OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4724

Core Mathematics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 2 4 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the quotient and the remainder when $x^4 + 11x^3 + 28x^2 + 3x + 1$ is divided by $x^2 + 5x + 2$. **[4]**

2 Points A, B and C have position vectors $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ respectively, where p is a constant.

(i) Given that angle $ABC = 90°$, find the value of p. **[4]**

(ii) Given instead that ABC is a straight line, find the value of p. **[2]**

3 By expressing $\cos 2x$ in terms of $\cos x$, find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x}\,\mathrm{d}x$. **[5]**

4 Use the substitution $u = 2 + \ln t$ to find the exact value of

$$\int_1^{\mathrm{e}} \frac{1}{t(2 + \ln t)^2}\,\mathrm{d}t.$$ **[6]**

5 **(i)** Expand $(1 + x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[2]**

(ii) **(a)** Hence, or otherwise, expand $(8 + 16x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[4]**

(b) State the set of values of x for which the expansion in part **(ii) (a)** is valid. **[1]**

6 A curve has parametric equations

$$x = 9t - \ln(9t), \quad y = t^3 - \ln(t^3).$$

Show that there is only one value of t for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$ and state that value. **[6]**

7 Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[8]**

8 **(i)** State the derivative of $\mathrm{e}^{\cos x}$. **[1]**

(ii) Hence use integration by parts to find the exact value of

$$\int_0^{\frac{1}{2}\pi} \cos x \sin x\, \mathrm{e}^{\cos x}\,\mathrm{d}x.$$ **[6]**

9 The equation of a straight line l is $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

(i) The point P on l is given by $t = 1$. Calculate the acute angle between OP and l. **[4]**

(ii) Find the position vector of the point Q on l such that OQ is perpendicular to l. **[4]**

(iii) Find the length of OQ. **[2]**

10 **(i)** Express $\dfrac{1}{(3-x)(6-x)}$ in partial fractions. **[2]**

(ii) In a chemical reaction, the amount x grams of a substance at time t seconds is related to the rate at which x is changing by the equation

$$\frac{dx}{dt} = k(3-x)(6-x),$$

where k is a constant. When $t = 0$, $x = 0$ and when $t = 1$, $x = 1$.

(a) Show that $k = \frac{1}{3}\ln\frac{5}{4}$. **[7]**

(b) Find the value of x when $t = 2$. **[4]**

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1		<u>Long division method</u>			
		Correct leading term x^2 in quotient	B1		
		Evidence of correct div process	M1		Sufficient to convince
		(Quotient =) x^2+6x-4	A1		
		(Remainder =) $11x+9$	A1		
		<u>Identity method</u>			
		$x^4+11x^3+28x^2+3x+1=Q(x^2+5x+2)+R$	M1		
		$Q=ax^2+bx+c$ or x^2+bx+c ; $R=dx+e$ & ≥ 3 ops	M1		N.B. $a=1 \Rightarrow$ 1 of the 3 ops
		$a=1, b=6, c=-4, d=11, e=9$ (for all 5)	A2		S.R. <u>B</u>1 for 3 of these
			4		
2	**(i)**	Find at least 2 of $(\overrightarrow{AB}$ or $\overrightarrow{BA}), (\overrightarrow{BC}$ or $\overrightarrow{CB}), (\overrightarrow{AC}$ or $\overrightarrow{CA})$	M1		irrespect of label; any notation
		Use correct method to find scal prod of any 2 vectors	M1		<u>or</u> use corr meth for modulus
		Use $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\dfrac{\overrightarrow{AB}.\overrightarrow{BC}}{\lvert AB\rvert\lvert BC\rvert}=0$	M1		or use $\lvert\overrightarrow{AB}\rvert^2+\lvert\overrightarrow{BC}\rvert^2=\lvert\overrightarrow{AC}\rvert^2$
		Obtain $p=1$ (dep 3 @ M1)	A1	**4**	
	(ii)	Use equal ratios of appropriate vectors	M1		or scalar product method
		Obtain $p=-8$	A1	**2**	
			6		
3		Use $\cos 2x =$ $a\cos^2 x+b$ / $\pm$ $\cos^2 x-\sin^2 x$ / $1-2\sin^2 x$	*M1		
		Obtain $\lambda+\mu\sec^2 x$	dep*M1		using 'reasonable' Pythag attempt
		$\int \lambda+\mu\sec^2 x\,\mathrm{d}x=\lambda x+\mu\tan x$	A1		(λ or μ may be 0 here/prev line)
		Obtain correct result $2x-\tan x$	A1		no follow-through
		$\frac{1}{6}\pi-\sqrt{3}+1$ ISW	A1		exact answer required
			5		
4		Attempt to connect du and dt or find $\frac{\mathrm{d}u}{\mathrm{d}t}$ or $\frac{\mathrm{d}t}{\mathrm{d}u}$	M1		not du = dt but no accuracy
		$\mathrm{d}u=\frac{1}{t}\mathrm{d}t$ or $\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{1}{t}$ or $\mathrm{d}t=\mathrm{e}^{u-2}\,\mathrm{d}u$ or $\frac{\mathrm{d}t}{\mathrm{d}u}=\mathrm{e}^{u-2}$	A1		
		Indef int $\rightarrow \int\frac{1}{u^2}(\mathrm{d}u)$	A1		no t or dt in evidence
		$=-\frac{1}{u}$	A1		
		Attempt to change limits if working with $\mathrm{f}(u)$	M1		or re-subst & use 1 and e
		$\frac{1}{6}$ ISW	A1		ln e must be changed to 1, ln 1 to 0
			6		

5	**(i)** $(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + ...$	B1		
	$... -\frac{1}{9}x^2$	B1	**2**	$-\frac{2}{18}x^2$ acceptable
	(ii) (a) $(8+16x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+2x)^{\frac{1}{3}}$	B1		not $16^{\frac{1}{3}}\left(\frac{1}{2}+x\right)^{\frac{1}{3}}$
	$(1+2x)^{\frac{1}{3}}$ = their **(i)** expansion with $2x$ replacing x	M1		not dep on prev B1
	$= 1+\frac{2}{3}x-\frac{4}{9}x^2 + ...$	√A1		$-\frac{8}{18}x^2$ acceptable
	Required expansion = 2 (expansion just found)	√B1	**4**	accept equiv fractions
	N.B. If not based on part **(i)**, award M1 for $8^{1/3}+\frac{1}{3}.8^{-2/3}(16x)+\frac{\frac{1}{3}.-\frac{2}{3}}{1.2}8^{-5/3}(16x)^2$, allowing $16x^2$ for $(16x)^2$, with 3 @ A1 for $2...+\frac{4}{3}x...-\frac{8}{9}x^2$, accepting equivalent fractions & ISW			
	(ii) (b) $-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x\rvert < \frac{1}{2}$	B1	**1**	no equality
		7		
6	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		quoted/implied
	$\frac{dx}{dt} = 9 - \frac{9}{9t}$ ISW	B1		
	$\frac{dy}{dt} = 3t^2 - \frac{3t^2}{t^3}$ ISW	B1		
	Stating/implying $\frac{3t^2-\frac{3}{t}}{9-\frac{1}{t}} = 3 \Rightarrow t^2 = 9$ or $t^3 - 9t = 0$	A1		WWW, totally correct at this stage
	$t = 3$ as final ans with clear log indication of invalidity of -3; ignore (non) mention of $t = 0$	A2		**S.R.** A1 if $t = \pm 3$ or $t = -3$ or ($t = 3$ **&** wrong/no indication)
		6		
7	Treat $\frac{d}{dx}(x^2y)$ as a product	M1		
	$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$	B1		
	$3x^2 + 2x^2\frac{dy}{dx} + 4xy = 3y^2\frac{dy}{dx}$	A1		Ignore $\frac{dy}{dx} =$ if not used
	Subst (2, 1) and solve for $\frac{dy}{dx}$ or vice-versa	M1		
	$\frac{dy}{dx} = -4$ WWW	A1		
	grad normal = $-\frac{1}{\text{their }\frac{dy}{dx}}$	√A1		stated or used
	Find eqn of line, through (2, 1), with either gradient	M1		using their $\frac{dy}{dx}$ or $-\frac{1}{\text{their }\frac{dy}{dx}}$
	$x - 4y + 2 = 0$	A1		AEF with integral coefficients
		8		

8	**(i)**	$-\sin x\, e^{\cos x}$	B1	**1**	
	(ii)	$\int \sin x\, e^{\cos x} dx = -e^{\cos x}$	B1		anywhere in part **(ii)**
		Parts with split $u = \cos x, dv = \sin x\, e^{\cos x}$	M1		result $f(x) +/- \int g(x)\,dx$
		Indef Integ, 1st stage $-\cos x\, e^{\cos x} - \int \sin x\, e^{\cos x} dx$	A1		accept $\ldots -\int -e^{\cos x}.-\sin x\, dx$
		Second stage = $-\cos x\, e^{\cos x} + e^{\cos x}$	*A1		
		Final answer = 1	dep*A2	**6**	
			7		
9	**(i)**	P is $\begin{pmatrix}3\\1\\1\end{pmatrix}+\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}$	B1		
		direction vector of ℓ is $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and of $\overrightarrow{OP}$ is their P	√B1		
		Use $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and their OP	M1		
		$\theta = 35.3$ or better (0.615… rad)	A1	**4**	
	(ii)	Use $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}=0$	M1		
		$1(3+t)-1(1-t)+2(1+2t)=0$	A1		
		$t=-\dfrac{2}{3}$	A1		
		Subst. into $\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}$ to produce $\begin{pmatrix}7/3\\5/3\\-1/3\end{pmatrix}$ ISW	A1	**4**	
	(iii)	Use $\sqrt{x^2+y^2+z^2}$ where $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is part **(ii)** answer	M1		
		Obtain $\sqrt{\dfrac{75}{9}}$ AEF, 2.89 or better (2.8867513….)	A1	**2**	
			10		

10 (i) $\dfrac{\frac{1}{3}}{3-x}$ ……. $-\dfrac{\frac{1}{3}}{6-x}$ B1+1 **2**

(ii) **(a)** Separate variables $\displaystyle\int \frac{1}{(3-x)(6-x)}\,dx = \int k\,dt$ M1 or invert both sides

<u>Style</u>: For the M1, dx & dt must appear on correct sides <u>or</u> there must be $\int$ sign on both sides

Change $\dfrac{1}{(3-x)(6-x)}$ into partial fractions from **(i)** √B1

$\displaystyle\int \frac{A}{3-x}\,dx = \left(-A \text{ or } -\frac{1}{A}\right)\ln(3-x)$ B1 or $\displaystyle\int \frac{B}{6-x}\,dx = \left(-B \text{ or } -\frac{1}{B}\right)\ln(6-x)$

$-\frac{1}{3}\ln(3-x)+\frac{1}{3}\ln(6-x) = kt\ (+c)$ √A1 f.t. from wrong multiples in **(i)**

Subst (x = 0, t = 0) & (x = 1, t = 1) into eqn with 'c' M1 and solve for 'k'

Use $\ln a + \ln b = \ln ab$ or $\ln a - \ln b = \ln \frac{a}{b}$ M1

Obtain $k = \frac{1}{3}\ln\frac{5}{4}$ with sufficient working & WWW A1 **7** **AG**

(b) Substitute $k = \frac{1}{3}\ln\frac{5}{4}$, $t = 2$ & their value of 'c' *M1

Reduce to an eqn of form $\dfrac{6-x}{3-x} = \lambda$ dep*M1 where λ is a const

Obtain $x = \dfrac{27}{17}$ or 1.6 or better (1.5882353…) A2 **4** S.R. A1√ for $x = \dfrac{3\lambda-6}{\lambda-1}$

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