{3}ma^2$. **[3]**

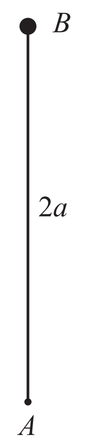


Fig. 1

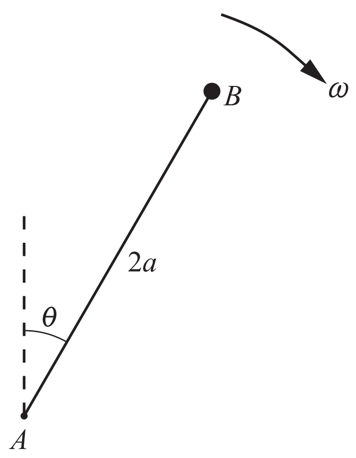


Fig. 2

The pendulum is initially held with B vertically above A (see Fig. 1) and it is slightly disturbed from this position. When the angle between the pendulum and the upward vertical is θ radians the pendulum has angular speed ω rad s^{-1} (see Fig. 2).

(ii) Show that

$$\omega^2 = \frac{6g}{5a}(1 - \cos\theta).$$ **[4]**

(iii) Find the angular acceleration of the pendulum in terms of g, a and θ. **[2]**

At an instant when $\theta = \frac{1}{3}\pi$, the force acting on the pendulum at A has magnitude F.

(iv) Find F in terms of m and g. **[7]**

It is given that $a = 0.735$ m.

(v) Show that the time taken for the pendulum to move from the position $\theta = \frac{1}{6}\pi$ to the position $\theta = \frac{1}{3}\pi$ is given by

$$k\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}\left(\tfrac{1}{2}\theta\right)\mathrm{d}\theta,$$

stating the value of the constant k. Hence find the time taken for the pendulum to rotate between these two points. (You may quote an appropriate result given in the List of Formulae (MF1).) **[6]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1		(i)	4 revolutions $\Rightarrow \theta = 8\pi$ $12.4^2 = 3^2 + 2\alpha(8\pi)$ $\alpha = 2.88 \text{ rad s}^{-2}$ (3 s f)	B1 M1 A1 **[3]**	 Using $\omega^2 = \omega_0^2 + 2\alpha\theta$ with $\text{cv}(\theta(\neq 4))$
		(ii)	$8\pi = \frac{1}{2}(3+12.4)t$ $t = 3.26$ s (3 s f)	M1 A1 **[2]**	Using $\theta = \frac{1}{2}(\omega_0 + \omega)t$ with $\text{cv}(\theta)$ (allow any θ)
2		(i)	$A = \int_1^2 kx^2 \text{d}x = \left[\frac{kx^3}{3}\right]_1^2 = \frac{7k}{3}$ $A\bar{x} = \int_1^2 x(kx^2)\text{d}x = \left[\frac{1}{4}kx^4\right]_1^2 = \frac{15k}{4}$ $\bar{x} = \left(\frac{15k}{4}\right)\left(\frac{3}{7k}\right)$ $\bar{x} = \frac{45}{28}$	*M1 A1 *M1 A1 M1 dep* A1 **[6]**	M1 attempt to integrate to find area. Limits not required for M mark M1 for $...\int x^3 \text{d}x$ and attempt to integrate. Limits not required for M mark M1 for $\bar{x} = \frac{A\bar{x}}{A}$
		(ii)	$A\bar{y} = \frac{1}{2}\int_1^2 (kx^2)^2 \text{d}x = \frac{1}{2}k^2\left[\frac{x^5}{5}\right]_1^2 = \frac{31k^2}{10}$ $\left(\frac{31k^2}{10}\right)\left(\frac{3}{7k}\right) = \frac{45}{28}$ $k = \frac{75}{62}$ Or $A\bar{y} = \frac{1}{4\sqrt{k}}\int_k^{4k} y^{\frac{3}{2}}\text{d}y = \frac{1}{4\sqrt{k}}\left[\frac{2}{5}y^{\frac{5}{2}}\right]_k^{4k} = \frac{31k^2}{10}$	*M1 A1 M1 dep* A1 **[4]**	M1 for $...\frac{1}{2}\int y^2 \text{d}x$ and attempt to integrate $\text{cv}(\bar{x}) = \text{cv}(\bar{y})$ - this mark is dependent on scoring all M marks in (i) and (ii) Cao M1 for $\frac{1}{4\sqrt{k}}\int y^{\frac{3}{2}}\text{d}y$ and attempt to integrate

Question			Answer	Marks	Guidance
3		(i)	$\cos\theta = 40/75 \Rightarrow \theta = 57.769...$ Bearing is $\theta + 180° = 237.8°$ Shortest distance $d = 400\cos(180-50-\theta)$ $d = 122$ m (3sf)	M1 A1 A1 M1 A1 **[5]**	A1 maybe implied (or A1 for 32.23095…) M1 for $d = 400\cos(180-50-\text{cv}(\theta))$ 122.07235…
		(ii)	Time to closest approach $= \dfrac{400\sin(180-50-\theta)}{\sqrt{75^2-40^2}}$ $t = 6.004…$ Total distance = $75t + 40t$ 690 m (3 sf)	*M1 B1 B1 A1 M1 dep* A1 **[6]**	M1 for use of $t = \frac{s}{u}$, B1 for numerator (380.91775…), B1 for denominator (63.44288…) 6.0041050… 115(cv(t)) 690.47207… if M0 then **SC** B1 for 450.307… or 240.164…
4		(i)	$\frac{1}{4}m(2a)^2 \quad (= ma^2)$	B1 **[1]**	
		(ii)	Mass per unit volume is $\rho = \dfrac{M}{\pi(2r)^2 h}$ MI of elemental disc about a diameter is $(4\pi r^2\delta x\rho)r^2$ MI of elemental disc about an end face is $4\rho\pi r^4\delta x + (4\rho\pi r^2\delta x)x^2$ $I = 4\rho\pi r^2\int_0^h (r^2+x^2)\mathrm{d}x$ $= 4\rho\pi r^2\left[r^2x + \frac{1}{3}x^3\right]_0^h = 4\left(\frac{M}{4\pi r^2 h}\right)\pi r^4 h + \frac{4}{3}\left(\frac{M}{4\pi r^2 h}\right)\pi r^2 h^3$ $= M\left(r^2 + \frac{1}{3}h^2\right)$ MI through the centre of the cylinder $= M\left(r^2+\frac{1}{3}h^2\right) - M\left(\frac{h}{2}\right)^2 \quad \left(= M\left(r^2 + \frac{1}{12}h^2\right)\right)$	B1 B1 *M1 A1 A1 M1 dep* A1 B1 **[8]**	 $(Mr^2/h)\delta x$ (condone lack of δx) $\frac{M}{h}\left(r^2+x^2\right)\delta x$ - M1 for using parallel axes rule – must be of the form $\lambda r^4 + \mu r^2 x^2$ (condone lack of δx) $\frac{M}{h}\int_0^h r^2 + x^2\mathrm{d}x$ Integrating and obtaining an expression for I in terms M, r and h – must be using the correct limits **AG www** correctly obtained – if δx is omitted throughout then do not award final A mark

Question			Answer	Marks	Guidance
5		(i)	$HB = 2r\cos\theta$ and $HP = a - 2r\cos\theta$ $V = -\lambda mg(HP) - mg(HB)\cos\theta$ $V = -\lambda mg(a - 2r\cos\theta) - 2mgr\cos^2\theta$ $V = mg(2\lambda r\cos\theta - 2r\cos^2\theta - \lambda a)$	B1 B1 M1 A1 A1 **[5]**	 Attempt at V with their HP and HB One term correct **AG www** correctly obtained If M0 then **SC** B1 for one term correct
		(ii)	$\frac{dV}{d\theta} = -2\lambda mgr\sin\theta + 4mgr\cos\theta\sin\theta = 0$ $2mgr\sin\theta(2\cos\theta - \lambda) = 0$ $\frac{1}{2} \le \cos\theta < 1$ $\Rightarrow 1 \le \lambda < 2$	M1 A1 M1 A1 **[4]**	Attempt at differentiation Correct derivative and equal to zero Comparing their $\cos\theta < 1$ or $\cos\theta \ge \frac{1}{2}$ - condone $\cos\theta \le 1$
		(iii)	$\frac{d^2V}{d\theta^2} = -2\left(\frac{3}{2}\right)mgr\cos\theta + 4mgr\cos 2\theta$ $\sin\theta = 0 \Rightarrow V'' = mgr > 0 \quad \therefore$ stable $\cos\theta = \frac{3}{4} \Rightarrow V'' = -\frac{7}{4}mgr < 0 \quad \therefore$ unstable	*M1 A1 M1 dep* A1 M1 dep* A1 **[6]**	M1 Attempt to differentiate V' (or first derivative test) – allow in terms of λ M1 sub. their first angle into V'' (maybe implied by later working (eg correct sign and conclusion)) A1 correct (unsimplified) value of V'' and > 0 M1 sub. their second angle into V'' (maybe implied by later working (eg correct sign and conclusion)) A1 correct (unsimplified) value of V'' and < 0 If both values of V'' correct and correct conclusion then award B1 if no consideration of sign seen

Question			Answer	Marks	Guidance
6		(i)	$I_{\text{rod}} = \frac{1}{3}(2m)a^2 + (2m)a^2 \quad \left(= \frac{8}{3}ma^2\right)$	B1	
			$I_{\text{particle}} = m(2a)^2$	B1	
			$I = 4ma^2 + \frac{8}{3}ma^2 \quad \left(= \frac{20}{3}ma^2\right)$	B1 **[3]**	
		(ii)		M1	Equation involving KE (must involve *I*) and PE (two terms)
			$\frac{1}{2}\left(\frac{20}{3}ma^2\right)\omega^2 = 4mga(1-\cos\theta)$	A1 A1	A1 for KE term, A1 for PE term
			$\omega^2 = \frac{6g}{5a}(1-\cos\theta)$	A1 **[4]**	**AG** Correctly obtained. If M0 then **SC** B1 for either KE or PE term correct
		(iii)	$2\omega\alpha = \frac{6g}{5a}(\sin\theta)\omega$	M1	Differentiating ω with respect to *t* or for applying $C = I\alpha$
			$\alpha = \frac{3g}{5a}\sin\theta$	A1 **[2]**	
		(iv)	Centre of mass of pendulum is $\frac{4a}{3}$ from *A*	B1	
			$Y + 3mg\cos\theta = 3mr\omega^2$	*M1	For radial acceleration $r\omega^2$ - must sub for ω^2 - allow incorrect *m* and *r* for M mark only
			$Y = 4ma\left\{\frac{6g}{5a}(1-\cos\theta)\right\} - 3mg\cos\theta$	A1	$Y = \pm\frac{9}{10}mg$
			$3mg\sin\theta - X = 3mr\alpha$	*M1	For transverse acceleration $r\alpha$ - must sub their α - allow incorrect *m* and *r* for M mark only
			$X = 3mg\sin\theta - 4ma\left(\frac{3g}{5a}\sin\theta\right)$	A1	$X = \pm\frac{3\sqrt{3}}{10}mg \quad (\pm 0.519615\ldots mg)$
			$F = \sqrt{X^2 + Y^2} = mg\sqrt{\frac{81}{100} + \frac{27}{100}}$	M1 dep*	Substituting $\theta = \frac{\pi}{3}$ into *X* and *Y* and applying formula for *F* This substitution could be done initially – must be using correct *m* and *r*
			$F = \frac{3}{5}mg\sqrt{3}$	A1 **[7]**	$1.039230\ldots mg$

Question			Answer	Marks	Guidance
		(v)	$\omega^2 = \frac{6g}{5a}(1-\cos\theta) \Rightarrow \frac{d\theta}{dt} = 4(1-\cos\theta)^{\frac{1}{2}} \Rightarrow \ldots$	M1	Re-writing ω as $\frac{d\theta}{dt}$ (maybe implied) and attempt to set up integral by separating variables
			$4t = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{d\theta}{(1-\cos\theta)^{\frac{1}{2}}}$	A1	Condone lack of limits on integral (may still be in terms of a)
			Re-write $(1-\cos\theta)^{\frac{1}{2}}$ as $\sqrt{2}\sin\left(\frac{\theta}{2}\right)$	B1	Applying the trigonometric identity $\sin^2 X = \frac{1}{2}(1-\cos 2X)$
			Leading to $\frac{1}{4\sqrt{2}}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{cosec}\left(\frac{\theta}{2}\right) d\theta$	A1	$k = \frac{1}{4\sqrt{2}}$ (= 0.1767766…) – condone lack of limits on integral oe for k eg $\sqrt{\frac{5a}{12g}}$
			$= \frac{1}{4\sqrt{2}}\left[2\ln\left\|\tan\left(\frac{\theta}{4}\right)\right\|\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	B1	Using the result $\int \text{cosec}\left(\frac{x}{2}\right) dx = 2\ln\left\|\tan\frac{1}{4}x\right\|$ $(+c)$ or $-2\ln\left\|\text{cosec}\frac{1}{2}x + \cot\frac{1}{2}x\right\|$ $(+c)$
			= 0.251 (3sf)	A1 **[6]**	0.2512461…