OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 4

QUESTION PAPER

4724

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4724
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Simplify $\dfrac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. **[4]**

2 Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. **[3]**

3 **(i)** Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is x. **[4]**

(ii) Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3}\,dx.$$ **[4]**

4 Use the substitution $x = \frac{1}{3}\sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{\left(1 - 9x^2\right)^{\frac{3}{2}}}\,dx.$$ **[6]**

5 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

(i) Show that l_1 and l_2 are skew. **[3]**

(ii) Find the acute angle between l_1 and l_2. **[4]**

(iii) The point A lies on l_1 and OA is perpendicular to l_1, where O is the origin. Find the position vector of A. **[3]**

6 Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1 + ax}{4 - x}},$$

giving your answer in terms of a. **[8]**

7 The gradient of a curve at the point (x, y), where $x > -2$, is given by

$$\frac{dy}{dx} = \frac{1}{3y^2(x + 2)}.$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q, giving your answer correct to 3 significant figures. **[7]**

8 A curve has parametric equations

$$x = \frac{1}{t+1}, \qquad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. **[2]**

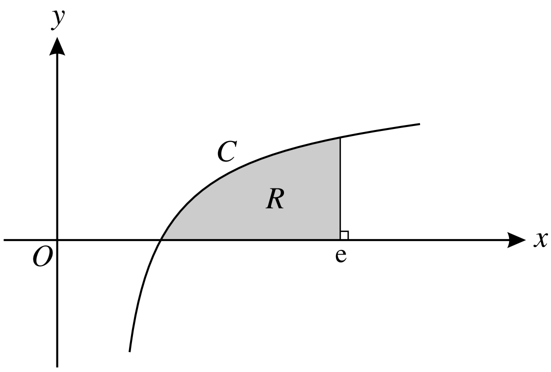
(ii) Find the equation of the normal to the curve at the point for which $t = -2$. **[6]**

(iii) Find the value of t at the point where this normal meets the curve again. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = \mathrm{f}(x)$. **[3]**

9 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$. **[3]**

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C, the x-axis and the line $x = \mathrm{e}$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x-axis. **[6]**

(b) The region R is rotated completely about the y-axis. Explain why the volume of the solid of revolution formed is given by

$$\pi \mathrm{e}^2 - \pi \int_0^1 \mathrm{e}^{2y}\,\mathrm{d}y,$$

and find this volume. **[4]**

1	Attempt to factorise **both** numerator & denominator	M1	completely or partially
	Num = e.g. $(x^2-1)(x^2-9)$ or $(x^2-2x-3)(x^2+2x-3)$	B1	or $(x-3)(x+3)(x-1)(x+1)$
	Denominator = e.g. $(x^2-2x-3)(x+5)(x+3)$	B1	or $(x-3)(x+1)(x+5)(x+3)$
	$\frac{x-1}{x+5}$ or $1-\frac{6}{x+5}$ WWW	A1 **4**	ISW but not if any further 'cancellation'
	Alternative start, attempting long division		
	Expand denom as quartic & attempt to divide $\frac{\text{numerator}}{\text{denominator}}$	M1	but not divide $\frac{\text{denominator}}{\text{numerator}}$
	Obtain quotient = 1 & remainder = $-6x^3-6x^2+54x+54$	B1	
	Final B1 A1 available as before		
			4
2	$2^2+(-3)^2+(\sqrt{12})^2$ soi e.g. 25 or 5	M1	Allow $2^2-3^2+\sqrt{12}^2$
	5	A1	May be implied by 5 or 1/5 in final answer
	$\frac{1}{5}\begin{pmatrix}2\\-3\\\sqrt{12}\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{5}\\-\frac{3}{5}\\\frac{\sqrt{12}}{5}\end{pmatrix}$ AEF	√A1 **3**	FT their '5'. Accept $-\frac{1}{5}\begin{pmatrix}\\\\\end{pmatrix}$ or $\frac{1}{\pm 5}\begin{pmatrix}\\\\\end{pmatrix}$
			3
3	**(i)** The words quotient and remainder need not be explicit		
	Long division For leading term $3x$ in quotient	B1	
	Suff evidence of div process ($3x$, mult back, attempt sub)	M1	
	(Quotient) = $3x-1$	A1	
	(Remainder) = x **AG**	A1 **4**	No wrong working, partic on penult line
	Identity $3x^3-x^2+10x-3=Q(x^2+3)+R$	*M1	
	$Q=ax+b, R=cx+d$ & attempt at least 2 operations	dep*M1	If $a=3$, this $\Rightarrow$ 1 operation
	$a=3, b=-1$	A1	
	$c=1, d=0$	A1	No wrong working anywhere
	Inspection $3x^3-x^2+10x-3=(x^2+3)(3x-1)+x$	B2	or state quotient $=3x-1$
	Clear demonstration of LHS = RHS	B2	
(ii)	Change integrand to 'their (i) quotient' + $\frac{x}{x^2+3}$	M1	
	Correct FT integration of 'their (i) quotient'	√A1	
	$\int \frac{x}{x^2+3}\,dx=\frac{1}{2}\ln(x^2+3)$	A1	
	Exact value of integral = $\frac{1}{2}+\frac{1}{2}\ln 4-\frac{1}{2}\ln 3$ AEF ISW	A1 **4**	Answer as decimal value (only) $\rightarrow$ A0
			8

4	<u>Indefinite integral</u> Attempt to connect $\mathrm{d}x$ and $\mathrm{d}\theta$	M1		Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}=, \frac{\mathrm{d}\theta}{\mathrm{d}x}=$, $\mathrm{d}x = ...\mathrm{d}\theta$; not $\mathrm{d}x=\mathrm{d}\theta$
	Denominator $\left(1-9x^2\right)^{\frac{3}{2}}$ becomes $\cos^3\theta$	B1		
	Reduce original integral to $\frac{1}{3}\int\frac{1}{\cos^2\theta}\,\mathrm{d}\theta$	A1		May be implied, seen only as $\frac{1}{3}\int\sec^2\theta\,\mathrm{d}\theta$
	Change $\int\frac{1}{\cos^2\theta}\,\mathrm{d}\theta$ to $\tan\theta$	B1		Ignore $\frac{1}{3}$ at this stage
	Use <u>appropriate</u> limits for θ (allow degrees) or x	M1		Integration need not be accurate
	$\frac{\sqrt{3}}{9}$ AEF, exact answer required, ISW	A1	**6**	
				6
5 (i)	Attempt to set up 3 equations	M1		of type $4+3s=1, 6+2s=t, 4+s=-t$
	$(s,t)=(-1,4)$ or $(-1,-3)$ or $(-\frac{10}{3},-\frac{2}{3})$	*A1		or $s=-1$ & $-\frac{10}{3}$ <u>or</u> $t=$ two of $\left(4,-3,-\frac{2}{3}\right)$
	Show clear contradiction e.g. $3\neq-4, 4\neq-3, -6\neq1$	dep*A1	**3**	Allow ✓ unsimpl contradictions. No ISW.
	<u>SC</u> If $s=\frac{-10}{3}$ found from 2nd & 3rd eqns and contradiction shown in 1st eqn, all 3 marks may be awarded.			
(ii)	Work with $\begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\\-1\end{pmatrix}$	M1		
	Clear method for scalar product of any 2 vectors	M1		
	Clear method for modulus of any vector	M1		
	$79.1^{(\circ)}$ or better (79.1066..) 1.38 (rad) (1.38067..) ISW	A1	**4**	(From $\frac{1}{\sqrt{14}.\sqrt{2}}$)
(iii)	Use $\begin{pmatrix}4+3s\\6+2s\\4+s\end{pmatrix}.\begin{pmatrix}3\\2\\1\end{pmatrix}=0$	M1		
	Obtain $s=-2$	A1		from $12+9s+12+4s+4+s=0$
	A is $\begin{pmatrix}-2\\2\\2\end{pmatrix}$ or $-2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ final answer	<u>B</u>1	**3**	Accept $(-2,2,2)$
				10

6 $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}ax \ \ldots\ldots + \frac{\frac{1}{2}\cdot\frac{-1}{2}}{2}(ax)^2$ B1,B1 N.B. third term = $-\frac{1}{8}a^2x^2$

Change $(4-x)^{-\frac{1}{2}}$ into $k\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$, where k is likely to be $\frac{1}{2}$ /2/4 /−2 , & work out expansion of $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\frac{1}{8}x \ \ldots. + \frac{\frac{-1}{2}\cdot\frac{-3}{2}}{2}\left(\frac{(-)x}{4}\right)^2$ B1,B1 N.B. third term = $\frac{3}{128}x^2$

OR Change $\{4-x\}^{\frac{1}{2}}$ into $l\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$, where l is likely to be $\frac{1}{2}$ / 2 / 4 /− 2,& work out expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 1-\frac{1}{8}x-\frac{1}{128}x^2$ B1 (for all 3 terms simplified)

$k=\frac{1}{2}$ (with possibility of M1 + A1 + A1 to follow) B1 $l=2$ (with no further marks available)

Multiply $(1+ax)^{\frac{1}{2}}$ by $(4-x)^{-\frac{1}{2}}$ or $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1 Ignore irrelevant products

The required three terms (with/without x^2) identified as

$-\frac{1}{16}a^2+\frac{1}{32}a+\frac{3}{256}$ or $\frac{-16a^2+8a+3}{256}$ AEF ISW A1+A1 **8** A1 for one correct term + A1 for other two

SC B1 for $\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$; B1 for $\left(1-\frac{x}{4}\right)^{-1} = 1+\frac{x}{4}+\frac{x^2}{16}$; M1 for multiplying $(1+ax)$ by their $(4-x)^{-1}$.

If result is $p+qx+rx^2$, then to find $\left(p+qx+rx^2\right)^{\frac{1}{2}}$ award B1 for $p^{\frac{1}{2}}(\ldots\ldots)$,

B1 correct 1st & 2nd terms of expansion, B1 correct 3rd term; A1,A1 as before, for correct answers.

8

7 Attempt to sep variables in format $\int py^2\,(\mathrm{d}y) = \int \frac{q}{x+2}(\mathrm{d}x)$ M1 where constants p and/or q may be wrong

Either y^3 & $\ln(x+2)$ or $\frac{1}{3}y^3$ & $\frac{1}{3}\ln(x+2)$ A1+A1 Accept $\frac{1}{3}\ln(3x+6)$ for $\frac{1}{3}\ln(x+2)$ & $|\ |$ for ( )

If indefinite integrals are being used (most likely scenario)

Substitute $x=1, y=2$ into an eqn containing '+const' M1

Sub y = 1.5 and their value of 'const' & solve for x or q M1

x or q = −1.97 only A2

[SC x or q = −1.970 or −1.971 or −1.9705 or −1.9706 A1] **7**

If definite integrals are used (less likely scenario)

Use $\int_{1.5}^{2}\ldots\mathrm{d}y = \int_{q}^{1}\ldots\mathrm{d}x$ where 2 corresponds with 1..... M2 & 1.5 corresp with q (at top/bottom or v.v.)

Then A2 or SC A1 as above

Use $\int_{1.5}^{2}\ldots\mathrm{d}y = \int_{1}^{q}\ldots\mathrm{d}x$ where 2 corresponds with q….. M1 & 1.5 corresp with 1 (at top/bottom or v.v.)

Then A1 for 1.97 only

7

8 **Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded**

(i)	Sub parametric eqns into $y = 3x$ & produce $t = -2$		
	OR sub $t = -2$ into para eqs, obtain $(-1, -3)$ & state $y = 3x$		
	OR other similar methods producing (or verifying) $t = -2$	B1	
	Value of t at other point is 2	B1 **2**	$t = \pm 2$ is sufficient for B1+B1
(ii)	Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	
	$= -(t+1)^2$	A1	or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$
	Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal	M1	
	Gradient normal = 1 cao	A1	
	Subst $t = -2$ into the parametric eqns.	M1	to find pt at which normal is drawn
	Produce $y = x - 2$ as equation of the normal WWW	A1 **6**	'A' marks in **(ii)** are dep on prev 'A'
(iii)	Substitute the parametric values into their eqn of normal	M1	
	Produce $t = 0$ as final answer cao	A1 **2**	This is dep on final A1 in (ii)
	N.B. If $y = x - 2$ is found fortuitously in (ii) (& ∴ given A0 in (ii)), you must award A0 here in (iii).		
(iv)	Attempt to eliminate t from the parametric equations	M1	
	Produce any correct equation	A1	e.g. $x = \frac{1}{y+2}$
	Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW	A1 **3**	Must be seen in (iv)
	{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}		
		13	

9 (i)	Treat $x\ln x$ as a product	M1	If $\int \ln x$, use parts $u = \ln x$, $dv = 1$
	Obtain $x.\frac{1}{x} + \ln x$	A1	$x\ln x - \int 1\,dx = x\ln x - x$
	Show $x.\frac{1}{x} + \ln x - 1 = \ln x$ WWW **AG**	A1 **3**	And state given result
(ii)(a)	Part (a) is mainly based on the indef integral $\int (\ln x)^2\,dx$		
	[A candidate stating e.g. $\int (\ln x)^2\,dx = \int 2\ln x\,dx$ or $= \int (\ln x - x)^2\,dx$ **is awarded 0 for (ii)(a)]**		
	Correct use of $\int \ln x\,dx = x\ln x - x$ anywhere in this part	B1	Quoted from (i) or derived
	Use integ by parts on $\int (\ln x)^2\,dx$ with $u = \ln x, dv = \ln x$	M1	or $u = (\ln x)^2, dv = 1$
	[For 'integration by parts, candidates must get to a 1st stage with format $f(x) +/- \int g(x)\,dx$]		
	1st stage = $\ln x(x\ln x - x) - \int \frac{1}{x}(x\ln x - x)\,dx$ soi	A1	$x(\ln x)^2 - \int x.\frac{2}{x}\ln x\,dx$
	2nd stage = $x(\ln x)^2 - 2x\ln x + 2x$ AEF (unsimplified)	A1	
	$\therefore$ Value of definite integral between 1 & e $= e - 2$ cao	A1	Use limits on 2nd stage & produce cao
	Volume = $\pi(e - 2)$ ISW	A1 **6**	Answer as decimal value (only) $\rightarrow$ A0
	Alternative method when subst. $u = \ln x$ used		
	Attempt to connect dx and du	M1	
	Becomes $\int u^2 e^u\,du$	A1	
	First stage $u^2 e^u - \int 2u\,e^u\,du$	A1	
	Third stage $(u^2 - 2u + 2)e^u$	A1	
	Final A1 A1 available as before		
(b)	Indication that reqd vol = vol cylinder – vol inner solid	M1	
	Clear demonstration of either vol of cylinder being πe^2 (including reason for height $= \ln e$) or rotation of $x = e$ about the y-axis (including upper limit of $y = \ln e$)	A1	Could appear as $\pi\int_0^1 e^2\,dy$
	$(\pi)\int x^2\,dy = (\pi)\int e^{2y}\,dy$	B1	
	$\frac{\pi(e^2+1)}{2}$ or 13.2 or 13.18 or better	B1 **4**	May be from graphical calculator
		13	

Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is.. e.g. in Qu.4, a candidate saying $\frac{dx}{d\theta} = -\frac{1}{3}\cos\theta$ is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol √ is sometimes used to indicate 'follow-through' in this scheme.