RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

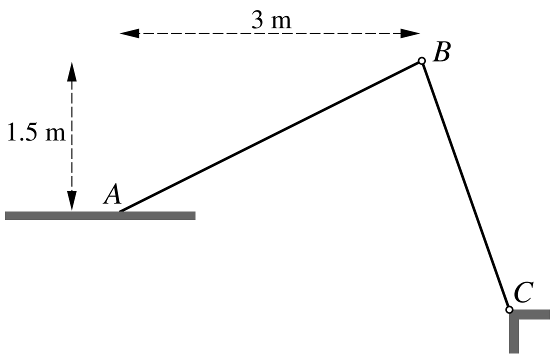
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A smooth sphere of mass 0.3 kg bounces on a fixed horizontal surface. Immediately before the sphere bounces the components of its velocity horizontally and vertically downwards are $4\,\text{m}\,\text{s}^{-1}$ and $6\,\text{m}\,\text{s}^{-1}$ respectively. The speed of the sphere immediately after it bounces is $5\,\text{m}\,\text{s}^{-1}$.

(i) Show that the vertical component of the velocity of the sphere immediately after impact is $3\,\text{m}\,\text{s}^{-1}$, and hence find the coefficient of restitution between the surface and the sphere. **[3]**

(ii) State the direction of the impulse on the sphere and find its magnitude. **[3]**

2

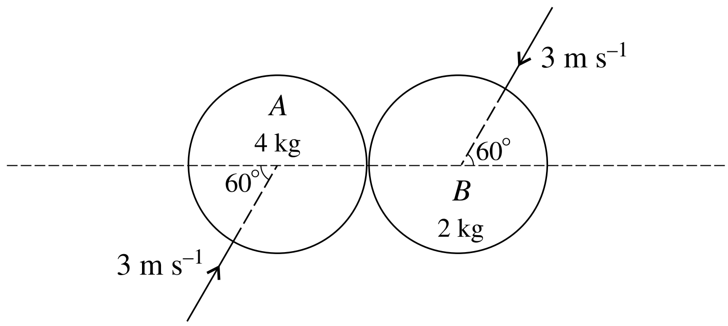


Two uniform rods, *AB* and *BC*, are freely jointed to each other at *B*, and *C* is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with *A* resting on a rough horizontal surface. This surface is 1.5 m below the level of *B* and the horizontal distance between *A* and *B* is 3 m (see diagram). The weight of *AB* is 80 N and the frictional force acting on *AB* at *A* is 14 N.

(i) Write down the horizontal component of the force acting on *AB* at *B* and show that the vertical component of this force is 33 N upwards. **[4]**

(ii) Given that the force acting on *BC* at *C* has magnitude 50 N, find the weight of *BC*. **[4]**

3



Two uniform smooth spheres *A* and *B*, of equal radius, have masses 4 kg and 2 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision both spheres have speed $3\,\text{m}\,\text{s}^{-1}$. The spheres are moving in opposite directions, each at 60° to the line of centres (see diagram). After the collision *A* moves in a direction perpendicular to the line of centres.

(i) Show that the speed of *B* is unchanged as a result of the collision, and find the angle that the new direction of motion of *B* makes with the line of centres. **[8]**

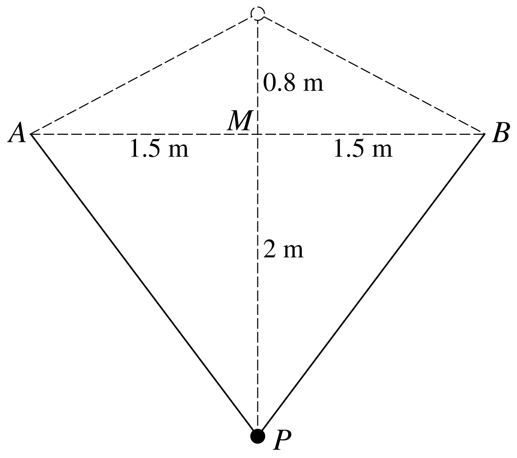
(ii) Find the coefficient of restitution between the spheres. **[2]**

4 A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point A with speed $40\,\text{m s}^{-1}$ and when it has travelled a distance of x m beyond A its speed is $v\,\text{m s}^{-1}$. The engine develops a constant power of 8 kW and resistances are modelled by a force of $0.25v^2$ N opposing the motion.

(i) Show that $\dfrac{480v^2}{v^3 - 32\,000}\dfrac{dv}{dx} = -1.$ **[5]**

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond A. **[6]**

5



Each of two identical strings has natural length 1.5 m and modulus of elasticity 18 N. One end of one of the strings is attached to A and one end of the other string is attached to B, where A and B are fixed points which are 3 m apart and at the same horizontal level. M is the mid-point of AB. A particle P of mass m kg is attached to the other end of each of the strings. P is held at rest at the point 0.8 m vertically above M, and then released. The lowest point reached by P in the subsequent motion is 2 m below M (see diagram).

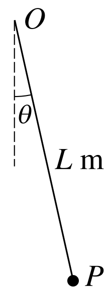
(i) Find the maximum tension in each of the strings during P's motion. **[3]**

(ii) By considering energy,

(a) show that the value of m is 0.42, correct to 2 significant figures, **[5]**

(b) find the speed of P at M. **[3]**

6



A particle P of mass m kg is attached to one end of a light inextensible string of length L m. The other end of the string is attached to a fixed point O. The particle is held at rest with the string taut and then released. P starts to move and in the subsequent motion the angular displacement of OP, at time t s, is θ radians from the downward vertical (see diagram). The initial value of θ is 0.05.

(i) Show that $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$. **[2]**

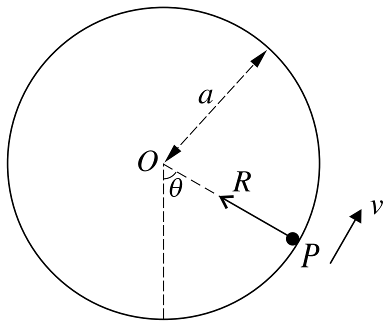
(ii) Hence show that the motion of P is approximately simple harmonic. **[2]**

(iii) Given that the period of the approximate simple harmonic motion is $\frac{4}{7}\pi$ s, find the value of L. **[2]**

(iv) Find the value of θ when $t = 0.7$ s, and the value of t when θ next takes this value. **[4]**

(v) Find the speed of P when $t = 0.7$ s. **[3]**

7



A hollow cylinder has internal radius a. The cylinder is fixed with its axis horizontal. A particle P of mass m is at rest in contact with the smooth inner surface of the cylinder. P is given a horizontal velocity u, in a vertical plane perpendicular to the axis of the cylinder, and begins to move in a vertical circle. While P remains in contact with the surface, OP makes an angle θ with the downward vertical, where O is the centre of the circle. The speed of P is v and the magnitude of the force exerted on P by the surface is R (see diagram).

(i) Find v^2 in terms of u, a, g and θ and show that $R = \frac{mu^2}{a} + mg(3\cos\theta - 2)$. **[7]**

(ii) Given that P just reaches the highest point of the circle, find u^2 in terms of a and g, and show that in this case the least value of v^2 is ag. **[4]**

(iii) Given instead that P oscillates between $\theta = \pm\frac{1}{6}\pi$ radians, find u^2 in terms of a and g. **[2]**

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1 i	Horiz. comp. of vel. after impact is $4ms^{-1}$ Vert. comp. of vel. after impact is $\sqrt{5^2 - 4^2} = 3ms^{-1}$ Coefficient of restitution is 0.5	B1 B1 B1 [3]	May be implied AG From e = 3/6
ii	Direction is vertically upwards Change of velocity is 3 – (-6) Impulse has magnitude 2.7Ns	B1 M1 A1 [3]	 From $m(\Delta v) = 0.3\times9$
2 i	Horizontal component is 14N $80\times1.5 = 14\times1.5 + 3Y$ or $3(80 - Y) = 80\times1.5 + 14\times1.5$ or $1.5(80 - Y) = 14\times0.75 + 14\times0.75 + 1.5Y$ Vertical component is 33N upwards	B1 M1 A1 A1 [4]	 For taking moments for *AB* about *A* or *B* or the midpoint of *AB* AG
ii	Horizontal component at *C* is 14N [Vertical component at *C* is $(\pm)\sqrt{50^2 - 14^2}$] [$W = (\pm)48 - 33$] Weight is 15N	B1 M1 DM1 A1 [4]	May be implied for using $R^2 = H^2 + V^2$ For resolving forces at *C* vertically
3 i	 $4\times3\cos60^o - 2\times3\cos60^o = 2b$ $b = 1.5$ **j** component of vel. of $B = (-)3\sin60^o$ $[v^2 = b^2 + (-3\sin60^o)^2]$ Speed ($3ms^{-1}$) is unchanged [Angle with l.o.c. = $\tan^{-1}(3\sin60^o/1.5)$] Angle is 60^o.	M1 A1 A1 B1ft M1 A1ft M1 A1ft [8]	For using the p.c.mmtm parallel to l.o.c. ft consistent sin/cos mix For using $v^2 = b^2 + v_y^2$ AG ft - allow same answer following consistent sin/cos mix. For using angle = $\tan^{-1}(\pm v_y/v_x)$ ft consistent sin/cos mix
ii	[$e(3\cos60^o + 3\cos60^o) = 1.5$] Coefficient is 0.5	M1 A1ft [2]	For using NEL ft - allow same answer following consistent sin/cos mix throughout.

4 i	$F - 0.25v^2 = 120v(dv/dx)$ $F = 8000/v$ $[32000 - v^3 = 480v^2(dv/dx)]$ $\frac{480v^2}{v^3 - 32000}\frac{dv}{dx} = -1$	M1 A1 B1 M1 A1 [5]	For using Newton's second law with $a = v(dv/dx)$ For substituting for F and multiplying throughout by $4v$ (or equivalent) AG
ii	$\int \frac{480v^2}{v^3 - 32000}dv = -\int dx$ $160\ln(v^3 - 32000) = -x \quad (+A)$ $160\ln(v^3 - 32000) = -x + 160\ln 32000$ or $160\ln(v^3 - 32000) - 160\ln 32000 = -500$ $(v^3 - 32000)/32000 = e^{-x/160}$ Speed of m/c is 32.2ms^{-1}	M1 A1 M1 A1ft B1ft B1 [6]	For separating variables and integrating For using $v(0) = 40$ or $[160\ln(v^3 - 32000)]_{40}^{v} = [-x]_0^{500}$ ft where factor 160 is incorrect but +ve, Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of A
5 i	$x_{max} = \sqrt{1.5^2 + 2^2} - 1.5$ (= 1) $[T_{max} = 18\times1/1.5]$ Maximum tension is 12N	B1 M1 A1 [3]	 For using $T = \lambda x/L$
      ii	**(a)** Gain in EE = $2[18(1^2 - 0.2^2)]/(2\times1.5)$ (11.52) Loss in GPE = 2.8mg (27.44m) $[2.8m \times 9.8 = 11.52]$ $m = 0.42$ **(b)** $\frac{1}{2}mv^2 = mg(0.8) + 2\times18\times0.2^2/(2\times1.5)$ or $\frac{1}{2}mv^2 = 2\times18\times1^2/(2\times1.5) - mg(2)$ Speed at M is 4.24ms^{-1}	M1 A1 B1 M1 A1 [5] M1 A1ft A1ft [3]	For using EE = $\lambda x^2/2L$ May be scored with correct EE terms in expressions for total energy on release and total energy at lowest point May be scored with correct GPE terms in expressions for total energy on release and total energy at lowest point For using the p.c.energy AG For using the p.c.energy KE, PE & EE must all be represented ft only when just one string is considered throughout in evaluating EE ft only for answer 4.10 following consideration of only one string

6 **i**	$[-mg\sin\theta = mL(d^2\theta/dt^2)]$ $d^2\theta/dt^2 = -(g/L)\sin\theta$	M1 A1 [2]	For using Newton's second law tangentially with $a = Ld^2\theta/dt^2$ AG
ii	$[d^2\theta/dt^2 = -(g/L)\ \theta]$ $d^2\theta/dt^2 = -(g/L)\ \theta$ ➔ motion is SH	M1 A1 [2]	For using $\sin\theta \approx \theta$ because θ is small ($\theta_{max} = 0.05$) AG
iii	$[4\pi/7 = 2\pi/\sqrt{9.8/L}\ ]$ $L = 0.8$	M1 A1 [2]	For using $T = 2\pi/n$ where $-n^2$ is coefficient of θ
iv	$[\theta = 0.05\cos 3.5\times 0.7]$ $\theta = -0.0385$ $t = 1.10$ (accept 1.1 or 1.09)	M1 A1ft M1 A1ft [4]	For using $\theta = \theta_o\cos nt$ {$\theta = \theta_o\sin nt$ not accepted unless the t is reconciled with the t as defined in the question} ft incorrect L {$\theta = 0.05\cos[4.9/(5L)^{½}]$} For attempting to find 3.5t ($\pi < 3.5t < 1.5\pi$) for which $0.05\cos 3.5t$ = answer found for θ or for using $3.5(t_1 + t_2) = 2\pi$ ft incorrect L {$t = [2\pi(5L)^{½}]/7 - 0.7$}
v	$\dot{\theta}^2 = 3.5^2(0.05^2 - (-0.0385)^2)$ or $\dot{\theta} = -3.5\times 0.05\sin(3.5\times 0.7)$ ($\dot{\theta} = -0.1116..$) Speed is 0.0893ms^{-1} (Accept answers correct to 2 s.f.)	M1 A1ft A1ft [3]	For using $\dot{\theta}^2 = n^2(\theta_o^2 - \theta^2)$ or $\dot{\theta} = -n\ \theta_o\sin nt$ {also allow $\dot{\theta} = n\theta_o\cos nt$ if $\theta = \theta_o\sin nt$ has been used previously} ft incorrect θ with or without 3.5 represented by $(g/L)^{½}$ using incorrect L in (iii) or for $\dot{\theta} = 3.5\times 0.05\cos(3.5\times 0.7)$ following previous use of $\theta = \theta_o\sin nt$ ft incorrect L ($L\times 0.089287/0.8$ with $n = 3.5$ used or from $\lvert 0.35\sin\{4.9/[5L]^{½}\}/[5L]^{½}\rvert$ **SR** for candidates who use $\dot{\theta}$ as v. (Max 1/3) For v = $\pm$0.112 B1

7 i	Gain in PE = $mga(1 - \cos\theta)$ [$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(1 - \cos\theta)$]	B1 M1	 For using KE loss = PE gain
	$v^2 = u^2 - 2ga(1 - \cos\theta)$ [$R - mg\cos\theta = m$(accel.)] $R = mv^2/a + mg\cos\theta$ [$R = m\{u^2 - 2ga(1 - \cos\theta)\}/a + mg\cos\theta$] $R = mu^2/a + mg(3\cos\theta - 2)$	A1 M1 A1 M1 A1 [7]	 For using Newton's second law radially For substituting for v^2 AG
ii	[$0 = mu^2/a - 5mg$] $u^2 = 5ag$ [$v^2 = 5ag - 4ag$] Least value of v^2 is ag	M1 A1 M1 A1 [4]	For substituting $R = 0$ and $\theta = 180°$ For substituting for u^2 (= $5ag$) and θ = 180° in v^2 (expression found in (i)) { but M0 if $v = 0$ has been used to find u^2} AG
iii	[$0 = u^2 - 2ga(1 - \sqrt{3}/2)$] $u^2 = ag(2 - \sqrt{3})$	M1 A1 [2]	For substituting $v^2 = 0$ and $\theta = \pi/6$ in v^2 (expression found in (i)) Accept $u^2 = 2ag(1 - \cos\pi/6)$