OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Monday **20 JUNE 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

[Turn over

1 Find the quotient and the remainder when $x^4 + 3x^3 + 5x^2 + 4x - 1$ is divided by $x^2 + x + 1$. [4]

2 Evaluate $\int_0^{\frac{1}{2}\pi} x \cos x \, dx$, giving your answer in an exact form. [5]

3 The line L_1 passes through the points $(2, -3, 1)$ and $(-1, -2, -4)$. The line L_2 passes through the point $(3, 2, -9)$ and is parallel to the vector $4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

(i) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Prove that L_1 and L_2 are skew. [5]

4 (i) Show that the substitution $x = \tan\theta$ transforms $\int \frac{1}{(1 + x^2)^2} \, dx$ to $\int \cos^2\theta \, d\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1 + x^2)^2} \, dx$. [4]

5 $ABCD$ is a parallelogram. The position vectors of A, B and C are given respectively by

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \qquad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}, \qquad \mathbf{c} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}.$$

(i) Find the position vector of D. [3]

(ii) Determine, to the nearest degree, the angle ABC. [4]

6 The equation of a curve is $xy^2 = 2x + 3y$.

(i) Show that $\frac{dy}{dx} = \frac{2 - y^2}{2xy - 3}$. [5]

(ii) Show that the curve has no tangents which are parallel to the y-axis. [3]

7 A curve is given parametrically by the equations

$$x = t^2, \qquad y = \frac{1}{t}.$$

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [3]

(ii) Show that the equation of the tangent at the point $P\left(4, -\frac{1}{2}\right)$ is

$$x - 16y = 12.$$ [3]

(iii) Find the value of the parameter at the point where the tangent at P meets the curve again. [4]

8 **(i)** Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B and C. [5]

(ii) Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. [1]

9 Newton's law of cooling states that the rate at which the temperature of an object is falling at any instant is proportional to the difference between the temperature of the object and the temperature of its surroundings at that instant. A container of hot liquid is placed in a room which has a constant temperature of 20 °C. At time t minutes later, the temperature of the liquid is θ °C.

(i) Explain how the information above leads to the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 20),$$

where k is a positive constant. [2]

(ii) The liquid is initially at a temperature of 100 °C. It takes 5 minutes for the liquid to cool from 100 °C to 68 °C. Show that

$$\theta = 20 + 80e^{-(\frac{1}{5}\ln\frac{5}{3})t}.$$ [8]

(iii) Calculate how much longer it takes for the liquid to cool by a further 32 °C. [3]

1	(Quotient =) $x^2 + 2x + 2$ (Remainder =) $0x - 3$ Allow without working	B1 M1 A1 A1 **4**	For correct leading term x^2 in quotient For evidence of division/identity process For correct quotient For correct remainder. The '$0x$' need not be written but must be clearly derived. **4**
2	 $x \sin x - \int \sin x \, dx$ (= $x \sin x + \cos x$) Answer = ½ π – 1	M1 A1 B1 M1 A1 **5**	For attempt at parts going correct way (u = x, dv = cos x and f(x) +/–∫g(x) (dx) For both terms correct Indic anywhere that $\int \sin x \, dx = -\cos x$ For correct method of limits For correct exact answer ISW **5**
3	(i) **r** = (2**i**-3**j**+**k** or –**i**–2**j**–4**k**) + t(3**i**–**j**+5**k**) (ii) *L*(2) (**r**) = 3**i**+2**j**–9**k**+s(4**i** – 4**j** + 5**k**) *L(1)&L(2)* must be of form **r** = **a** + t**b** 2+3t=3+4s, –3–t=2–4s, 1+5t= – 9+5s or suitable equivalences (t,s) = (+/-3,2) or (–/+1,1) or (–/+9,-7) or (+/–4,2) or (0,1) or (–/+8,-7) Basic check other eqn & interp √	M1 A1 **2** M1 M1 M1 A1 B1 **5**	For (either point) + t(diff betw vectors) Completely correct including **r** =. AEF For point + (s or t) direction vector For 2/3 eqns with 2 different parameters For solving any relevant pair of eqns For both parameters correct **7**
4	(i) dx = sec²θ dθ AEF Indefinite integral = ∫ cos²θ dθ (ii) = k∫ +/– 1 +/– cos 2θ dθ ½[θ + ½ sin 2θ] Limits = ¼π(accept 45) and 0 (π + 2)/8 AEF	M1 A1 A1 **3** M1 A1 M1 A1 **4**	Attempt to connect dx,dθ (not dx = dθ) For dx = sec²θ dθ or equiv correctly used With at least one intermed step **AG** "Satis" attempt to change to double angle Correct attempt + correct integration New limits for θ or resubstituting Ignore decimals after correct answer **7** Single 'parts' + sin²θ=1–cos²θ acceptable
5	(i)**OD**=**OA**+**AD** or **OB**+**BC**+**CD** AEF **AD** = **BC** or **CD** = **BA** (**a** + **c** – **b**) = 2**j** + **k** (ii) **AB.CB** = |**AB**||**CB**| cos θ Scalar product of <u>any</u> 2 vectors Magnitude of <u>any</u> vector 94°(94.386...) or 1.65 (1.647...)	M1 A1 A1 **3** M1 M1 M1 A1 **4**	Connect **OD** & 2/3/4 vectors in their diag Or similar ,from their diag [i.e.if diag mislabelled, M1A1A0 possible] Or **AB.BC** i.e.scalar prod for correct pair 2 + 3 – 6 = – 1 is expected √19 or 3 expected Accept 86°(85.614...) or 1.49(424..) **7**
6	(i) For d/dx (y^2) = 2y dy/dx Using d(uv) = u dv + v du 2xy dy/dx + y^2 = 2 + 3 dy/dx dy/dx = (2 – y^2)/(2xy – 3)	B1 M1 A1 M1 A1 **5**	 Solving an equation,with at least 2 dy/dx terms, for dy/dx; dy/dx on one side, non dy/dx on other. **AG**

	(ii) Stating/using $2xy - 3 = 0$ Attempt to eliminate x or y $8x^2 = -9$ or $y^2 = -2$	B1 M1 A1 **3**	No use of 2 - y^2 in this part. Between $2xy - 3 = 0$ & eqn of curve Together with suitable finish **8**
7	(i)$dy / dx = (dy/dt) / (dx/dt)$ $= (-1/t^2) / 2t$ as unsimplified expression $= -1 / 2t^3$ as simplified expression (ii) $(4,-1/2) \rightarrow t = -2$ <u>only</u> Satis attempt to find equation of tgt $x - 16y = 12$ <u>only</u> (iii) $t^3 - 12t - 16 = 0$ <u>or</u> $16y^3 + 12y^2 - 1 = 0$ <u>or</u> $x^3 - 24x^2 + 144x - 256 = 0$ $t = 4$ (only) ISW giving cartesian coords	M1 A1 A1 **3** B1 M1 A1 **3** M1 A1 B2 **4**	(S.R.Award M1 for attempt to change to cartesian eqn & differentiate + A1 for dy/dx or dx/dy in terms of x or y) Not $1/-2t^3$. Not in terms of x &/or y. Using $t = -2$ or 2 **AG** For substituting $(t^2, 1/t)$ into tgt eqn <u>or</u> solving simult tgt & their cartes eqns For simplified equiv non-fract cubic S.R. Award B1 for "4 or –2". S.R. If B0, award M1 for clear indic of method of soln of correct eqn. **10**
8	(i) $3x+4 \equiv A(2+x)^2+B(2+x)(1+x) + C(1+x)$ $A = 1$ $C = 2$ $A+B = 0$ or $4A+3B+C=3$ or $4A+2B+C = 4$ $B = -1$ (ii) $1 - x + x^2$ $1 - \frac{1}{2}x + \frac{1}{4}x^2$ $1 - x$ $+ \frac{3}{4}x^2$ $1 - 5/4\,x + 5/4\,x^2$ (iii) $-1 < x < 1$ AEF	M1 A/B1 A/B1 A1 A1 **5** B1 B1 B1 B1 B1 **5** B1 **1**	Accept ≡ or = If identity used, award 'A' mark, if cover-up rule used, award 'B' mark. <u>Any</u> correct eqn for B from identity Expansion of $(1 + x)^{-1}$ Expansion of $(1 + \frac{1}{2}x)^{-1}$ First 2 terms of $(1 + \frac{1}{2}x)^{-2}$ Third term of $(1 + \frac{1}{2}x)^{-2}$ Complete correct expansion <u>If partial fractions not used</u> Award B1 for expansion of $(1+x)^{-1}$ B1+B1 for expansion of $(1 + \frac{1}{2}x)^{-2}$, and B1 for $1-5/4x$... & B1 for...$+5/4x^2$ <u>Or</u> if denom expanded to give $a+bx+cx^2$ with a=4.b=8,c=5,award B1 Expansion of $[1+(b/a\}x+(c/a)x^2]^{-1}$ = $1-(b/a)x+ \ldots (-c/a + b^2/a^2)x^2$ B1+B1 Final ans = $(1 - 5/4x\ldots + 5/4x^2)$B1+B1 Other inequalities to be discarded. **11**
9	k = const of proportionality $-$ = falling, $d\theta/dt$ = rate of change $\theta - 20$ = diff betw obj & surround temp (ii) $\int 1/(\theta - 20)\,d\theta = -k \int dt$ $\ln(\theta - 20) = -kt + c$ Subst $(\theta,t) = (100,0)$ or $(68,5)$	B2 **2** M1 A1A1 M1 A1	All 4 items (first two may be linked) S.R. Award B1 for any 2 items For separating variables For integ each side (c not essential) Dep on 'c' being involved [or M2 for limits (100,0) (68,5) + A1 for

	c = ln 80 k = 1/5 ln 5/3 $\theta = 20 + 80e^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}$	A1 M1 A1 **8**	k] *AG*
	(iii) Substitute θ = 68 – 32 *t* = 15.75 Extra time = 10.75, √their 15.75 – 5	M1 A1 B1 **3**	Subst into AEF of given eqn & solve Accept 15.7 or 15.8 f.t. only if θ = their (68 – 32) or 32 **13**