OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4723**

Core Mathematics 3

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Solve the inequality $|2x+1| > |x-1|$. [5]

2 **(i)** Prove the identity

$$\sin(x+30°)+(\sqrt{3})\cos(x+30°) \equiv 2\cos x\,,$$

where x is measured in degrees. [4]

(ii) Hence express $\cos 15°$ in surd form. [2]

3 The sequence defined by the iterative formula

$$x_{n+1} = \sqrt[3]{(17-5x_n)}\,,$$

with $x_1 = 2$, converges to α.

(i) Use the iterative formula to find α correct to 2 decimal places. You should show the result of each iteration. [3]

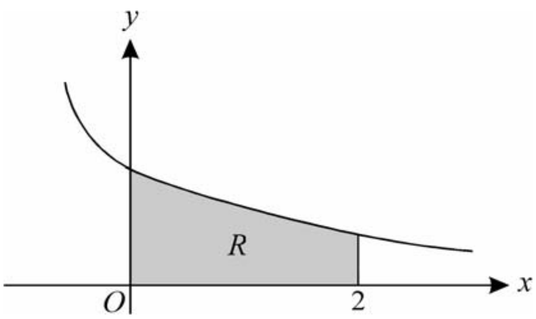
(ii) Find a cubic equation of the form

$$x^3 + cx + d = 0$$

which has α as a root. [2]

(iii) Does this cubic equation have any other real roots? Justify your answer. [2]

4



The diagram shows the curve

$$y = \frac{1}{\sqrt{(4x+1)}}\,.$$

The region R (shaded in the diagram) is enclosed by the curve, the axes and the line $x = 2$.

(i) Show that the exact area of R is 1. [4]

(ii) The region R is rotated completely about the x-axis. Find the exact volume of the solid formed. [4]

5 At time t minutes after an oven is switched on, its temperature θ °C is given by

$$\theta = 200 - 180\mathrm{e}^{-0.1t}.$$

(i) State the value which the oven's temperature approaches after a long time. [1]

(ii) Find the time taken for the oven's temperature to reach 150 °C. [3]

(iii) Find the rate at which the temperature is increasing at the instant when the temperature reaches 150 °C. [4]

6 The function f is defined by

$$\mathrm{f} : x \mapsto 1 + \sqrt{x} \qquad \text{for } x \geqslant 0.$$

(i) State the domain and range of the inverse function f^{-1}. [2]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(iii) By considering the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, show that the solution to the equation

$$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$$

is $x = \frac{1}{2}(3 + \sqrt{5})$. [4]

7 **(i)** Write down the formula for $\tan 2x$ in terms of $\tan x$. [1]

(ii) By letting $\tan x = t$, show that the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

becomes

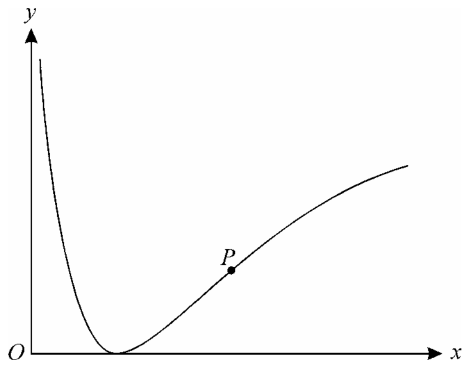
$$3t^4 - 8t^2 - 3 = 0.$$ [4]

(iii) Hence find all the solutions of the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

which lie in the interval $0 \leqslant x \leqslant 2\pi$. [4]

8

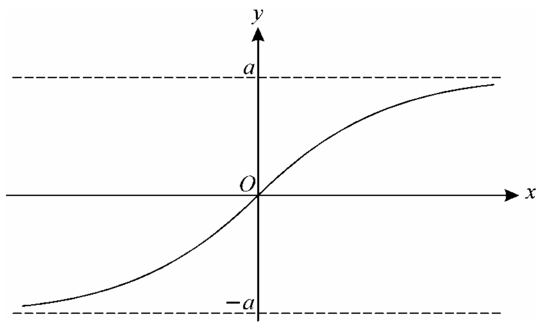


The diagram shows the curve $y = (\ln x)^2$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) The point P on the curve is the point at which the gradient takes its maximum value. Show that the tangent at P passes through the point $(0, -1)$. [6]

9



The diagram shows the curve $y = \tan^{-1} x$ and its asymptotes $y = \pm a$.

(i) State the exact value of a. [1]

(ii) Find the value of x for which $\tan^{-1} x = \frac{1}{2}a$. [2]

The equation of another curve is $y = 2\tan^{-1}(x-1)$.

(iii) Sketch this curve on a copy of the diagram, and state the equations of its asymptotes in terms of a. [3]

(iv) Verify by calculation that the value of x at the point of intersection of the two curves is 1.54, correct to 2 decimal places. [2]

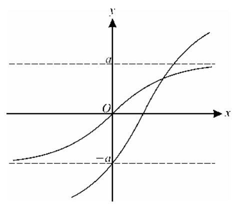
Another curve (which you are *not* asked to sketch) has equation $y = \left(\tan^{-1} x\right)^2$.

(v) Use Simpson's rule, with 4 strips, to find an approximate value for $\int_0^1 \left(\tan^{-1} x\right)^2 \, dx$. [3]

1 *EITHER*: $4x^2+4x+1 > x^2-2x+1$	M1	For squaring both sides
i.e. $3x^2+6x>0$	A1	For reduction to correct quadratic
So $x(x+2)>0$	M1	For factorising, or equivalent
Hence $x<-2$ or $x>0$	A1	For both critical values correct
	A1	For completely correct solution set
OR: Critical values where $2x+1=\pm(x-1)$	M1	For considering both cases, or from graphs
i.e. where $x=-2$ and $x=0$	B1	For the correct value -2
	A1	For the correct value 0
Hence $x<-2$ or $x>0$	M1	For any correct method for solution set using two critical values
	A1 **5**	For completely correct solution set
	[5]	
2 **(i)** $\sin x(\frac{1}{2}\sqrt{3})+\cos x(\frac{1}{2})+(\sqrt{3})(\cos x(\frac{1}{2}\sqrt{3})-\sin x(\frac{1}{2}))$	M1	For expanding both compound angles
	A1	For completely correct expansion
	M1	For using exact values of $\sin 30°$ and $\cos 30°$
$=\frac{1}{2}\cos x+\frac{3}{2}\cos x=2\cos x$, as required	A1 **4**	For showing given answer correctly
(ii) $\sin 45°+(\sqrt{3})\cos 45°=2\cos 15°$	M1	For letting $x=15°$ throughout
Hence $\cos 15°=\dfrac{1+\sqrt{3}}{2\sqrt{2}}$	A1 **2**	For any correct exact form
	[6]	
3 **(i)** $x_2=\sqrt[3]{7}=1.9129...$	B1	For 1.91… seen or implied
$x_3=1.9517...,\quad x_4=1.9346...$	M1	For continuing the correct process
$\alpha=1.94$ to 2dp	A1 **3**	For correct value reached, following x_5 and x_6 both 1.94 to 2dp
(ii) $x=\sqrt[3]{(17-5x)}\Rightarrow x^3+5x-17=0$	M1	For letting $x_n=x_{n+1}=x$ (or α)
	A1 **2**	For correct equation stated
(iii) *EITHER*: Graphs of $y=x^3$ and $y=17-5x$ only cross once	M1	For argument based on sketching a pair of graphs, or a sketch of the cubic by calculator
Hence there is only one real root	A1√	For correct conclusion for a valid reason
OR: $\dfrac{d}{dx}(x^3+5x-17)=3x^2+5>0$	M1	For consideration of the cubic's gradient
Hence there is only one real root	A1√ **2**	For correct conclusion for a valid reason
	[7]	
4 **(i)** $\int_0^2 (4x+1)^{-\frac{1}{2}}\,dx=\left[\frac{1}{2}(4x+1)^{\frac{1}{2}}\right]_0^2=\frac{1}{2}(3-1)=1$	M1	For integral of the form $k(4x+1)^{\frac{1}{2}}$
	A1	For correct indefinite integral
	M1	For correct use of limits
	A1 **4**	For given answer correctly shown
(ii) $\pi\int_0^2 \dfrac{1}{4x+1}\,dx=\pi\left[\frac{1}{4}\ln(4x+1)\right]_0^2=\frac{1}{4}\pi\ln 9$	M1	For integral of the form $k\ln(4x+1)$
	A1	For correct $\frac{1}{4}\ln(4x+1)$, with or without π
	M1	Correct use of limits and π
	A1 **4**	For correct (simplified) exact value
	[8]	

5	**(i)** 200 °C	B1	**1**	For value 200
	(ii) $150 = 200 - 180\mathrm{e}^{-0.1t} \Rightarrow \mathrm{e}^{-0.1t} = \frac{50}{180}$	M1		For isolating the exponential term
	Hence $-0.1t = \ln\frac{5}{18} \Rightarrow t = 12.8$	M1		For taking logs correctly
		A1	**3**	For correct value 12.8 (minutes)
	(iii) $\frac{\mathrm{d}\theta}{\mathrm{d}t} = 18\mathrm{e}^{-0.1t}$	M1		For differentiation attempt
		A1		For correct derivative
	Hence rate is $18\mathrm{e}^{-0.1\times 12.8} = 5.0$ °C per minute	M1		For using their value from **(ii)** in their $\dot{\theta}$
		A1	**4**	For value 5.0(0)
			8	
6	**(i)** Domain of f^{-1} is $x \geqslant 1$	B1		For the correct set, in any notation
	Range is $x \geqslant 0$	B1	**2**	Ditto
	(ii) If $y = 1 + \sqrt{x}$, then $x = (y-1)^2$	M1		For changing the subject, or equivalent
	Hence $\mathrm{f}^{-1}(x) = (x-1)^2$	A1	**2**	For correct expression in terms of x
	(iii) The graphs intersect on the line $y = x$	B1		For stating or using this fact
	Hence x satisfies $x = (x-1)^2$	B1		For either $x = \mathrm{f}(x)$ or $x = \mathrm{f}^{-1}(x)$
	i.e. $x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$	M1		For solving the relevant quadratic equation
	So $x = \frac{1}{2}(3 + \sqrt{5})$ as x must be greater than 1	A1	**4**	For showing the given answer fully
			8	
7	**(i)** $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	B1	**1**	For correct RHS stated
	(ii) $\frac{8t}{1-t^2} + 3 \times \frac{1}{t} \times (1+t^2) = 0$	B1		For $\cot x = \frac{1}{t}$ seen
		B1		For $\sec^2 x = 1 + t^2$ seen
	Hence $8t^2 + 3(1-t^2)(1+t^2) = 0$	M1		For complete substitution in terms of t
	i.e. $3t^4 - 8t^2 - 3 = 0$, as required	A1	**4**	For showing given equation correctly
	(iii) $(3t^2 + 1)(t^2 - 3) = 0$	M1		For factorising or other solution method
	Hence $t = \pm\sqrt{3}$	A1		For $t^2 = 3$ found correctly
	So $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$	A1		For any two correct angles
		A1	**4**	For all four correct and no others
			9	

[Turn over

8	(i)	$\frac{dy}{dx} = \frac{2\ln x}{x}$	M1		For relevant attempt at the chain rule
			A1		For correct result, in any form
		$\frac{d^2y}{dx^2} = \frac{x(2/x) - 2\ln x}{x^2} = \frac{2 - 2\ln x}{x^2}$	M1		For relevant attempt at quotient rule
			A1	4	For correct simplified answer
	(ii)	For maximum gradient, $2 - 2\ln x = 0 \Rightarrow x = e$	M1		For equating second derivative to zero
			A1		For correct value e
		Hence P is $(e, 1)$	A1√		For stating or using the y-coordinate
		The gradient at P is $\frac{2}{e}$	A1√		For stating or using the gradient at P
		Tangent at P is $y - 1 = \frac{2}{e}(x - e)$	M1		For forming the equation of the tangent
		Hence, when $x = 0$, $y = -1$ as required	A1	6	For correct verification of $(0, -1)$
				10	
9	(i)	$a = \frac{1}{2}\pi$	B1	1	For correct exact value stated
	(ii)	$x = \tan(\frac{1}{4}\pi) = 1$	M1		For use of $x = \tan(\frac{1}{2}a)$
			A1√	2	For correct answer, following their a
	(iii)		B1		For x-translation of (approx) +1
			B1		For y-stretch with (approx) factor 2
		Asymptotes are $y = \pm 2a$	B1	3	For correct statement of asymptotes
		x $\tan^{-1} x$ $2\tan^{-1}(x-1)$			
	(iv)	1.535 0.993 0.983	M1		For relevant evaluations at 1.535, 1.545
		1.545 0.996 0.998			
		Hence graphs cross between 1.535 and 1.545	A1	2	For correct details and explanation
	(v)	Relevant values of $\left(\tan^{-1} x\right)^2$ are (approximately)	M1		For the relevant function values seen or
		0, 0.0600, 0.2150, 0.4141, 0.6169			implied; must be radians, not degrees
		$\frac{1}{12}\{0 + 4(0.0600 + 0.4141) + 2\times 0.2150 + 0.6169\}$	M1		For use of correct formula with $h = \frac{1}{4}$
		Hence required approximation is 0.245	A1	3	For correct (2 or 3sf) answer
				4	