OCR
RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find $\int x \cos 3x \, dx$. **[4]**

2 Find the first three terms in the expansion of $(9 - 16x)^{\frac{3}{2}}$ in ascending powers of x, and state the set of values for which this expansion is valid. **[5]**

3 The equation of a curve is $xy^2 = x^2 + 1$. Find $\frac{dy}{dx}$ in terms of x and y, and hence find the coordinates of the stationary points on the curve. **[7]**

4 The equations of two lines are

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$

(i) Show that these lines meet, and find the coordinates of the point of intersection. **[5]**

(ii) Find the acute angle between these lines. **[3]**

5 The parametric equations of a curve are

$$x = 2 + 3\sin\theta \quad \text{and} \quad y = 1 - 2\cos\theta \quad \text{for} \quad 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. **[5]**

(ii) Find the cartesian equation of the curve. **[2]**

6 Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} \, dx$. **[7]**

7 **(i)** Given that $y = \ln(1 + \sin x) - \ln(\cos x)$, show that $\frac{dy}{dx} = \frac{1}{\cos x}$. **[4]**

(ii) Using this result, evaluate $\int_0^{\frac{1}{3}\pi} \sec x \, dx$, giving your answer as a single logarithm. **[3]**

8 The points A (3, 2, 1), B (5, 4, −3), C (3, 17, −4) and D (1, 6, 3) form a quadrilateral $ABCD$.

(i) Show that $AB = AD$. **[2]**

(ii) Find a vector equation of the line through A and the mid-point of BD. **[3]**

(iii) Show that C lies on the line found in part **(ii)**. **[1]**

(iv) What type of quadrilateral is $ABCD$? **[1]**

9 The temperature of a freezer is –20 °C. A container of a liquid is placed in the freezer. The rate at which the temperature, θ °C, of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta + 20),$$

where time t is in minutes and k is a positive constant.

(i) Express θ in terms of t, k and an arbitrary constant. **[3]**

Initially the temperature of the liquid in the container is 40 °C and, at this instant, the liquid is cooling at a rate of 3 °C per minute. The liquid freezes at 0 °C.

(ii) Find the value of k and find also the time it takes (to the nearest minute) for the liquid to freeze. **[5]**

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is 90 °C. After 19 minutes its temperature is 0 °C.

(iii) Without any further calculation, explain what you can deduce about the value of k in this case. **[1]**

10 **(i)** Use algebraic division to express $\frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \frac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants. **[4]**

(ii) Hence find $\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,dx$, giving your answer in the form $a + \ln b$. **[7]**

Question			Answer	Marks	Guidance	
1			$u = x$ and $dv = \cos 3x$	M1	integration by parts as far as $f(x) \pm \int g(x)\,dx$	Check if labelled v,du
			$x \times \frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x\,dx$	A2	**A1** for $x \times k\sin 3x - \int k\sin 3x\,dx$; $k \neq \frac{1}{3}$ or 0	k may be negative
			$\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x$ [+ c] cao www ISW	A1	Not ….. $\frac{1}{3}\left(\frac{1}{3}\cos 3x\right)$ or ….. $--\frac{1}{9}\cos 3x$	
				[4]		
2			<u>The first 3 marks refer to the expansion…</u>	……...	<u>of $\left(1 - \frac{16x}{9}\right)^{3/2}$ and to no other expansion</u>	
			First 2 terms = $1 - \frac{8}{3}x$	B1	Allow any equiv fraction for the $-\frac{8}{3}$ and ISW	$\frac{3}{2}.-\frac{16}{9}$ is not an equiv fraction
			3rd term = $\frac{\frac{3}{2}.\frac{1}{2}}{1.2}\left(-\frac{16x}{9}\right)^2$	M1	Allow clear evidence of intention,e.g. $\frac{\frac{3}{2}.\frac{1}{2}}{1.2}\frac{-16x^2}{9}$	
			$= \frac{32}{27}x^2$	A1	Allow any equiv fraction for the $\frac{32}{27}$ and ISW	
			Complete expansion $\approx 27 - 72x + 32x^2$	A1	cao No equivalents. Ignore any further terms	If expansion $(a+b)^n$ used, award B1,B1,B1 for $27, -72x, 32x^2$
			valid for $\frac{-9}{16} < x < \frac{9}{16}$ or $\lvert x \rvert < \frac{9}{16}$	B1	oe Beware, e.g. $x < \left\lvert \frac{9}{16} \right\rvert$	condone $\leq$ instead of $<$
				[5]		

Question			Answer	Marks	Guidance	
3			For attempt at product rule on xy^2	M1	or changing equation to $y^2 = x + x^{-1}$	
			$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	soi in the differentiating process	
			$\frac{dy}{dx} = \frac{2x - y^2}{2xy}$ or $\frac{1-x^{-2}}{2y}$	A1	Award <u>B</u>1 for $(\pm)\frac{1}{2}(x + x^{-1})^{-\frac{1}{2}}(1 - x^{-2})$	
			Stationary point $\rightarrow$ (their) $\frac{dy}{dx} = 0$ soi	M1		
			$x^2 = 1$ <u>or</u> $y^2 = 2$ <u>or</u> $y^4 = 4$	A1	Ignore any other values	
			$(1, \sqrt{2})$, $(1, -\sqrt{2})$	A1,A1	Accept 1.41 or $4^{\frac{1}{4}}$ for $\sqrt{2}$	SR. Award **A1** only if extra co-ordinates presented with both correct answers
				[7]		
4	(i)		Produce (at least 2) relevant equations	M1	e.g. $1 + 2\lambda = 6 + \mu$, $2 + \lambda = 8 + 4\mu$, $3\lambda = 1 - 5\mu$	
			Eliminate either λ or μ from 2 of them and solve for the other (μ or λ)	M1	soi by correct (λ, μ)	
			$\lambda = 2$ <u>and</u> $\mu = -1$ cao	A1	or e.g. $\lambda = 2$ from 2 different pairs	
			Check that $(\lambda, \mu) = (2, -1)$ satisfies all eqns	B1	<u>This must be convincing</u>. Check unusual arguments	Dep previous M1M1A1 earned
			P is (5, 4, 6) cao www	A1	Allow any reasonable vector notation	
				[5]		
4	(ii)		Using $\begin{pmatrix}2\\1\\3\end{pmatrix}$ and $\begin{pmatrix}1\\4\\-5\end{pmatrix}$	M1	i.e. correct parts for direction vectors	
			Using $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ giving value $\frac{n}{\sqrt{a}\sqrt{b}}$	M1	for any 2 meaningful vectors in this question using meaningful scalar product & modulus	Expect $\frac{-9}{\sqrt{14}\sqrt{42}}$
			68.2°... (not 111.8…)	A1	or 1.19 (radians)	
				[3]		

Question			Answer	Marks	Guidance	
5	(i)		their $\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1		
			$\dfrac{dy}{dx} = \dfrac{2\sin\theta}{3\cos\theta}$	A1		
			their $\dfrac{dy}{dx} = \dfrac{1}{2}$	M1		
			$\tan\theta = \dfrac{3}{4}$	A1	If $\tan\theta = \dfrac{3}{4}$ not seen, award this A1 only if coords are correct	
			$(3.8, -0.6)$ or $\left(\dfrac{19}{5}, -\dfrac{3}{5}\right)$ or $x = 3.8,\ y = -0.6$	A1 **[5]**		
5	(ii)		Manipulating equations into form $\sin\theta = f(x)$ and $\cos\theta = g(y)$ and then using $\sin^2\theta + \cos^2\theta = 1$	M1	<u>If part (ii) is attempted first, and then part (i), allow</u> B1 for obtaining $\dfrac{dy}{dx} = \dfrac{4(x-2)}{9(y-1)}$ M1 for equating their $\dfrac{dy}{dx}$ to $\dfrac{1}{2}$ A1 for obtaining $9y - 8x = -7$ M1 for eliminating x or y from above eqn... A1 for $(3.8, -0.6))$	<u>the following marks in part (i):-</u> ….and their Cartesian equation
			$\dfrac{(x-2)^2}{9} + \dfrac{(1-y)^2}{4} = 1$ oe www ISW Accept e.g. $\left(\dfrac{x-2}{3}\right)^2$ $4x^2 + 9y^2 - 16x - 18y - 11 = 0$	A1 **[2]**		

Question			Answer	Marks	Guidance	
6			Attempt diff to connect du & dx Correct result e.g. $\frac{du}{dx}=2$ or $du = 2\,dx$	M1 A1	or find $\frac{du}{dx}$ or $\frac{dx}{du}$	
			Indef integ in terms of $u = \frac{1}{2}\int \frac{2u-3}{u^5}(du)$	A1	Must be completely in terms of u.	
			Integrate to $\frac{u^{-3}}{-3} - \frac{3u^{-4}}{-8}$ oe	A1A1	or (using 'by parts') $\frac{(2u-3)u^{-4}}{-8} - \frac{u^{-3}}{12}$	Award B1,B1 for $\frac{4u^{-3}}{-3} - \frac{3u^{-4}}{-2}$ or for $\frac{2u^{-3}}{-3} - \frac{3u^{-4}}{-4}$ or for $\frac{(2u-3)u^{-4}}{-2} - \frac{u^{-3}}{3}$ or for $\frac{(2u-3)u^{-4}}{-4} - \frac{u^{-3}}{6}$
			Use correct variable & correct values for limits	M1	Provided minimal attempt at $\int f(u)du$ made	
			$= \frac{-23}{384}$ oe $(-0.059895\ldots..)$	A1	Accept decimal answer only if minimum of first 3 marks scored	
			[ISW,e.g. changing to $\frac{23}{384}$]	**[7]**		

Question			Answer	Marks	Guidance	
7	(i)	I	$\frac{\cos x}{1+\sin x} - \frac{-\sin x}{\cos x}$ or $\frac{\cos x}{1+\sin x} + \frac{\sin x}{\cos x}$	B2	Each half (including 'middle' sign) scores B1	
			$\frac{+/-\cos^2 x + /-\sin x(1+\sin x)}{(1+\sin x)\cos x}$	M1	Combine, <u>provided</u> derivative was of form $\frac{f'(x)}{f(x)}$	Allow only variations num signs
			$\frac{1+\sin x}{\cos x(1+\sin x)} = \frac{1}{\cos x}$ <u>www</u> **AG**	A1	$\cos^2 x + \sin^2 x = 1$ in intermediate step required	
		II	Change to $\ln\left(\frac{1+\sin x}{\cos x}\right)$	B1		
			Change to ln(sec x + tan x)	B1	<u>Not</u> $\ln(\frac{1}{\cos x} + \tan x)$	
			Diff as $\frac{\text{attempt at } \frac{d}{dx}(\sec x + \tan x}{\sec x + \tan x}$	M1		
			Reduce to $\sec x = \frac{1}{\cos x}$	A1		
		III	Change to $\ln\left(\frac{1+\sin x}{\cos x}\right)$	B1		
			Diff as $\frac{\text{attempt at quotient differentiation}}{\frac{1+\sin x}{\cos x}}$	M1		
			Fully correct differentiation	A1		
			Correct reduction to $\frac{1}{\cos x}$	A1 **[4]**		
7	(ii)		Indef integral = $\ln(1+\sin x) - \ln(\cos x)$ [Method I]	B1	or $\ln(\sec x + \tan x)$ [Method II]	
			Substitute limits & use log manipulation	M1	Use of ln A – ln B = $\ln\frac{A}{B}$ anywhere in question	
			Answer = ln(2 + √3)	B1 **[3]**	Accept ln 3.73 or $\ln\frac{2+\sqrt{3}}{1}$ …..but not $\ln\frac{1+\sqrt{3}/2}{1/2}$	Answer has <u>not</u> been given

Question			Answer	Marks	Guidance	
8	(i)		$AB=\sqrt{(+/-2)^2+(+/-2^2+(+/-4)^2)}$ $AD=\sqrt{(+/-2)^2+(+/-4)^2+(+/-2)^2}$	B1 B1 **[2]**	oe oe	If $AB^2=AD^2=24$, then SR B1 $AB=AD$ to be stated for 2nd B1
8	(ii)		midpoint is (3, 5, 0) Clear method for finding direction vector $\mathbf{r}=3\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(3\mathbf{j}-\mathbf{k})$ oe or e.g. $\mathbf{r}=3\mathbf{i}+5\mathbf{j}+\mu(-3\mathbf{j}+\mathbf{k})$ cao	B1 M1 A1 **[3]**	Accept any reasonable vector notation. Expect $3\mathbf{j}-\mathbf{k}$ or $-3\mathbf{j}+\mathbf{k}$ **"r ="** is essential. No f.t. for wrong mid-point.	
8	(iii)		substitution of $\lambda=+/-5$ or $\mu=+/-4$	M1 **[1]**	Based on correct answer to (ii)	
8	(iv)		Kite	B1 **[1]**		

Question			Answer	Marks	Guidance	
9	(i)		Separating variables $\int \frac{1}{\theta+20}\,d\theta = \int -k\,dt$	M1	or invert each side: $\frac{dt}{d\theta} = -\frac{1}{k(\theta+20)}$	Must see $\frac{1}{\theta+20}$; ignore posn 'k'
			$\ln(\theta + 20) = -kt$ (+ c) or equivalent	A1	"Eqn A"	
			$\theta = Ae^{-kt} - 20$ oe (i.e. $\theta = e^{-kt+c} - 20$)	A1	"Eqn B"	
				[3]		
9	(ii)		$(-)3 = -k(40+20)$	M1	Using $t = 0, \theta = 40, \frac{d\theta}{dt} = (-)3$ in <u>given</u> equation	
			$k = \frac{1}{20}$ oe	*A1	Not $k = -\frac{1}{20}$	
			Subst $t = 0, \theta = 40$ & their k (where necessary) into their Eqn A or their Eqn B and solve for the arbitrary constant	M1		
			Subst $\theta = 0$ & their values of k and the arbitrary constant into their Eqn A or their Eqn B	M1		
			$t =$ 21.9722 = 22 minutes cao www	dep*A1		
				[5]		
9	(iii)		k is larger	B1 [1]		

Question			Answer	Marks	Guidance	
10	(i)		Clear start to algebraic division (Quotient) = $x-1$ (Remainder) = $x+7$ Final answer: $x-1+\dfrac{x+7}{x^2-x-6}$	M1 A1 A1 A1 **[4]**	at least as far as x term in quot & subseq mult back final answer in correct form This must be shown in part (i) or, if not, then implied in part (ii) If no long division shown but only comparison of coefficients or otherwise, SR M0 B1 B1 B1	& attempt at subtraction Accept $A=1, B=-1, C=1, D=7$
10	(ii)		Convert their $\dfrac{Cx+D}{x^2-x-6}$ to Partial Fracts $\dfrac{x+7}{x^2-x-6}=\dfrac{2}{x-3}-\dfrac{1}{x+2}$ Their …. $\int Ax+B\,dx=\dfrac{1}{2}Ax^2+Bx$ or $\dfrac{(Ax+B)^2}{2A}$ $\int \dfrac{E}{x-3}+\dfrac{F}{x+2}\,dx=E\ln(x-3)+F\ln(x+2)$ Using limits in a correct manner $8+\ln\dfrac{27}{4}$ $\left(8+\ln\dfrac{54}{8}\right)$ isw	M1 A1A1 B1 ft B1 ft M1 A1 **[7]**	 Correct fraction converted to correct PFs Tolerate some wrong signs provided intention clear Answer required in the form $a+\ln b$, so giving only a decimalised form is awarded A0	