OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4730

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 24 January 2011
Morning

Duration: 1 hour 30 minutes

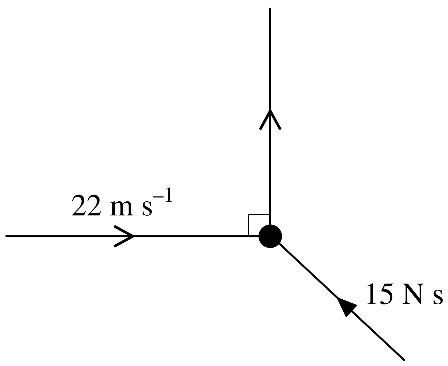
INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



A ball of mass 0.5 kg is moving with speed 22 m s^{-1} in a straight line when it is struck by a bat. The impulse exerted by the bat has magnitude 15 N s and the ball is deflected through an angle of 90° (see diagram). Find

(i) the direction of the impulse, **[3]**

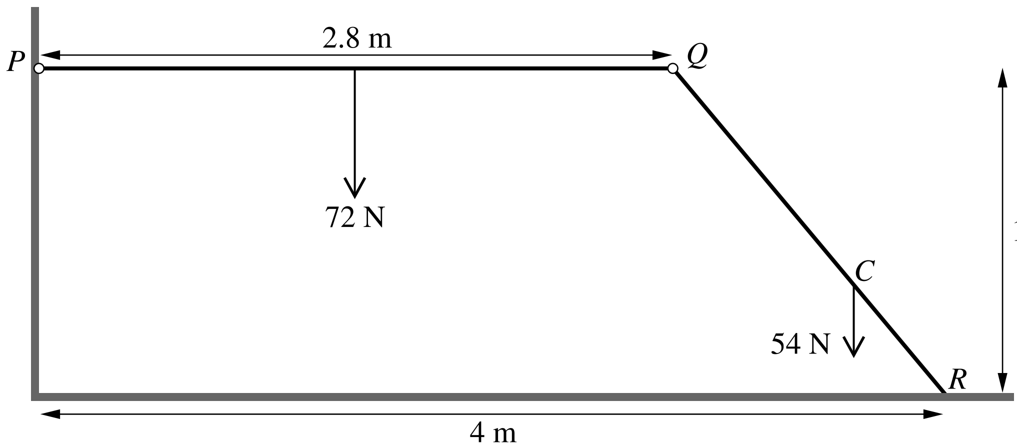
(ii) the speed of the ball immediately after it is struck. **[3]**

2 A particle of mass 0.4 kg is attached to a fixed point O by a light inextensible string of length 0.5 m. The particle is projected horizontally with speed 6 m s^{-1} from the point 0.5 m vertically below O. The particle moves in a complete circle. Find the tension in the string when

(i) the string is horizontal,

(ii) the particle is vertically above O. **[6]**

3



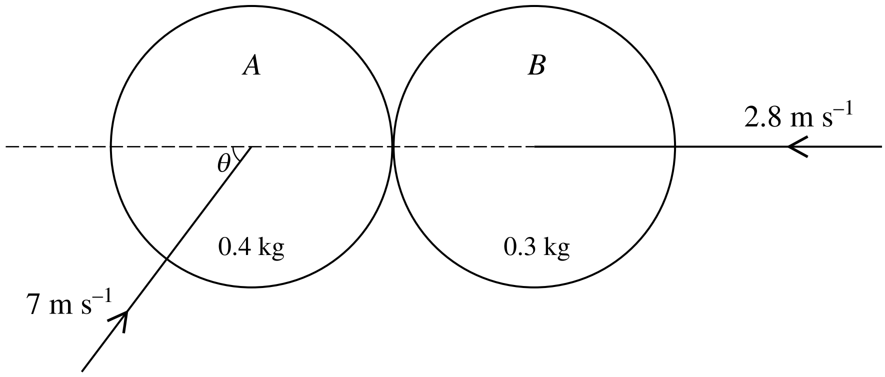
A uniform rod PQ has weight 72 N. A non-uniform rod QR has weight 54 N and its centre of mass is at C, where $QC = 2CR$. The rods are freely jointed to each other at Q. The rod PQ is freely jointed to a fixed point of a vertical wall at P and the rod QR rests on horizontal ground at R. The rod PQ is 2.8 m long and is horizontal. The point R is 1.44 m below the level of PQ and 4 m from the wall (see diagram).

(i) Find the vertical component of the force exerted by the wall on PQ. **[2]**

(ii) Hence show that the normal component of the force exerted by the ground on QR is 90 N. **[2]**

(iii) Given that the friction at R is limiting, find the coefficient of friction between the rod QR and the ground. **[5]**

4



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.4 kg and B has mass 0.3 kg. Immediately before the collision A is moving with speed $7\,\text{m}\,\text{s}^{-1}$ at an acute angle θ to the line of centres, where $\cos\theta = 0.6$, and B is moving with speed $2.8\,\text{m}\,\text{s}^{-1}$ along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.7. Find

(i) the speed of B immediately after the collision, **[6]**

(ii) the angle turned through by the direction of motion of A as a result of the collision. **[5]**

5 A particle P of mass 0.05 kg is suspended from a fixed point O by a light elastic string of natural length 0.5 m and modulus of elasticity 2.45 N.

(i) Show that the equilibrium position of P is 0.6 m below O. **[3]**

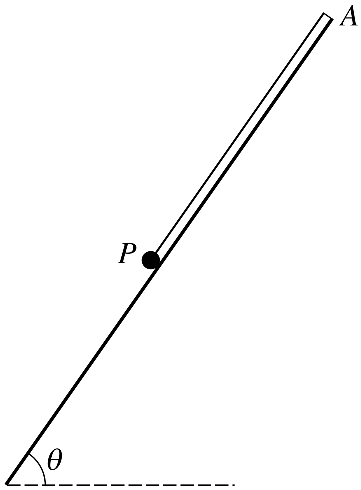
P is held at rest at a point 0.675 m vertically below O and then released. At time t s after P is released, its downward displacement from the equilibrium position is x m.

(ii) Show that $\dfrac{d^2x}{dt^2} = -98x$. **[3]**

(iii) Find the value of x and the magnitude and direction of the velocity of P when $t = 0.2$. **[7]**

[Questions 6 and 7 are printed overleaf.]

6



A particle P, of mass 3.5 kg, is in equilibrium suspended from the top A of a smooth slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{40}{49}$, by an elastic rope of natural length 4 m and modulus of elasticity 112 N (see diagram). Another particle Q, of mass 0.5 kg, is released from rest at A and slides freely downwards until it reaches P and becomes attached to it.

(i) Find the value of V^2, where $V\,\text{m s}^{-1}$ is the speed of Q immediately before it becomes attached to P, and show that the speed of the combined particles, immediately after Q becomes attached to P, is $\frac{1}{2}\sqrt{5}\,\text{m s}^{-1}$. **[6]**

The combined particles slide downwards for a distance of X m, before coming instantaneously to rest at B.

(ii) Show that $28X^2 - 8X - 5 = 0$. **[6]**

7 A particle P of mass 0.2 kg is released from rest at a point O and falls vertically. Air resistance of magnitude $\frac{v^2}{2000}$ N acts upwards on P, where $v\,\text{m s}^{-1}$ is the velocity of P when it has fallen a distance of x m.

(i) Show that $\left(\frac{400v}{3920 - v^2}\right)\frac{dv}{dx} = 1$. **[2]**

(ii) Find v^2 in terms of x and hence show that $v^2 < 3920$ for all values of x. **[7]**

(iii) Find the work done against the air resistance while P is falling, from O, to the point where its downward acceleration is $5.8\,\text{m s}^{-2}$. **[6]**

1 i	(-)15cosα = (0 –) 0.5x22 or 15sinβ = 0.5x22 Impulse makes angle 42.8^o (0.748 rads) with negative x-axis	M1 A1 A1 [3]	M1 for using I = Δ(mv) in 'x' direction or for sketching Δ reflecting **I** = m(**v** – **u**) AEF, but angle must be clear
ii	15sinα = 0.5v or 15cosβ = 0.5v or $(0.5v)^2 = 15^2 - 11^2$ Correct explicit expression for v Speed is 20.4 ms^{-1}	M1 A1 A1 [3]	For using I = Δ(mv) in 'y' direction or using sketched Δ

2	½ (m)(v^2 - 6^2) = -(m)g x 0.5 in (i) or ½ (m)(v^2 – 6^2) = -(m)g x 1 in (ii) v^2 = 26.2 in (i) and 16.4 in (ii) T = 0.4v^2/0.5 in (i) or T + 0.4g = 0.4v^2/0.5 Tension is 21.0N in (i) (20.96) 9.2N in (ii)	M1 A1 M1 A1 A1 A1 [6]	For using the principle of conservation of energy in (i) or (ii) soi For using Newton's second law with a = v^2/L. M1 for either attempt, A1 for both right

3 i	2.8V = 1.4x72 Vertical component at P is 36 N	M1 A1 [2]	For taking moments about Q for PQ or for using symmetry
ii	36 + N = 72 + 54 Normal component at R is 90 N	M1 A1 [2]	For resolving forces vertically on both rods AG
iii	1.44F = 1.2x90 – 0.8x54 or 72x1.4 + 54x3.6 + 1.44F = 90x4 with not more than 1 error in either case Equation correct and leading to F = 45 For using F = μR Coefficient is 0.5	 M1 A1 A1 M1 A1 [5]	For taking moments about Q for QR or about P for the whole structure (all terms needed)

4 i	0.4(7x0.6) – 0.3x2.8 = 0.4a + 0.3b 0.7(7x0.6 + 2.8) = b – a Speed of *B* is 4ms^{-1}	M1 A1 M1 A1 M1 A1 [6]	For using the principle of conservation of momentum For using e(Δ u) = Δ v For eliminating a from equations
ii	a = (-)0.9 Component perp. to l.o.c. is 5.6 tan α = 5.6/0.9 α = 80.9° Angle turned through is 46.0° (0.803^c)	B1 B1 M1 A1 A1ft [5]	For attempting to find α - the angle between the direction of motion of A after collision and the l.o.c. to the left, or 90° − α 126.9° − α

5 i	2.45*e*/0.5 = 0.05*g* (*e* = 0.1) Distance from O is 0.5 + 0.1 = 0.6m	M1 A1 A1 [3]	For using $T = \lambda e/L$ and resolving forces vertically accept use of 0.1 to show both sides equal to 0.49 AG
ii	$mg - T = m\ddot{x}$ $0.05g - 2.45(0.1 + x)/0.5 = 0.05\ddot{x}$ $\ddot{x} = -98x$	M1 A1 A1 [3]	For using Newton's second law with 3 terms AG
iii	$a = 0.075$ $n = 7\sqrt{2}$ oe $x = 0.075\cos(7\sqrt{2}\,t)$ $x(0.2) = -0.0298$ $v = -0.075(7\sqrt{2})\sin(7\sqrt{2}\,t)$ $v(0.2) = -0.681$ ➔ velocity is 0.681ms^{-1} upwards	B1 B1 M1 A1 M1 A1ft A1 [7]	 accept 9.90 For using $x = a\cos nt$ oe For differentiating x = *a*cos*nt* and using it ft incorrect *a* and/or *n* If from $v^2 = n^2(a^2 - x^2)$ the direction must be clearly established

6 i	$112e/4 = 3.5 \times 9.8 \times \frac{40}{49}$ $V^2 = 2 \times 8 \times (4 + 1)$ $V^2 = 80$ $0.5\sqrt{80} = (0.5 + 3.5)u$ Initial speed of combined particles is $\frac{1}{2}\sqrt{5}$ ms^{-1}	M1 A1 M1 A1 M1 A1 [6]	For using $mg\sin\theta$ and $\lambda e/L$ For using $s = 4 + e$ and a = 8 in $v^2 = 2as$, or by energy For using the principle of conservation of momentum AG
ii	Gain in EE = $(112/(2 \times 4))\{(X + 1)^2 - 1^2\}$ Loss of KE = ½ (0.5 + 3.5) x 5/4 Loss of PE = $(0.5 + 3.5) \times 9.8 \times \frac{40}{49}X$ $14(X^2 + 2X) = 2.5 + 32X$ $28X^2 - 8X - 5 = 0$	M1 A1 B1 B1 M1 A1 [6]	For using EE = $\lambda x^2/2L$ For using the principle of conservation of energy AG
OR	$T - mg\sin\theta = -ma$ $\frac{112(x+1)}{4} - 4g\frac{40}{49} = -4a$ $\int (7x-1)dx = -\int v dv \ (+c)$ $\frac{7x^2}{2} - x = -\frac{v^2}{2} + c$ $c = \frac{5}{8}$ $28X^2 - 8X - 5 = 0$	M1 A1 M1 A1 A1 A1 [6]	For use of $F = ma$ allow one sign slip for A1 Using a = $v\frac{dv}{dx}$ and integrating AG Convincingly

7 i	$0.2g - v^2/2000 = 0.2v(dv/dx)$ $\left(\frac{400v}{3920 - v^2}\right)\frac{dv}{dx} = 1.$	M1 A1 [2]	For using Newton's second law with $a = v(dv/dx)$ AG Convincing, with no slips.
ii	$-200\ln(3920 - v^2) = x + (A)$ $-200\ln(3920) = A$ $x = 200\ln\left(\frac{3920}{3920 - v^2}\right)$ $e^{x/200} = 3920/(3920 - v^2)$ $v^2 = 3920(1 - e^{-x/200})$ $0 < e^{-x/200} \Rightarrow v^2 < 3920$	M1 A1 M1 A1 M1 A1 B1 [7]	For separating variables and integrating For using v(0) = 0 For using inverse ln process AG Convincingly – dep on correct answer
iii	Using $0.2g - v^2/2000 = 0.2a$ $v = 40$ Gain in KE = ½ 0.2x1600 (=160J) $x = 200\ln\left(\frac{3920}{3920 - 1600}\right)$ (= 104.90) 0.2g x (104.9) – 160 Work done is 45.6 J	M1 A1 B1ft B1ft M1 A1 [6]	 For using WD = loss of PE – gain in KE
OR	Using $0.2g - v^2/2000 = 0.2a$ $v = 40$ $x = 200\ln\left(\frac{3920}{3920 - 1600}\right)$ (= 104.90...) WD = $\int \frac{v^2}{2000}dx + c$ = $\int \frac{3920}{2000}(1 - e^{-x/200})\,dx$ = $3920/2000(x + 200e^{(-x/200)} - 392$ Work done is 45.6 J	M1 A1 B1ft M1 A1 A1 [6]	 Use of WD = $\int F dx$ and subst for v^2