OCR RECOGNISING ACHIEVEMENT

Friday 25 January 2013 – Afternoon

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

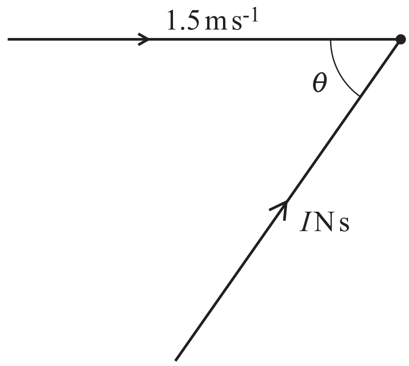
This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

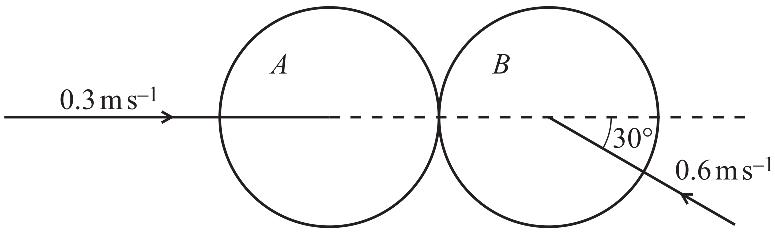
Turn over

1



A ball of mass 0.6 kg is moving with speed $1.5\,\mathrm{m\,s^{-1}}$ in a straight line. It is struck by an impulse I N s acting at an acute angle θ to its direction of motion (see diagram). The impulse causes the direction of motion of the ball to change by an acute angle α, where $\sin\alpha = \frac{8}{17}$. After the impulse acts the ball is moving with a speed of $3.4\,\mathrm{m\,s^{-1}}$. Find I and θ. **[5]**

2 Two uniform smooth spheres A and B, of equal radius and equal mass, are moving towards each other on a horizontal surface. Immediately before they collide, A has speed $0.3\,\mathrm{m\,s^{-1}}$ along the line of centres and B has speed $0.6\,\mathrm{m\,s^{-1}}$ at an angle of 30° to the line of centres (see diagram).



After the collision, the direction of motion of B is at right angles to its original direction of motion. Find

(i) the speed of B after the collision, **[3]**

(ii) the speed and direction of motion of A after the collision, **[3]**

(iii) the coefficient of restitution between A and B. **[3]**

3 At time $t = 0$ s a particle P, of mass 0.3 kg, is 1 m away from a point O on a smooth horizontal plane and is moving away from O with speed $\sqrt{5}\,\mathrm{m\,s^{-1}}$. The only horizontal force acting on P has magnitude $1.5x$ N, where x is the distance OP, and acts away from O.

(i) Show that the speed of P, $v\,\mathrm{m\,s^{-1}}$, is given by $v = \sqrt{5}x$. **[4]**

(ii) Find an expression for v in terms of t. **[4]**

4 A smooth cylinder of radius a m is fixed with its axis horizontal and O is the centre of a cross-section. Particle P, of mass 0.4 kg, and particle Q, of mass 0.6 kg, are connected by a light inextensible string of length πa m. The string is held at rest with P and Q at opposite ends of the horizontal diameter of the cross-section through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is $v\,\text{m s}^{-1}$ (see Fig. 2).

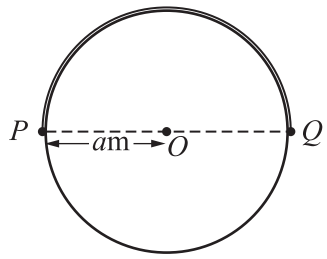


Fig. 1

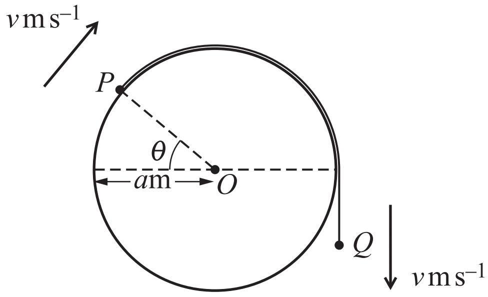


Fig. 2

(i) Show that $v^2 = 3.92a(3\theta - 2\sin\theta)$ and find an expression in terms of θ for the normal force of the cylinder on P at this time. **[9]**

(ii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $\sin\alpha = k\alpha$ where k is a constant to be found. **[2]**

5 A particle P, of mass 2.5 kg, is in equilibrium suspended from a fixed point A by a light elastic string of natural length 3 m and modulus of elasticity 36.75 N. Another particle Q, of mass 1 kg, is released from rest at A and falls freely until it reaches P and becomes attached to it.

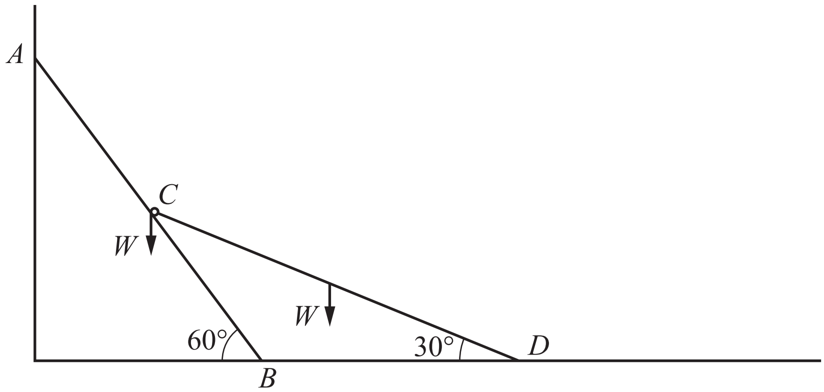
(i) Show that the speed of the combined particles, immediately after Q becomes attached to P, is $2\sqrt{2}\,\text{m s}^{-1}$. **[6]**

The combined particles fall a further distance X m before coming to instantaneous rest.

(ii) Find a quadratic equation satisfied by X, and show that it simplifies to $35X^2 - 56X - 80 = 0$. **[6]**

[Questions 6 and 7 are printed overleaf]

6 A uniform rod AB, of weight W and length $2l$ is in equilibrium at 60° to the horizontal with A resting against a smooth vertical plane and B resting on a rough section of a horizontal plane. Another uniform rod CD, of length $\sqrt{3}\,l$ and weight W, is freely jointed to the mid-point of AB at C; its other end D rests on a smooth section of the horizontal plane. CD is inclined at 30° to the horizontal (see diagram).



(i) Show that the force exerted by the horizontal plane on CD is $\frac{1}{2}W$. Find the normal component of the force exerted by the horizontal plane on AB. **[5]**

(ii) Find the magnitude and direction of the force exerted by CD on AB. **[3]**

(iii) Given that AB is in limiting equilibrium, find the coefficient of friction between AB and the horizontal plane. **[5]**

7 A simple pendulum consists of a light inextensible string of length 0.8 m and a particle P of mass m kg. The pendulum is hanging vertically at rest from a fixed point O when P is given a horizontal velocity of $0.3\,\text{m s}^{-1}$.

(i) Show that, in the subsequent motion, the maximum angle between the string and the downward vertical is 0.107 radians, correct to 3 significant figures. **[3]**

(ii) Show that the motion may be modelled as simple harmonic motion, and find the period of this motion. **[5]**

(iii) Find the time after the start of the motion when the velocity of the particle is first $-0.2\,\text{m s}^{-1}$ and find the angular displacement of OP from the downward vertical at this time. **[6]**

Answer			Marks	Guidance	
1			M1		Use of cos rule; condone + for − / missing 2/ missing '0.6'; angle as 'θ' for M1
		$I^2 = 2.04^2 + 0.9^2 - 2\text{x}2.04\text{x}0.9\text{x}\frac{15}{17}$	A1	And attempt to square root	Condone + for −
		1.32 (N)	A1	CAO	(1.3159)
			M1	Correct use of sin rule from their diagram oe	Can be in terms of I α and θ
		46.8(°) with initial direction of ball	A1	CAO	(46.8476) (0.8176 rads) Accept 46.7 from using I = 1.32
			[5]	OR $0.9 + I\cos\theta = 0.6\text{x}3.4\text{x}15/17$ M1 $I\sin\theta = 0.6\text{x}3.4\text{x}8/17$ M1 square and add to find I^2; or divide to find θ M1 I, θ A1 A1 CAO	Allow missing 0.6 and/or sign or trig error for these 2 marks, then M0A0A0
2	**(i)**	Vel unchanged perp to L o C $0.6\sin 30° = v\cos 30°$ $0.2\sqrt{3}$ (ms^{-1})	M1 M1 A1 **[3]**		Stated or used Allow 1 sign or trig error (0.34641)
2	**(ii)**	Use momentum equation $0.3m - 0.6m\cos 30° = am + 0.2\sqrt{3}m\cos 60°$ (a =) 0.393 to left	M1 A1ft A1 **[3]**	 Follow through on v Direction must be clearly stated or implied from working. WWW	Allow their v; allow sign errors / omission of m m's not necessary; (0.39282) Away from B/opp direction to before
2	**(iii)**	Use of NLR $(0.2\sqrt{3})\cos 60° - (-0.393) = e(0.6\cos 30° + 0.3)$ 0.691	M1 A1ft A1 **[3]**	 Ft on a and v CAO	Allow sign error and/or trig error (0.69082 or 0.6905679)

Answer			Marks	Guidance	
3	(i)	Use of $F = ma$, using $v\frac{dv}{dx}$	M1*		Allow sign error / 0.3 omitted
		$0.3v\frac{dv}{dx} = 1.5x$	A1		
		Attempt to rearrange and integrate	*M1	$0.3v^2 = 1.5x^2 (+c)$	No need for c. At least one side integrated correctly
		$v = \sqrt{5}x$ **AG**	A1 **[4]**	correct derivation WWW	
3	(ii)	Integrate to find x in terms of t	M1	$dx/x = \sqrt{5}dt$ and int 1 side correctly	Need to separate variables
		$\ln x = \sqrt{5}t + c$	A1		No need for c for first 2 marks
		$x = e^{\sqrt{5}t}$	A1		Must include showing c = 0.
		$v = \sqrt{5}\, e^{\sqrt{5}t}$	A1 **[4]**	CAO	
		OR Integrate to find v in terms of t	M1	Use jn $0.3\frac{dv}{dt} = 1.5x$ and int 1 side correctly	No need for c for first 2 marks
		$\frac{dv}{v} = \sqrt{5dt}$			
		$\ln v = \sqrt{5}t + c$	A1		
		$\ln v = \sqrt{5}t + \ln(\sqrt{5})$	A1		Must include showing c = ln(√5)
		$v = \sqrt{5}\, e^{\sqrt{5}t}$	A1	CAO	

		Answer	Marks	Guidance	
4	(i)	Conservation of energy	M1 M1		Need 4 terms; allow sign & trig errors Both KE or both PE correct
		$\frac{1}{2}0.4v^2 + \frac{1}{2}0.6v^2 + 0.4ga\sin\theta - 0.6ga\theta = 0$	A1		completely correct
		$v^2 = 3.92a(3\theta - 2\sin\theta)$	M1 A1	Attempt to find v^2 dep both earlier M1s **AG**	Allow with sign and trig errors No errors
		F = ma radially for P	M1*		Allow sign and trig errors
		$0.4g\sin\theta - R = \frac{0.4v^2}{a}$	A1		
		$R = -4.704\theta + 7.056\sin\theta$	*M1 A1 **[9]**	Manipulation attempted, leading to $a\theta + b\sin\theta$	Allow sign and trig errors $2.352(-2\theta + 3\sin\theta)$
4	(ii)	Using $R = 0$ (k =) $\frac{2}{3}$	M1 A1 **[2]**	$0 = -4.704\theta + 7.056\sin\theta$	Must be from correct expression in (i)
5	(i)	2.5g = 36.75 e/3 $e = 2$ $v^2 = 0^2 + 2g(3 + e)$ $v = 7\sqrt{2}$ 1 x v = 3.5 V Combined speed = $2\sqrt{2}$ (ms-1)	M1 A1 M1 A1 M1 A1 **[6]**	P in equilibrium **AG**	Allow missing g May be implied by $v^2 = 98$ Convincing derivation, no errors

Answer			Marks	Guidance	
5	(ii)	change in PE is $3.5gX$ change in KE is $0.5\text{x}3.5\,(2\sqrt{2})^2$ change in EE is $36.75(X+2)^2/(2\times3) - 36.75\times2^2/(2\times3)$ Use conservation of energy $35X^2 - 56X - 80 = 0$	B1 B1 M1 A1 M1 A1 **[6]**	$34.3X$ 14 $\frac{36.75(X+2)^2}{2\times3} = \frac{36.75\times2^2}{2\times3} + 3.5gX + \frac{3.5}{2}V^2$ **AG**	 Allow sign errors / omission of 2; Allow 'x' or '$x + 5$' for '$x + 2$'; 2 terms or difference Allow sign errors; at least PE, KE, EE term Convincing derivation, no errors may see $36.75X^2 - 58.8X - 84 = 0$
6	(i)	Moments about C for CD $Wl\sqrt{3}/2(\cos30°) = Ql\sqrt{3}(\cos30°)$ $(Q =)\ W/2$ Resolve vert $(R =)\ \frac{3}{2}W$	M1 A1 A1 M1 A1 **[5]**	 **AG** CAO	allow M if sin/cos wrong
6	(ii)	$X = 0$ Resolve vert for CD or AB $Y = W/2$ Vertically downwards	B1 B1* *B1 **[3]**	 $Y + Q = W$ or $Y + W = R$	

Answer			Marks	Guidance	
6	(iii)	Moments about C for AB	M1		Allow M if sin/cos wrong or sign errors; need all terms
		$Pl\cos30° + Fl\cos30° = Rl\sin30°$	A1	Correct	
		Use P in terms of F	M1	$F = P$ or other correct 2nd step	Allow if missing term above
		Find F in terms of W, or in terms of R	M1	$F = \frac{\sqrt{3}}{4}W$	Or getting 'their' F oe, ie putting $F = \mu R$ in moment equation.
		$\mu = (F/R) = \sqrt{3}/6$	A1 **[5]**	Accept decimal answers from 0.288675	
		OR Moments about A for AB	M1		Allow M if sin/cos wrong or sign errors; need all terms
		$Wl\sin30° + (Y)l\sin30° + F2l\cos30° = R2l\sin30°$	A1		May have X term if not 0 in (ii)
		Write Y (and X) in terms of W	M1		
		Find F in terms of W, or in terms of R, oe	M1	$F = \frac{\sqrt{3}}{4}W$	
		$\mu = (F/R) = \sqrt{3}/6$	A1	Accept decimal answers from 0.288675	
7	(i)	Use of energy equation	M1		Allow M1 if sign error and/or 9.8 missing and/or missing m or l
		$0.5\text{ m }(0.3)^2 = m\text{x}9.8\text{x}0.8\text{x}(1 - \cos\theta)$	A1		
		$\theta = 0.107$	A1 **[3]**	No errors **AG**	0.107194171
7	(ii)	Use $F = ma$	M1	$m \times 9.8 \sin\theta = - m \times 0.8\ \ddot{\theta}$	allow M1 if sign error, or 9.8 missing
		$\ddot{\theta} = - 12.25\ \theta$	A1		Allow fraction
		small θ	B1	Dep on having seen acc = $k\sin\theta$	Rigorous
		Use of $T = \frac{2\pi}{\omega}$	M1	or sight of $\omega = 3.5$	
		$T = 1.80$	A1 **[5]**		accept $\frac{4\pi}{7}$ (1.795195)

Answer			Marks	Guidance	
7	(iii)	identifying amplitude as 0.107 Use of ($\dot{\theta}$) = 0.107x3.5xcos(3.5t) Use of $\dot{\theta}$ = −0.25 t = 0.658 Use of θ = 0.107 sin(3.5t) (θ =) 0.0797rads	B1 M1 A1 A1 M1 A1 **[6]**	 or sin(3.5t+ε), ε not 0 Consistent angle or length ft from velocity equation (matches, ignore sign) accept 5.20°	ft from (i) ft for a and ω; allow sign error (0.6576339) (0.0796678 or 0.079576)