RECOGNISING ACHIEVEMENT

ADVANCED GCE

4730/01

MATHEMATICS

Mechanics 3

MONDAY 2 JUNE 2008

Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.8 m and modulus of elasticity $1.35mg$ N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P is held at rest at a point on the surface 3 m from O. The particle is then released. Find

(i) the initial acceleration of P, [3]

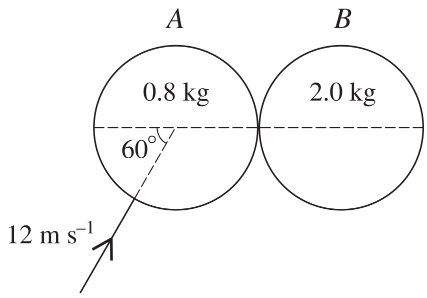
(ii) the speed of P at the instant the string becomes slack. [3]

2 A particle P of mass 0.2 kg is moving with speed 8 m s^{-1} when it hits a horizontal smooth surface. The direction of motion of P immediately before impact makes an angle of 27° with the surface. Given that the coefficient of restitution between the particle and the surface is 0.6, find

(i) the vertical component of the velocity of P immediately after impact, [3]

(ii) the magnitude of the impulse exerted on P. [3]

3



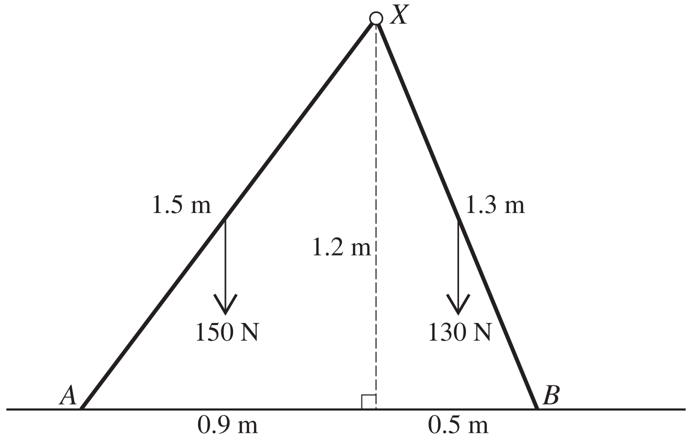
Two uniform smooth spheres A and B, of equal radius, have masses 0.8 kg and 2.0 kg respectively. The spheres are on a horizontal surface. A is moving with speed 12 m s^{-1} at 60° to the line of centres when it collides with B, which is stationary (see diagram). The coefficient of restitution between the spheres is 0.75. Find the speed and direction of motion of A immediately after the collision. [10]

4 A particle P of mass m kg is held at rest at a point O on a fixed plane inclined at an angle $\sin^{-1}\left(\frac{4}{7}\right)$ to the horizontal. P is released and moves down the plane. The total resistance acting on P is $0.2mv$ N, where $v\text{ m s}^{-1}$ is the velocity of P at time t s after leaving O.

(i) Show that $5\frac{dv}{dt} = 28 - v$ and hence find an expression for v in terms of t. [8]

(ii) Find the acceleration of P when $t = 10$. [2]

5



Two uniform rods XA and XB are freely jointed at X. The lengths of the rods are 1.5 m and 1.3 m respectively, and their weights are 150 N and 130 N respectively. The rods are in equilibrium in a vertical plane with A and B in contact with a rough horizontal surface. A and B are at distances horizontally from X of 0.9 m and 0.5 m respectively, and X is 1.2 m above the surface (see diagram).

(i) The normal components of the contact forces acting on the rods at A and B are R_A N and R_B N respectively. Show that $R_A = 125$ and find R_B. [4]

(ii) Find the frictional components of the contact forces acting on the rods at A and B. [4]

(iii) Find the horizontal and vertical components of the force exerted on XA at X, stating their directions. [3]

6 A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal surface. A force of $(0.36 - 0.144x)$ N acts on P in the direction from O to P, where x m is the displacement of P from a point O on the surface at time t s.

(i) By using the substitution $x = y + 2.5$, or otherwise, show that P moves with simple harmonic motion of period 5.24 s, correct to 3 significant figures. [5]

The maximum value of x during the motion is 3.

(ii) Write down the amplitude of P's motion and find the two possible values of x for which P's speed is $0.48\,\text{m}\,\text{s}^{-1}$. [4]

(iii) On each of the first two occasions when P has speed $0.48\,\text{m}\,\text{s}^{-1}$, P is moving towards O. Find the time interval between

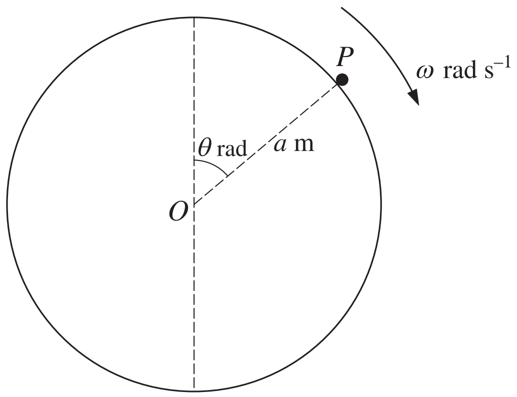
(a) these first two occasions,

(b) the second and third occasions when P has speed $0.48\,\text{m}\,\text{s}^{-1}$.

[5]

[Question 7 is printed overleaf.]

7



A particle P of mass m kg is slightly disturbed from rest at the highest point on the surface of a smooth fixed sphere of radius a m and centre O. The particle starts to move downwards on the surface. While P remains on the surface OP makes an angle of θ radians with the upward vertical and has angular speed $\omega \text{ rad s}^{-1}$ (see diagram). The sphere exerts a force of magnitude R N on P.

(i) Show that $a\omega^2 = 2g(1 - \cos\theta)$. [3]

(ii) Find an expression for R in terms of m, g and θ. [4]

At the instant that P loses contact with the surface of the sphere, find

(iii) the transverse component of the acceleration of P, [4]

(iv) the rate of change of R with respect to time t, in terms of m, g and a. [4]

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1 (i) $T = (1.35mg)(3 - 1.8) \div 1.8$		B1		
[0.9mg = ma]		M1		For using T = ma
Acceleration is $8.82ms^{-2}$		A1	3	
(ii) Initial EE =				
	$(1.35mg)(3 - 1.8)^2 \div (2x1.8)$	B1		
$[\frac{1}{2}mv^2 = 0.54mg]$		M1		For using $\frac{1}{2}mv^2$ = Initial EE
Speed is $3.25ms^{-1}$		A1	3	

2 **(i)**		M1		For using NEL vertically
Component is $8e\sin27^o$		A1		
Component is $2.18ms^{-1}$		A1	3	
(ii) Change in velocity vertically =				
	$8\sin27^o(1 + e)$	B1ft		ft $8\sin27^o$ + candidate's ans. in (i)
|I| = 0.2 x 5.81		M1		For using |I| = m x change in velocity
Magnitude of Impulse is $1.16 kgms^{-1}$		A1ft	3	ft incorrect ans. in (i) providing both M marks are scored.

3		M1		For using the principle of conservation of momentum in the **i** direction
$0.8x12\cos60^o = 0.8a + 2b$		A1		
		M1		For using NEL
$0.75x12\cos60^o = b - a$		A1		
[4.8 = 0.8a + 2(a + 4.5)]		DM1		For eliminating b; depends on at least one previous M mark
a = -1.5		A1		
Comp. of vel. perp. to l.o.c. after impact is				
	$12\sin60^o$	B1		
		M1		For correct method for speed or direction
The speed of A is $10.5ms^{-1}$		A1ft		ft $v^2 = a^2 + 108$
Direction of A is at 98.2^o to l.o.c.		A1ft	10	Accept $\theta = 81.8^o$ if θ is clearly and appropriately indicated; ft $\tan^{-1}\theta = (12\sin60^o)/\|a\|$)

4	**(i)** [$mg\sin\alpha$ – 0.2mv = ma]	M1		For using Newton's second law
	$5\frac{dv}{dt} = 28 - v$	A1		AG
	$[\int \frac{5}{28 - v}dv = \int dt]$	M1		For separating variables and integrating
	(C) - 5ln(28 –v) = t	A1		
		M1		For using v = 0 when t = 0
	ln[(28 – v)/28] = -t/5	A1ft		ft for ln[(28 – v)/28] = t/A from C + Aln(28 –v) = t previously
	[28 – v = $28e^{-t/5}$]	M1		For expressing v in terms of t
	v = $28(1 - e^{-t/5})$	A1ft	8	ft for v = $28(1 - e^{t/A})$ from ln[(28 – v)/28] = t/A previously
	(ii) [a = $28e^{-2}/5$]	M1		For using a = (28 – v(t))/5 or a = d(28 – $28e^{-t/5}$)dt and substituting t = 10.
	Acceleration is 0.758ms^{-2}	A1ft	2	ft from incorrect v in the form a + be^{ct} ($b \neq 0$); Accept $5.6/e^2$

5	**(i)**	M1		For taking moments about B or about A for the whole or For taking moments about X for the whole and using $R_A + R_B$ = 280 and $F_A = F_B$
	1.4R_A = 150x0.95 + 130x0.25 or 1.4R_B = 130x1.15 + 150x0.45 or 1.2F – 0.9(280 – R_B) + 0.45x150 – 1.2F + 0.5R_B – 0.25x130 = 0	A1		
	R_A = 125N	A1		AG
	R_B = 155N	B1	4	
	(ii)	M1		For taking moments about X for XA or XB
	1.2F_A = -150x0.45 + 0.9R_A or 1.2F_B = 0.5R_B – 130x0.25	A1		
	F_A or F_B = 37.5N	A1ft		F_B = (1.25R_B – 81.25)/3
	F_B or F_A = 37.5N	B1ft	4	
	(iii) Horizontal component is 37.5N to the left	B1ft		ft H = F or H = 56.25 – 0.75V or 12H = 325 + 5V
	[Y + R_A = 150]	M1		For resolving forces on XA vertically
	Vertical component is 25N upwards	A1ft	3	ft 3V = 225 – 4H or V = 2.4H -65

6	**(i)**			For applying Newton's second law
	[0.36 – 0.144x = 0.1a]	M1		
	$\ddot{x} = 3.6 - 1.44x$	A1		
	$\ddot{y} = -1.44y$ ➔ SHM or $d^2(x-2.5)/dt^2 = -1.44(x-2.5)$ ➔SHM	B1		
		M1		For using $T = 2\pi / n$
	Of period 5.24s	A1	5	AG
	(ii) Amplitude is 0.5m	B1		
		M1		For using $v^2 = n^2(a^2 - y^2)$
	$0.48^2 = 1.2^2(0.5^2 - y^2)$	A1ft		
	Possible values are 2.2 and 2.8	A1	4	
	(iii) [$t_0 = (\sin^{-1}0.6)/1.2$; $t_1 = (\cos^{-1}0.6)/1.2$]	M1		For using y = 0.5sin1.2t to find t_0 or y = 0.5cos1.2t to find t_1
	t_0 = 0.53625 .... or t_1 = 0.7727…..	A1		Principal value may be implied
	(a) [$2(\sin^{-1}0.6)/1.2$ or $(\pi - 2\cos^{-1} 0.6)/1.2$]	M1		For using $\Delta t = 2t_0$ or $\Delta t = T/2 - 2t_1$
	Time interval is 1.07s	A1ft		ft incorrect t_0 or t_1
	(b)			From $\Delta t = T/2 - 2t_0$ or $\Delta t = 2t_1$; ft 2.62 – ans(a) or
	Time interval is 1.55s	B1ft	5	incorrect t_0 or t_1

7	**(i)**	M1		For using KE gain = PE loss
	$\frac{1}{2} mv^2 = mga(1 - \cos\theta)$	A1		
	$aw^2 = 2g(1 - \cos\theta)$	B1	3	AG From v = wr
	(ii)	M1		For using Newton's second law radially (3 terms required) with accel = v^2/r or w^2r
	$mv^2/a = mg\cos\theta - R$ or $maw^2 = mg\cos\theta - R$	A1		
	[$2mg(1 - \cos\theta) = mg\cos\theta - R$]	DM1		For eliminating v^2 or w^2; depends on at least one previous M1
	$R = mg(3\cos\theta - 2)$	A1ft	4	ft sign error in N2 equation
	(iii) [$mg\sin\theta$ = m(accel.) or $2a(\dot{\theta})\ddot{\theta} = 2g\sin\theta(\dot{\theta})$]	M1		For using Newton's second law tangentially or differentiating $aw^2 = 2g(1 - \cos\theta)$ w.r.t. t
	Accel. (=$a\ddot{\theta}$) = $g\sin\theta$	A1		
	[$\theta = \cos^{-1}(2/3)$]	M1		For using R = 0
	Acceleration is 7.30ms^{-2}	A1ft	4	ft from incorrect R of the form mg(Acos□ +B), A $\neq$ 0, B $\neq$ 0; accept $g\sqrt{5}/3$
	(iv)	M1		For using rate of change = (dR/dθ)(dθ/dt)
	$dR/dt = (-3mg\sin\theta)\sqrt{2g(1-\cos\theta)/a}$	A1ft		ft from incorrect R of the form mg(Acos□ +B), A $\neq$ 0
		M1		For using $\cos\theta = 2/3$
	Rate of change is $-mg\sqrt{\frac{10g}{3a}}$ Ns^{-1}	A1ft	4	Any correct form of $\dot{R}$ with $\cos\theta = 2/3$ used; ft with □ from incorrect R of the form mg(Acos□ +B), A $\neq$ 0, B $\neq$ 0