OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4730/01**

MATHEMATICS

Mechanics 3

MONDAY 21 MAY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 A particle P is moving with simple harmonic motion in a straight line. The period is 6.1 s and the amplitude is 3 m. Calculate, in either order,

(i) the maximum speed of P, [3]

(ii) the distance of P from the centre of motion when P has speed $2.5\ \text{m s}^{-1}$. [3]

2 A tennis ball of mass 0.057 kg has speed $10\ \text{m s}^{-1}$. The ball receives an impulse of magnitude 0.6 N s which reduces the speed of the ball to $7\ \text{m s}^{-1}$. Using an impulse-momentum triangle, or otherwise, find the angle the impulse makes with the original direction of motion of the ball. [7]

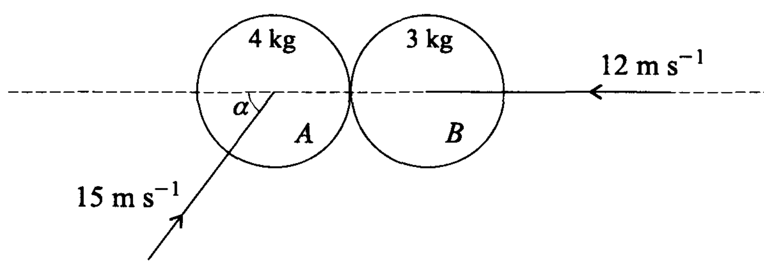
3 A particle P of mass 0.2 kg is projected horizontally with speed $u\ \text{m s}^{-1}$ from a fixed point O on a smooth horizontal surface. P moves in a straight line and, at time t s after projection, P has speed $v\ \text{m s}^{-1}$ and is x m from O. The only force acting on P has magnitude $0.4v^2$ N and is directed towards O.

(i) Show that $\frac{1}{v}\frac{dv}{dx} = -2$. [2]

(ii) Hence show that $v = ue^{-2x}$. [4]

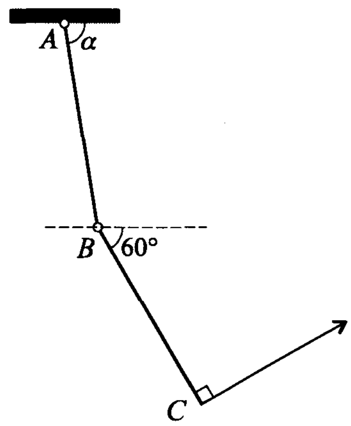
(iii) Find u, given that $x = 2$ when $t = 4$. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 3 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, A is moving with speed $15\ \text{m s}^{-1}$ at an angle α to the line of centres, where $\sin\alpha = 0.8$, and B is moving along the line of centres with speed $12\ \text{m s}^{-1}$ (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [10]

5

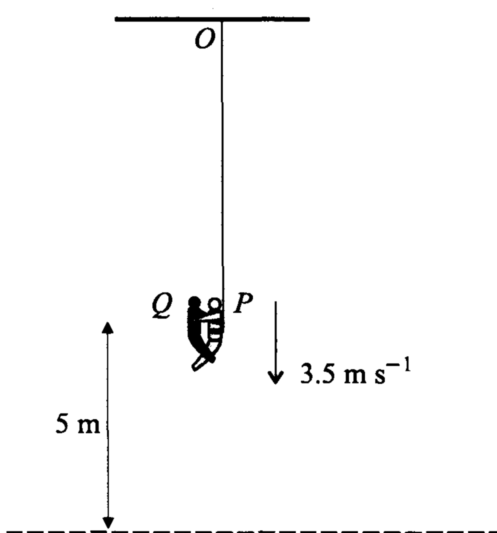


Two uniform rods AB and BC, each of length 1.4 m and weight 80 N, are freely jointed to each other at B, and AB is freely jointed to a fixed point at A. They are held in equilibrium with AB at an angle α to the horizontal, and BC at an angle of 60° to the horizontal, by a light string, perpendicular to BC, attached to C (see diagram).

(i) By taking moments about B for BC, calculate the tension in the string. Hence find the horizontal and vertical components of the force acting on BC at B. [7]

(ii) Find α. [4]

6



A circus performer P of mass 80 kg is suspended from a fixed point O by an elastic rope of natural length 5.25 m and modulus of elasticity 2058 N. P is in equilibrium at a point 5 m above a safety net. A second performer Q, also of mass 80 kg, falls freely under gravity from a point above P. P catches Q and together they begin to descend vertically with initial speed 3.5 m s^{-1} (see diagram). The performers are modelled as particles.

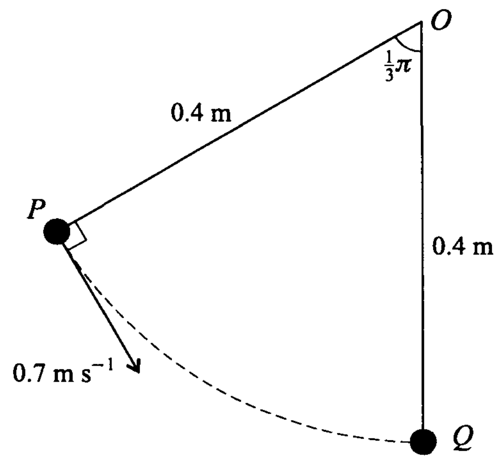
(i) Show that, when P is in equilibrium, $OP = 7.25$ m. [3]

(ii) Verify that P and Q together just reach the safety net. [5]

(iii) At the lowest point of their motion P releases Q. Prove that P subsequently just reaches O. [3]

(iv) State two additional modelling assumptions made when answering this question. [2]

7



A particle P of mass 0.8 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. A particle Q is suspended from O by an identical string. With the string OP taut and inclined at $\frac{1}{3}\pi$ radians to the vertical, P is projected with speed 0.7 m s^{-1} in a direction perpendicular to the string so as to strike Q directly (see diagram). The coefficient of restitution between P and Q is $\frac{1}{7}$.

(i) Calculate the tension in the string immediately after P is set in motion. [4]

(ii) Immediately after P and Q collide they have equal speeds and are moving in opposite directions. Show that Q starts to move with speed 0.15 m s^{-1}. [4]

(iii) Prove that before the second collision between P and Q, Q is moving with approximate simple harmonic motion. [5]

(iv) Hence find the time interval between the first and second collisions of P and Q. [2]

1	(i) $[\omega = 2\pi/6.1 = 1.03]$	M1		For using $T = 2\pi/\omega$
		M1		For using $v_{max} = a\omega$
	Speed is $3.09 ms^{-1}$	A1	3	
	(ii)	M1		For using $v^2 = \omega^2(A^2 - x^2)$ or for using $v = A\omega\cos\omega t$ and $x = A\sin\omega t$
	$2.5^2 = 1.03^2(3^2 - x^2)$ or $x = 3\sin(1.03 \times 0.60996..)$	A1ft		ft incorrect ω
	Distance is 1.76m	A1	3	

2	[Magnitudes 0.6, 0.057 x 7, 0.057 x 10]	M1		For triangle with magnitudes shown
	For magnitudes of 2 sides correctly marked	A1		
	For magnitudes of all 3 sides correctly marked	A1		
		M1		For attempting to find angle (α) opposite to the side of magnitude 0.057 x 7
		M1		For correct use of the cosine rule or equivalent
	$0.399^2 = 0.57^2 + 0.6^2 - 2 \times 0.57 \times 0.6\cos\alpha$	A1ft		
	Angle is 140°	A1	7	$(180 - 39.8)^o$

2	ALTERNATIVE METHOD			
		M1		For using I= Δmv parallel to the initial direction of motion or parallel to the impulse
	$-0.6\cos\alpha = 0.057 \times 7\cos\beta - 0.057 \times 10$ or $0.6 = 0.057 \times 10\cos\alpha + 0.057 \times 7\cos\gamma$	A1		
		M1		For using I= Δmv perpendicular to the initial direction of motion or perpendicular to the impulse
	$0.6\sin\alpha = 0.057 \times 7\sin\beta$ or $0.057 \times 10\sin\alpha = 0.057 \times 7\sin\gamma$	A1		
		M1		For eliminating β *or γ
	$0.399^2 = (0.57 - 0.6\cos\alpha)^2 + (0.6\sin\alpha)^2$ or $0.399^2 = (0.6 - 0.57\cos\alpha)^2 + (0.057\sin\alpha)^2$	A1ft		
	Angle is 140°	A1	7	$(180 - 39.8)^o$

3	(i) $[0.2v\ dv/dx = -0.4v^2]$	M1		For using Newton's second law with a = v dv/dx
	(1/v) dv/dx = -2	A1	2	AG
	(ii) $[\int (1/v)dv = \int -2dx]$	M1		For separating variables and attempting to integrate
	ln v = -2x (+C)	A1		
	[ln v = -2x + ln u]	M1		For using v(0) = u
	$v = ue^{-2x}$	A1	4	AG
	(iii) $[\int e^{2x}dx = \int u dt]$	M1		For using v = dx/dt and separating variables
	$e^{2x}/2 = ut$ (+C)	A1		
	$[e^{2x}/2 = ut + ½]$	M1		For using x(0) = 0
	u = 6.70	A1	4	Accept $(e^4 – 1)/8$

	ALTERNATIVE METHOD FOR PART (iii)			
	$[\int \frac{1}{v^2}dv = -2\int dt$ ➔ -1/v = -2t + A, and A = -1/u]	M1		For using a = dv/dt, separating variables, attempting to integrate and using v(0) = u
		M1		For substituting $v = ue^{-2x}$
	$-e^{2x}/u = -2t – 1/u$	A1		
	u = 6.70	A1	4	Accept $(e^4 – 1)/8$

4	$y= 15\sin\alpha$ (=12)	B1		
	$[4(15\cos\alpha) – 3 \times 12 = 4a + 3b]$	M1		For using principle of conservation of momentum in the direction of l.o.c.
	Equation complete with not more than one error	A1		
	4a + 3b = 0	A1		
		M1		For using NEL in the direction of l.o.c.
	$0.5(15\cos\alpha + 12) = b - a$	A1		
	[a = -4.5, b = 6]	M1		For solving for a and b
	[Speed = $\sqrt{(-4.5)^2 + 12^2}$, Direction $\tan^{-1}(12/(-4.50))$]	M1		For correct method for speed or direction of A
	Speed of A is $12.8ms^{-1}$ and direction is 111° anticlockwise from '**i**' direction	A1		Direction may be stated in any form , including θ = 69° with θ clearly and appropriately indicated
	Speed of B is $6ms^{-1}$ to the right	A1	10	Depends on first three M marks

5	(i)	M1		For taking moments of forces on BC about B
	80 x $0.7\cos 60^o = 1.4T$	A1		
	Tension is 20N	A1		
	[$X = 20\cos 30^o$]	M1		For resolving forces horizontally
	Horizontal component is 17.3N	A1ft		ft $X = T\cos 30^o$
	[$Y = 80 - 20\sin 30^o$]	M1		For resolving forces vertically
	Vertical component is 70N	A1ft	7	ft $Y = 80 - T\sin 30^o$
	(ii)	M1		For taking moments of forces on AB, or on ABC, about A
	$17.3 \times 1.4\sin\alpha = (80 \times 0.7 + 70 \times 1.4)\cos\alpha$ or $80\times 0.7\cos\alpha + 80(1.4\cos\alpha + 0.7\cos 60^o) =$ $20\cos 60^o(1.4\cos\alpha + 1.4\cos 60^o) +$ $20\sin 60^o(1.4\sin\alpha + 14\sin 60^o)$	A1ft		
	[$\tan\alpha = (\frac{1}{2}\,80 + 70)/17.3 = 11/\sqrt{3}$]	M1		For obtaining a numerical expression for $\tan\alpha$
	$\alpha = 81.1^o$	A1	4	

ALTERNATIVE METHOD FOR PART (i)			
	M1		For taking moments of forces on BC about B
$H\times 1.4\sin 60^o + V\times 1.4\cos 60^o = 80\times 0.7\cos 60^o$	A1		Where H and V are components of T
	M1		For using $H = V\sqrt{3}$ and solving simultaneous equations
Tension is 20N	A1		
Horizontal component is 17.3N	B1ft		ft value of H used to find T
[$Y = 80 - V$]	M1		For resolving forces vertically
Vertical component is 70N	A1ft	7	ft value of V used to find T

6 (i)	[T = 2058x/5.25]	M1		For using T = λx/L
	2058x/5.25 = 80 x 9.8 (x = 2)	A1		
	OP = 7.25m	A1	3	AG From 5.25 + 2
(ii)	Initial PE = (80 + 80)g(5) (= 7840) or (80 + 80)gX used in energy equation	B1		
	Initial KE = ½ (80 + 80)3.5^2 (= 980)	B1		
	[Initial EE = $2058x2^2/(2x5.25)$ (= 784), Final EE = $2058x7^2/(2x5.25)$ (= 9604), or $2058(X + 2)^2/(2x5.25)$]	M1		For using EE = $\lambda x^2/2L$
	[Initial energy = 7840 + 980 + 784, final energy = 9604 or 1568X + 980 + 784 = $196(X^2 + 4X + 4)$ ➔ $196X^2 – 784X – 980 = 0$]	M1		For attempting to verify compatibility with the principle of conservation of energy, or using the principle and solving for X
	Initial energy = final energy or X = 5 ➔P&Q just reach the net	A1	5	AG
(iii)	[PE gain = 80g(7.25 + 5)]	M1		For finding PE gain from net level to O
	PE gain = 9604	A1		
	PE gain = EE at net level ➔ P just reaches O	A1	3	AG
(iv)	For any one of 'light rope', 'no air resistance', 'no energy lost in rope'	B1		
	For any other of the above	B1	2	

FIRST ALTERNATIVE METHOD FOR PART (ii)			
[160g – 2058x/5.25 = 160v dv/dx]	M1		For using Newton's second law with a = v dv/dx, separating the variables and attempting to integrate
$v^2/2 = gx – 1.225x^2$ (+ C)	A1		Any correct form
	M1		For using v(2) = 3.5
C = -8.575	A1		
$[v(7)^2]/2$ = 68.6 – 60.025 – 8.575 = 0 ➔P&Q just reach the net	A1	5	AG

SECOND ALTERNATIVE METHOD FOR PART (ii)			
$\ddot{x} = g - 2.45x$ (= -2.45(x – 4))	B1		
	M1		For using $n^2 = 2.45$ and $v^2 = n^2(A^2 – (x – 4)^2)$
$3.5^2 = 2.45(A^2 – (-2)^2)$ (A = 3)	A1		
[(4 – 2) + 3]	M1		For using 'distance travelled downwards by P and Q = distance to new equilibrium position + A
distance travelled downwards by P and Q = 5 ➔P&Q just reach the net	A1	5	AG

7	(i) [$a = 0.7^2/0.4$]	M1		For using $a = v^2/r$
	For not more than one error in $T - 0.8g\cos 60^o = 0.8 \times 0.7^2/0.4$	A1		
	Above equation complete and correct	A1		
	Tension is 4.9N	A1	4	
	(ii)	M1		For using the principle of conservation of energy
	$\frac{1}{2}\, 0.8v^2 =$ $\frac{1}{2}\, 0.8(0.7)^2 + 0.8g0.4 - 0.8g0.4\cos 60^o$	A1		(v = 2.1)
	$(2.1 - 0)/7 = 2u$	M1		For using NEL
	Q's initial speed is 0.15ms^{-1}	A1	4	AG
	(iii)	M1		For using Newton's second law transversely
	$(m)0.4\ddot{\theta} = -(m)g\sin\theta$	A1		*Allow m = 0.8 (or any other numerical value)
	[$0.4\ddot{\theta} \approx -g\theta$]	M1		For using $\sin\theta \approx \theta$
	[$\frac{1}{2}\, m0.15^2 = mg0.4(1 - \cos\theta_{max})$ ➔ $\theta_{max} = 4.34^o$ (0.0758rad)]	M1		For using the principle of conservation of energy to find θ_{max}
	θ_{max} small justifies $0.4\ddot{\theta} \approx -g\theta$, and this implies SHM	A1	5	
	(iv) [$T = 2\pi/\sqrt{24.5} = 1.269..$] [$\sqrt{24.5}\, t = \pi$]	M1		For using $T = 2\pi/n$ or for solving either sin nt = 0 (non-zero t) (considering displacement) or cos nt = -1 (considering velocity)
	Time interval is 0.635s	A1ft	2	From t = ½ T