OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

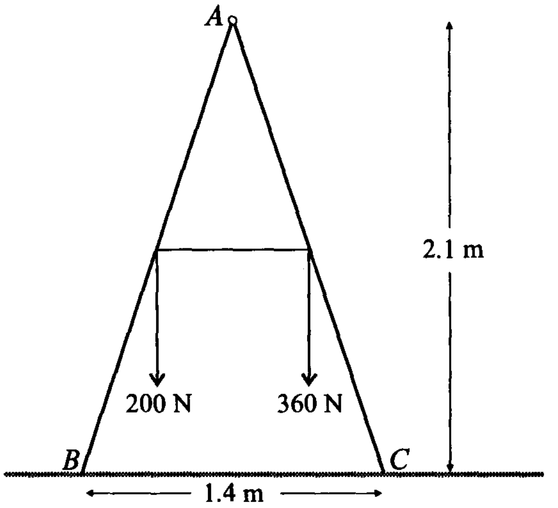
1 A ball of mass 0.4 kg is moving in a straight line, with speed 25 m s^{-1}, when it is struck by a bat. The bat exerts an impulse of magnitude 20 N s and the ball is deflected through an angle of 90°. Calculate

(i) the direction of the impulse, [3]

(ii) the speed of the ball immediately after it is struck. [3]

2 A duck of mass 2 kg is travelling with horizontal speed 4 m s^{-1} when it lands on a lake. The duck is brought to rest by the action of resistive forces, acting in the direction opposite to the duck's motion and having total magnitude $(2v + 3v^2)$ N, where v m s^{-1} is the speed of the duck. Show that the duck comes to rest after travelling approximately 1.30 m from the point of its initial contact with the surface of the lake. [8]

3



Two uniform rods AB and AC, of equal lengths, and of weights 200 N and 360 N respectively, are freely jointed at A. The mid-points of the rods are joined by a taut light inextensible string. The rods are in equilibrium in a vertical plane with B and C in contact with a smooth horizontal surface. The point A is 2.1 m above the surface and BC = 1.4 m (see diagram).

(i) Show that the force exerted on AB at B has magnitude 240 N and find the tension in the string. [6]

(ii) Find the horizontal and vertical components of the force exerted on AB at A and state their directions. [3]

4 A particle is connected to a fixed point by a light inextensible string of length 2.45 m to make a simple pendulum. The particle is released from rest with the string taut and inclined at 0.1 radians to the downward vertical.

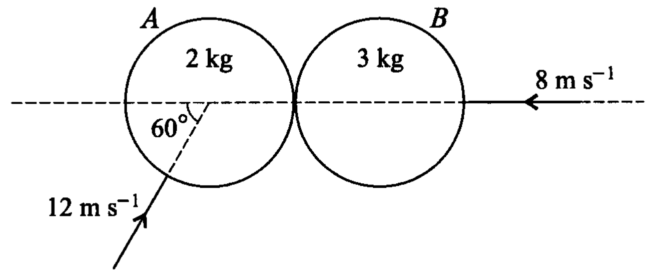
(i) Show that the motion of the particle is approximately simple harmonic with period 3.14 s, correct to 3 significant figures. [5]

Calculate, in either order,

(ii) the angular speed of the pendulum when it has moved 0.04 radians from the **initial** position, [3]

(iii) the time taken by the pendulum to move 0.04 radians from the **initial** position. [3]

5



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision A is moving with speed 12 m s^{-1} at 60° to the line of centres, and B is moving with speed 8 m s^{-1} along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [12]

6 A bungee jumper of mass 70 kg is joined to a fixed point O by a light elastic rope of natural length 30 m and modulus of elasticity 1470 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Find the distance fallen by the jumper when maximum speed is reached. [4]

(ii) Show that this maximum speed is 26.9 m s^{-1}, correct to 3 significant figures. [4]

(iii) Find the extension of the rope when the jumper is at the lowest position. [4]

[Question 7 is printed overleaf.]

[Turn over

7

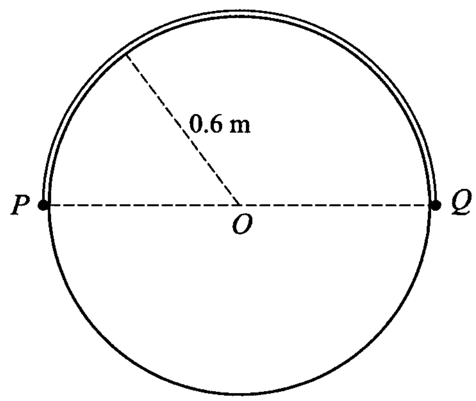


Fig. 1

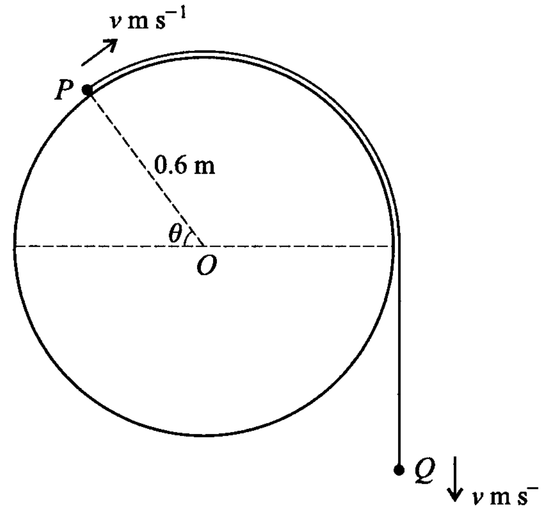


Fig. 2

A smooth horizontal cylinder of radius 0.6 m is fixed with its axis horizontal and passing through a fixed point O. A light inextensible string of length 0.6π m has particles P and Q, of masses 0.3 kg and 0.4 kg respectively, attached at its ends. The string passes over the cylinder and is held at rest with P, O and Q in a straight horizontal line (see Fig. 1). The string is released and Q begins to descend. When the line OP makes an angle θ radians, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, with the horizontal, the particles have speed v m s^{-1} (see Fig. 2).

(i) By considering the total energy of the system, or otherwise, show that

$$v^2 = 6.72\theta - 5.04\sin\theta.$$ [5]

(ii) Show that the magnitude of the contact force between P and the cylinder is

$$(5.46\sin\theta - 3.36\theta) \text{ newtons.}$$

Hence find the value of θ for which the magnitude of the contact force is greatest. [6]

(iii) Find the transverse component of the acceleration of P in terms of θ. [3]

1	**(i)**		M1		For using I = Δ(mv) in the direction of the original motion (or equivalent from use of relevant vector diagram).
		$20\cos\theta = 0.4\text{x}25$	A1		
		Direction at angle 120^o to original motion	A1	3	Accept $\theta = 60^o$ with θ correctly identified.
	(ii)		M1		For using I = Δ(mv) perp. to direction of the original motion (or equivalent from use of relevant vector diagram).
		$20\sin60^o = 0.4v$	A1ft		
		Speed is 43.3 ms^{-1}	A1	3	

2			M1		For applying Newton's 2^{nd} Law.
			M1		For using a = v(dv/dx).
		$2v(dv/dx) = -(2v + 3v^2)$	A1		
			M1		For separating variables and attempting to integrate.
		$2/3\ln(2 + 3v) = -x$ (+C)	A1ft		ft absence of minus sign,
		[$2/3\ln14 = C$]	M1		For using v(0) = 4.
		[$2/3\ln2 = -x + 2/3\ln14$]	M1		For attempting to solve v(x) = 0 for x.
		Comes to rest after travelling 1.30m	A1	8	AG

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3	**(i)**		M1		For taking moments about C for the whole structure.
		1.4R = 0.35x360 + 1.05x200	A1		
		Magnitude is 240N	A1		AG
			M1		For taking moments about A for the rod AB.
		0.7x240 = 0.35x200 + 1.05T	A1		
		Tension is 93.3N	A1	6	
	OR **(i)**		M1		For taking moments about A for AB and AC.
		$0.7R_B = 70 + 1.05T$ and $0.7R_C = 126 + 1.05T$	A1		
			M1		For eliminating T or for adding the equations, and then using $R_B + R_C = 560$.
		$0.7(560 – R_B) – 0.7R_B = 126 – 70$ or $0.7\text{x}560 = 70 + 126 + 2.1T$	A1		For a correct equation in R_B only or T only
		Magnitude is 240N	A1		AG
		Tension is 93.3N	A1	6	
	(ii)	Horizontal component is 93.3 N to the left	B1ft		
		Y = 240 - 200	M1		For resolving forces vertically.
		Vertical component is 40 N downwards	A1	3	

4	**(i)**		M1		For using Newton's 2nd Law perp. to string with a = L$\ddot{\theta}$.
		L(m)$\ddot{\theta}$ = -(m)gsinθ or (m) $\ddot{s}$ = -(m)gsin(s/L)	A1		
		$\ddot{\theta} \approx$ -kθ or $\ddot{s}$ = -ks [and motion is therefore approx. simple harmonic]	B1		
			M1		For using T = 2π/n and k = w^2 or T = $2\pi\sqrt{L/g}$ for simple pendulum.
		Period is 3.14s.	A1	5	AG
	(ii)		M1		For using $\dot{\theta}^2 = n^2(\theta_0{}^2 - \theta^2)$ or the principle of conservation of energy
		$\dot{\theta}^2 = 4(0.1^2 – 0.06^2)$ or ½ m(2.45$\dot{\theta}$)^2 = 2.45mg(cos0.06 – cos0.1)	A1		
		Angular speed is 0.16 rad s^{-1}.	A1	3	(0.1599… from energy method)
	OR	(in the case for which (iii) is attempted before (ii))			
	(ii)	[$\dot{\theta}$ = -0.2sin2t]	M1		For using $\dot{\theta}$ = d(Acos nt)/dt
		$\dot{\theta}$ = -0.2sin(2x0.464)	A1ft		
		Angular speed is 0.16 rad s^{-1}.	A1	3	
	(iii)		M1		For using θ = Acos nt or Asin(π/2 – nt) or for using θ = Asin nt and T =$t_{0.1} – t_{0.06}$
		0.06 = 0.1cos2t or 0.1sin(π/2 – 2t) or 2T = π/2 – sin^{-1}0.6	A1ft		ft angular displacement of 0.04 instead of 0.06
		Time taken is 0.464s	A1	3	

5			M1		Σ mv conserved in **i** direction.
		2x12cos60^o – 3x8 = 2a + 3b	A1		
			M1		For using NEL
		For LHS of equation below	A1		
		0.5(12cos60^o + 8) = b - a	A1		Complete equation with signs of a and b consistent with previous equation.
			M1		For eliminating a or b.
		Speed of B is 0.4ms^{-1} in **i** direction	A1		
		a = -6.6	A1		
		Component of A's velocity in **j** direction is 12sin60^o	B1		May be shown on diagram or implied in subsequent work.
		Speed of A is 12.3ms^{-1}	B1ft		
			M1		For using θ = tan^{-1}(**j**comp/$\pm$ **i** comp)
		Direction is at 122.4^o to the **i** direction	A1ft	12	Accept θ = 57.6^o with θ correctly identified.

6	**(i)**	T = 1470x/30	B1		
		[49x = 70x9.8]	M1		For using T = mg
		x = 14	A1		
		Distance fallen is 44m	A1ft	4	
	(ii)	PE loss = 70g(30 + 14)	B1ft		
		EE gain = $1470 \times 14^2/(2 \times 30)$	B1ft		
		[½ $70v^2$ = 30184 – 4802]	M1		For a linear equation with terms representing KE, PE and EE changes.
		Speed is 26.9ms^{-1}	A1	4	AG
	OR (ii)	[0.5 v^2 = 14g – 68.6 + 30g]	M1		For using Newton's 2nd law (vdv/dx = g – 0.7x), integrating (0.5 v^2 = gx – 0.35x^2 + k), using $v(0)^2$ = 60g➔ k = 30g, and substituting x = 14.
		For 14g + 30g	B1ft		
		For $\mp$ 68.6	B1ft		Accept in unsimplified form.
		Speed is 26.9ms^{-1}	A1	4	AG
	(iii)	PE loss = 70g(30 + x)	B1ft		
		EE gain = $1470x^2/(2 \times 30)$	B1ft		
		[x^2 – 28x – 840 = 0]	M1		For using PE loss = KE gain to obtain a 3 term quadratic equation.
		Extension is 46.2m	A1	4	
	OR (iii)		M1		For identifying SHM with $n^2 = 1470/(70 \times 30)$
			M1		For using v_{max} = An
		$A = 26.9/\sqrt{0.7}$	A1		
		Extension is 46.2m	A1	4	

7	**(i)**	$\frac{1}{2}\,0.3v^2 + \frac{1}{2}\,0.4v^2$	B1		
		$\pm\,0.3g(0.6\sin\theta)$	B1		
		$\pm\,0.4g(0.6\theta)$	B1		
		$[0.35v^2 = 2.352\theta - 1.764\sin\theta]$	M1		For using the principle of conservation of energy.
		$v^2 = 6.72\theta - 5.04\sin\theta$	A1	5	AG
	(ii)		M1		For applying Newton's 2nd Law radially to P and using $a = v^2/r$
		$0.3(v^2/0.6) = 0.3g\sin\theta - R$	A1		
		$[\frac{1}{2}(6.72\theta - 5.04\sin\theta) = 0.3g\sin\theta - R]$	M1		For substituting for v^2.
		Magnitude is $(5.46\sin\theta - 3.36\theta)$N	A1		AG
		$[5.46\cos\theta - 3.36 = 0]$	M1		For using $dR/d\theta = 0$
		Value of θ is 0.908	A1	6	
	(iii)	$[T - 0.3g\cos\theta = 0.3a]$	M1		For applying Newton's 2nd Law tangentially to P
		$[0.4g - T = 0.4a]$	M1		For applying Newton's 2nd Law to Q [If $0.4g - 0.3g\cos\theta = 0.3a$ is seen, assume this derives from $T - 0.3g\cos\theta = 0.3a$ ……. M1 and $T = 0.4g$ ……. M0]
		Component is $5.6 - 4.2\cos\theta$	A1	3	
	OR (iii)	$0.4g - 0.3g\cos\theta = (0.3 + 0.4)a$	B2		
		Component is $5.6 - 4.2\cos\theta$	B1	3	
	OR (iii)	$[2v(dv/d\theta) = 6.72 - 5.04\cos\theta]$	M1		For differentiating v^2 (from (i)) w.r.t. θ
		$2(0.6a) = 6.72 - 5.04\cos\theta$	M1		For using $v(dv/d\theta) = ar$
		Component is $5.6 - 4.2\cos\theta$	A1	3	