OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4724/01**

MATHEMATICS

Core Mathematics 4

WEDNESDAY 21 MAY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

 [Turn over

1 **(a)** Simplify $\dfrac{(2x^2-7x-4)(x+1)}{(3x^2+x-2)(x-4)}$. [2]

(b) Find the quotient and remainder when x^3+2x^2-6x-5 is divided by x^2+4x+1. [4]

2 Find the exact value of $\displaystyle\int_1^{e} x^4 \ln x \, dx$. [5]

3 The equation of a curve is $x^2y - xy^2 = 2$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y^2-2xy}{x^2-2xy}$. [3]

(ii) **(a)** Show that, if $\dfrac{dy}{dx} = 0$, then $y = 2x$. [2]

(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

4 Relative to an origin O, the points A and B have position vectors $3\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B. [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB. [5]

5 **(i)** Show that $\sqrt{\dfrac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$, for $|x| < 1$. [5]

(ii) By taking $x = \frac{2}{7}$, show that $\sqrt{5} \approx \frac{111}{49}$. [3]

6 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

7 **(i)** Show that, if $y = \operatorname{cosec} x$, then $\dfrac{dy}{dx}$ can be expressed as $-\operatorname{cosec} x \cot x$. [3]

(ii) Solve the differential equation

$$\frac{dx}{dt} = -\sin x \tan x \cot t,$$

given that $x = \frac{1}{6}\pi$ when $t = \frac{1}{2}\pi$. [5]

8 **(i)** Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

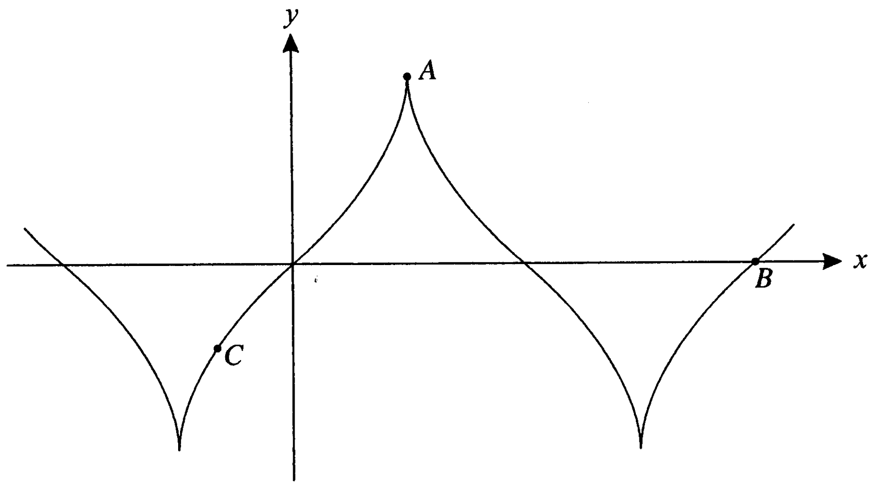
(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x+\sqrt{2x-1}}\,dx$ to $\int \frac{2t}{(t+1)^2}\,dt$. [4]

(iii) Hence find the exact value of $\int_1^5 \frac{1}{x+\sqrt{2x-1}}\,dx$. [4]

9 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 4\sin\theta,$$

and part of its graph is shown below.



(i) Find the value of θ at A and the value of θ at B. [3]

(ii) Show that $\frac{dy}{dx} = \sec\theta$. [5]

(iii) At the point C on the curve, the gradient is 2. Find the coordinates of C, giving your answer in an exact form. [3]

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1	(a)	$2x^2-7x-4=(2x+1)(x-4)$ or $3x^2+x-2=(3x-2)(x+1)$	B1	
		$\frac{2x+1}{3x-2}$ as final answer; this answer only	B1	Do not ISW
			2	
	(b)	For correct leading term x in quotient	B1	Identity method
		For evidence of correct division process	M1	M1: $x^3+2x^2-6x-5=Q(x^2+4x+1)+R$
		Quotient = $x-2$	A1	M1: $Q=ax+b$ or $x+b, R=cx+d$ & ≥ 2 ops [N.B. If $Q=x+b$, this $\Rightarrow$ 1 of the 2 ops]
		Remainder = $x-3$	A1	A2: $a=1, b=-2, c=1, d=-3$ SR: B1 for two
			4	
2		Parts with correct split of $u=\ln x, \frac{dv}{dx}=x^4$	*M1	obtaining result $f(x)+/-\int g(x)\,dx$
		$\frac{x^5}{5}\ln x-\int\frac{x^5}{5}.\frac{1}{x}(dx)$	A1	
		$\frac{x^5}{5}\ln x-\frac{x^5}{25}$	A1	
		Correct method with the limits	dep*M1	Decimals acceptable here
		$\frac{4e^5}{25}+\frac{1}{25}$ ISW (Not '+c')	A1	Accept equiv fracts; like terms amalgamated
			5	
3	(i)	$\frac{d}{dx}(x^2y)=x^2\frac{dy}{dx}+2xy$ or $\frac{d}{dx}(xy^2)=2xy\frac{dy}{dx}+y^2$	*B1	
		Attempt to solve their differentiated equation for $\frac{dy}{dx}$	dep*M1	
		$\frac{dy}{dx}=\frac{y^2-2xy}{x^2-2xy}$ only	A1	WWW **AG** Must have intermediate line &… …could imply "=0" on 1st line
			3	
	(ii)(a)	Attempt to solve **only** $y^2-2xy=0$ & derive $y=2x$	B1	**AG** Any effort at solving $x^2-2xy=0\rightarrow$ B0
		Clear indication why $y=0$ is not acceptable	B1	Substituting $y=2x\rightarrow$ B0, B0
			2	
	(b)	Attempt to solve $y=2x$ simult with $x^2y-xy^2=2$	M1	
		Produce $-2x^3=2$ or $y^3=-8$	A1	AEF
		$(-1, -2)$ or $x=-1, y=-2$ **only**	A1	
			3	

4	**(i)**	For (either point) + t(difference between vectors)	M1	't' can be 's', 'λ' etc.
		r = (3**i** + 2**j** + 3**k** or **i** + 3**j** + 4**k**)+ t(–2**i** + **j** + **k** or 2**i** – **j** – **k**)	A1	'**r**' must be '**r**' but need not be bold Check other formats, e.g. $ta+(1-t)b$
			2	
	(ii)	State/imply that their **r** and their –2**i** + **j** + **k** are perpendicular	*M1	N.B.This *M1 is dep on M1 being earned in (i)
		Consider scalar product = 0	dep*M1	
		Obtain $t=-\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$	A1	
		Subst their t into their equation of AB	M1	
		Obtain $\frac{1}{6}$(16**i** + 13**j** + 19**k**) AEF	A1	Accept decimals if clear
			5	
5	**(i)**	$(1-x)^{\frac{1}{2}}=1-\frac{1}{2}x-\frac{1}{8}x^2$ ignoring x^3 etc	B2	SR Allow B1 for $1-\frac{1}{2}x+kx^2$, $k\neq-\frac{1}{8}$ or 0
		$(1+x)^{-\frac{1}{2}}=1-\frac{1}{2}x+\frac{3}{8}x^2$ ignoring x^3 etc	B2	SR Allow B1 for $1-\frac{1}{2}x+kx^2$, $k\neq\frac{3}{8}$ or 0
		Product = $1-x+\frac{1}{2}x^2$ ignoring x^3 etc	B1	**AG**; with (at least) 1 intermediate step (cf x^2)
			5	
	(ii)	$\sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$ seen	B1	
		$\frac{37}{49}$ or $1-\frac{2}{7}+\frac{1}{2}\left(\frac{2}{7}\right)^2$ seen	B1	
		$\frac{\sqrt{5}}{3}\approx\frac{37}{49}\Rightarrow\sqrt{5}\approx\frac{111}{49}$	B1	AG
			3	
6	**(i)**	Produce at least 2 of the 3 relevant equations in t and s	M1	1 + 2t = 12 + s, 3t = –4s, –5 + 4t = 5 – 2s
		Solve for t and s	M1	
		(t, s) = (4, –3) AEF	*A1	
		Subst (4, –3) into suitable equation(s) & show consistency	dep*A1	Either into "3rd" eqn or into all 3 coordinates. N.B. Intersection coords not asked for
			4	
	(ii)	Method for finding magnitude of any vector	*M1	Expect $\sqrt{29}$ and $\sqrt{21}$
		Method for finding scalar product of any 2 vectors	*M1	Expect -18
		Using $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ AEF for the correct 2 vectors	dep*M1	Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$
		137 (136.8359) or 43.2(43.164…)	A1	2.39 (2.388236..) or 0.753(0.75335…) rads
			4	

7	**(i)**	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	**M1**	
		Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2}\cos x$	**A1**	
		Show manipulation to –cosec x cot x (or vice-versa)	**A1** 3	WWW **AG** with ≥ 1 line intermed working
	(ii)	Separate variables, $\int(-)\frac{1}{\sin x\tan x}\,dx = \int \cot t\,dt$	**M1**	or $\int\frac{1}{\sin x\tan x}\,dx = \int(-)\cot t\,dt$
		Style: For the M1 to be awarded, dx and dt must appear on correct sides or there must be $\int$ sign on both sides		
		$\int -\operatorname{cosec} x\cot x\,dx = \operatorname{cosec} x$ (+c)	**A1**	or $\int \operatorname{cosec} x\cot x\,dx = -\operatorname{cosec} x$
		$\int \cot t\,dt = \ln\sin t$ or $\ln\lvert\sin t\rvert$ (+c)	**B1**	or $\int -\cot t\,dt = -\ln\sin t$ or $-\ln\lvert\sin t\rvert$
		Subst $(t, x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing 'c'	**M1**	and attempt to find 'c'
		$\operatorname{cosec} x = \ln\sin t + 2$ or $\ln\lvert\sin t\rvert + 2$	**A1** 5	WWW ISW; cosec $\frac{\pi}{6}$ to be changed to 2
8	**(i)**	$A(t + 1) + B = 2t$ $A = 2$ B = –2	**M1** **A1** **A1** 3	Beware: correct values for A and/or B can be ... … obtained from a wrong identity Alt method: subst suitable values into given… …expressions
	(ii)	Attempt to connect dx and dt dx = t dt s.o.i. AEF	**M1** **A1**	But not just dx = dt. As **AG**, look carefully.
		$x+\sqrt{2x-1} \to \frac{t^2+1}{2}+t=\frac{(t+1)^2}{2}$ s.o.i.	**B1**	Any wrong working invalidates
		$\int\frac{2t}{(t+1)^2}\,dt$	**A1** 4	**AG** WWW The 'dt' must be present
	(iii)	$\int\frac{1}{t+1}\,dt = \ln(t+1)$	**B1**	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t + 1$
		$\int\frac{1}{(t+1)^2}\,dt = -\frac{1}{t+1}$	**B1**	
		Attempt to change limits (expect 1 & 3) and use f(t)	**M1**	or re-substitute and use 1 and 5 on g(x)
		$\ln 4 - \frac{1}{2}$	**A1** 4	AEF (like terms amalgamated); if A0 A0 in (i), then final A0

9	(i)	$A: \theta = \frac{1}{2}\pi$ (accept 90°)	B1	
		$B: \theta = 2\pi$ (accept 360°)	B2	SR If B0 awarded for point *B*, allow B1 SR for any angle s.t. $\sin\theta = 0$
			3	
	(ii)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1	or $\frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$ Must be used, not just quoted
		$\frac{dx}{d\theta} = 2 + 2\cos 2\theta$	B1	
		$2 + 2\cos 2\theta = 4\cos^2\theta$ with ≥ 1 line intermed work	*B1	
		$\frac{dy}{dx} = \frac{4\cos\theta}{2 + 2\cos 2\theta}$ s.o.i.	A1	This & previous line are interchangeable
		$= \sec\theta$	dep*A1	WWW **AG**
			5	
	(iii)	Equating $\sec\theta$ to 2 and producing at least one value of θ	M1	degrees or radians
		$(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$	A1	'Exact' form required
		$(y =) -2\sqrt{3}$	A1	'Exact' form required
			3	