RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4725/01**

MATHEMATICS

Further Pure Mathematics 1

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and $\mathbf{I}$ is the 2×2 identity matrix. Find

(i) $\mathbf{A} - 3\mathbf{I}$, [2]

(ii) $\mathbf{A}^{-1}$. [2]

2 The complex number $3 + 4\mathrm{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = |a|$, [2]

(b) $\arg(z - 3) = \arg a$. [3]

3 **(i)** Show that $\dfrac{1}{r!} - \dfrac{1}{(r+1)!} = \dfrac{r}{(r+1)!}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}.$$ [4]

4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{A}^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}.$$ [6]

5 Find $\displaystyle\sum_{r=1}^{n} r^2(r-1)$, expressing your answer in a fully factorised form. [6]

6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots $(3 + \mathrm{i})$ and 2.

(i) Write down the other root of the equation. [1]

(ii) Find the values of a, b and c. [6]

7 Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$, [1]

(ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, [2]

(iii) $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, [2]

(iv) $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$. [2]

8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

9 **(i)** Use an algebraic method to find the square roots of the complex number 5 + 12i. [5]

(ii) Find $(3 - 2\text{i})^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

10 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix **B** is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

(i) Show that **AB** is non-singular. [2]

(ii) Find $(\mathbf{AB})^{-1}$. [4]

(iii) Find $\mathbf{B}^{-1}$. [5]

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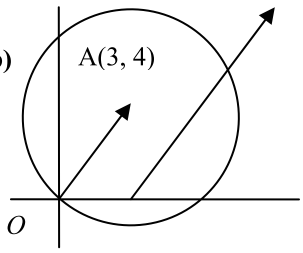
1	(i)	$\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$	B1	Two elements correct
			B1	All four elements correct
			2	
	(ii)	*EITHER*	B1	Both diagonals correct
		$\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$	B1	Divide by determinant
			2	
		OR	B1	Solve sim. eqns. 1st column correct
			B1	2nd column correct
2	(i)	5	B1	Correct modulus
		0.927 or 53.1°	B1	Correct argument, any equivalent form
			2	
	(ii)(a)		B1	Circle centre A (3, 4)
			B1	Through O, allow if centre is (4, 3)
			2	
	(b)		B1	Half line with +ve slope
			B1	Starting at (3, 0)
			B1	Parallel to OA, (implied by correct arg shown)
			3	
3	(i)	$\frac{r}{(r+1)!}$	M1	Common denominator of $(r + 1)!$ or $r!(r + 1)!$
			A1	Obtain given answer correctly
			2	
	(ii)	$1-\frac{1}{(n+1)!}$	M1	Express terms as differences using (i)
			A1	At least 1st two and last term correct
			M1	Show pairs cancelling
			A1	Correct answer a.e.f.
			4	
4			B1	Establish result is true, for n = 1 (or 2 or 3)
			M1	Attempt to multiply **A** and $\mathbf{A}^n$, or vice versa
			M1	Correct process for matrix multiplication
			A1	Obtain 3^{n+1}, 0 and 1
			A1	Obtain $\frac{1}{2}(3^{n+1} - 1)$
			A1	Statement of Induction conclusion, only if 5 marks earned, but may be in body of working
			6	

5			**M1**	Express as difference of two series
			M1	Use standard results
		$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	**A1**	Correct unsimplified answer
			M1	Attempt to factorise
			A1	At least factor of $n(n+1)$
		$\frac{1}{12}n(n+1)(3n+2)(n-1)$	**A1**	Obtain correct answer
			6	
6	**(i)**	$3 - \mathrm{i}$	**B1**	Conjugate stated
			1	
	(ii)	*EITHER*	**M1**	Use sum of roots
			A1	Obtain correct answer
			M1	Use sum of pairs of roots
			A1	Obtain correct answer
			M1	Use product of roots
		$a = -8,\ b = 22,\ c = -20$	**A1**	Obtain correct answers
			6	
		OR	**M1**	Attempt to find a quadratic factor
			A1	Obtain correct factor
			M1	Expand linear and quadratic factors
		$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
		OR		
			M1	Substitute 1 imaginary & the real root into eqn
			M1	Equate real and imaginary parts
			M1	Attempt to solve 3 eqns.
		$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
7	**(i)**		**B1**	Enlargement (centre *O*) scale factor 6
			1	
	(ii)		**B1**	Reflection
			B1	Mirror line is $y = x$
			2	
	(iii)		**B1**	Stretch in y direction
			B1	Scale factor 6, must be a stretch
			2	
	(iv)		**B1**	Rotation
			B1	36.9° clockwise or equivalent
			2	

8	$\alpha+\beta=-k$	**B1**	State or use correct value
	$\alpha\beta=2k$	**B1**	State or use correct value
		M1	Attempt to express sum of new roots in terms of $\alpha+\beta,\quad \alpha\beta$
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	**A1**	Obtain correct expression
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{1}{2}(k-4)$	**A1**	Obtain correct answer a.e.f.
	$\alpha'\beta'=1$	**B1**	Correct product of new roots seen
	$x^2-\frac{1}{2}(k-4)x+1=0$	**B1ft**	Obtain correct answer, must be an eqn.
		7	
			Alternative for last 5 marks
		M1	Obtain expression for $u=\frac{\alpha}{\beta}$ in terms of k and α or k and β
		A1	Obtain a correct expression
		A1	rearrange to get α in terms of u
		M1	Substitute into given equation
		A1	Obtain correct answer
9 (i)		**M1**	Attempt to equate real and imaginary parts of $(x+iy)^2$ and 5 + 12i
	$x^2-y^2=5$ and $xy=6$	**A1**	Obtain both results
		M1	Eliminate to obtain a quadratic in x^2 or y^2
	±(3 + 2i)	**M1**	Solve a 3 term quadratic & obtain x or y
		A1	Obtain correct answers as complex nos.
		5	
(ii)	5 – 12i	**B1B1**	Correct real and imaginary parts
		2	
(iii)		**M1**	Attempt to solve a quadratic equation
	$x^2=5\pm12i$	**A1**	Obtain correct answers
	$x=\pm(3\pm2i)$	**A1A1**	Each pair of correct answers a.e.f.
		4	

10 (i)		**M1**	Find value of det **AB**
		A1	Correct value 2 seen
		2	
(ii)		**M1**	Show correct process for adjoint entries
		A1	Obtain at least 4 correct entries in adjoint
		B1	Divide by their determinant
	$(\mathbf{AB})^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$	**A1**	Obtain completely correct answer
		4	
(iii)	*EITHER*	**M1**	State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
		A1	Obtain $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$
		M1	Correct multiplication process seen
		A1	Obtain three correct elements
	$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$	**A1**	All elements correct
		5	
	OR	**M1**	Attempt to find elements of **B**
		A1	All correct
		M1	Correct process for $\mathbf{B}^{-1}$
		A1	3 elements correct
		A1	All elements correct