OCR

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$ [6]

2 The matrices $\mathbf{A}$ and $\mathbf{I}$ are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

(i) Find $\mathbf{A}^2$ and verify that $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$. [4]

(ii) Hence, or otherwise, show that $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$. [2]

3 The complex numbers 2 + 3i and 4 − i are denoted by z and w respectively. Express each of the following in the form $x + \mathrm{i}y$, showing clearly how you obtain your answers.

(i) $z + 5w$, [2]

(ii) z^*w, where z^* is the complex conjugate of z, [3]

(iii) $\frac{1}{w}$. [2]

4 Use an algebraic method to find the square roots of the complex number 21 − 20i. [6]

5 (i) Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}.$$ [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}.$$ [4]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$. [1]

6 The loci C_1 and C_2 are given by

$$|z - 2\mathrm{i}| = 2 \qquad \text{and} \qquad |z + 1| = |z + \mathrm{i}|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2. [2]

7 The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

(i) Given that $\mathbf{B}$ is singular, show that $a = -\frac{2}{3}$. [3]

(ii) Given instead that $\mathbf{B}$ is non-singular, find the inverse matrix $\mathbf{B}^{-1}$. [4]

(iii) Hence, or otherwise, solve the equations

$$\begin{aligned} -x + y + 3z &= 1, \\ 2x + y - z &= 4, \\ y + 2z &= -1. \end{aligned}$$ [3]

8 **(a)** The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p, $2p$ and $3p$.

(i) Find the value of p. [2]

(ii) Hence find the value of a. [2]

9 **(i)** Write down the matrix $\mathbf{C}$ which represents a stretch, scale factor 2, in the x-direction. [2]

(ii) The matrix $\mathbf{D}$ is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by $\mathbf{D}$. [2]

(iii) The matrix $\mathbf{M}$ represents the combined effect of the transformation represented by $\mathbf{C}$ followed by the transformation represented by $\mathbf{D}$. Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}.$$ [2]

(iv) Prove by induction that $\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$, for all positive integers n. [6]

1.	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	**6**	Obtain given answer correctly
2.	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1		Attempt to find A^2, 2 elements correct
		A1		All elements correct
	$4A = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1		Use correct matrix 4**A**
	$\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$	A1	4	Obtain given answer correctly
	(ii)	M1		Multiply answer to (i) by $\mathbf{A}^{-1}$ or obtain $\mathbf{A}^{-1}$ or factorise $\mathbf{A}^2 - 4\mathbf{A}$
	$\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$	A1	2	Obtain given answer correctly
			6	
3.	(i) $22 - 2i$	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$	B1		Correct conjugate seen or implied
	$5 - 14i$	B1B1	3	Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1		Attempt to use w^*
		A1	2	Obtain correct answer in any form
			7	

4.	$x^2 - y^2 = 21$ and $xy = -10$ $\pm(5 - 2i)$	M1 A1A1 M1 M1 A1	 6 **6**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and 21 –20i Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,2$ Obtain correct answers as complex numbers
5.	(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$ $\dfrac{1}{(r+1)(r+2)}$ (ii) *EITHER* $\frac{2}{3} - \frac{1}{2} + \frac{3}{4} - \frac{2}{3} \ldots \frac{n+1}{n+2} - \frac{n}{n+1}$ $\frac{n+1}{n+2} - \frac{1}{2}$ *OR* (iii) $\frac{1}{2}$	M1 A1 M1 A1 M1 A1 M2 A1A1 B1 ft	 2 4 1 **7**	Show correct process for subtracting fractions Obtain given answer correctly Express terms as differences using (i) At least first two and last term correct Show or imply that pairs of terms cancel Obtain correct answer in any form State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$ Each term correct Obtain value from their sum to n terms
6.	(i) Circle Centre (0, 2) Radius 2 Straight line Through origin with positive slope (ii) 0 or 0 +0i and 2 + 2i	B1 B1 B1 B1 B1 B1ftB1ft	 5 2 **7**	Sketch(s) showing correct features, each mark independent Obtain intersections as complex numbers
7.	(i) det $\boldsymbol{B} = 0$ $a(2+1) - 2(2-3) = 0$ $3a = -2$ $a = -\frac{2}{3}$. (ii) det $\boldsymbol{B} = 3a + 2$ matrix of cofactors: $\begin{pmatrix} 3 & -4 & 2 \\ 1 & 2a & -a \\ -4 & a+6 & a-2 \end{pmatrix}$ so $\boldsymbol{B}^{-1} = \dfrac{1}{3a-2}\begin{pmatrix} 3 & 1 & -4 \\ -4 & 2a & a+6 \\ 2 & -a & a-2 \end{pmatrix}$			(iii) $\begin{pmatrix} -1 & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ With $a = -1$, $\boldsymbol{B}^{-1} = \begin{pmatrix} -3 & -1 & 4 \\ 4 & 2 & -5 \\ -2 & -1 & 3 \end{pmatrix}$ so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 & -1 & 4 \\ 4 & 2 & -5 \\ -2 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -11 \\ 17 \\ -9 \end{pmatrix}$

8.	(a) (i) $\alpha + \beta = 2 \quad \alpha\beta = 4$	B1B1	2	Values stated
	(ii) *EITHER* $\alpha^2 + \beta^2 = -4$	M1 A1	2	Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly
	OR	M1 A1		Find numeric values of roots, square and add Obtain given answer correctly
	(iii) $x^2 + 4x + 16 = 0$	B1		State or use $\alpha^2\beta^2 = 16$
		M1		Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square
		A1	3	Obtain $x^2 + 4x + 16 = 0$
	(b) (i) $p = 2$	M1		Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$
	(ii) $a = 44$	A1	2	Obtain $p = 2$
		M1		Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation
		A1ft	2	Obtain $11p^2$
			11	
9.	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	(iii) **M** = $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$	M1 A1	2	Attempt to find **DC** (not **CD**) Obtain given answer
	(iv)	B1		Explicit check for $n = 1$or $n = 2$
	$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1		Induction hypothesis that result is true for $\mathbf{M}^k$
		M1		Attempt to multiply $\mathbf{MM}^k$ or vice versa
	$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1 A1		Element $3(2^{k+1} - 1)$ derived correctly All other elements correct
		A1	6	Explicit statement of induction conclusion
			12	