OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4723**

Core Mathematics 3

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

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1 Find

(i) $\int 6e^{2x+1}\,dx$,

(ii) $\int 10(2x+1)^{-1}\,dx$. **[5]**

2 The curve $y = \ln x$ is transformed by:

a reflection in the x-axis,
followed by a stretch with scale factor 3 parallel to the y-axis,
followed by a translation in the positive y-direction by $\ln 4$.

Find the equation of the resulting curve, giving your answer in the form $y = \ln(f(x))$. **[4]**

3 **(a)** Given that $7\sin 2\alpha = 3\sin\alpha$, where $0° < \alpha < 90°$, find the exact value of $\cos\alpha$. **[3]**

(b) Given that $3\cos 2\beta + 19\cos\beta + 13 = 0$, where $90° < \beta < 180°$, find the exact value of $\sec\beta$. **[5]**

4 **(i)** Show by means of suitable sketch graphs that the equation

$$(x-2)^4 = x + 16$$

has exactly 2 real roots. **[3]**

(ii) State the value of the smaller root. **[1]**

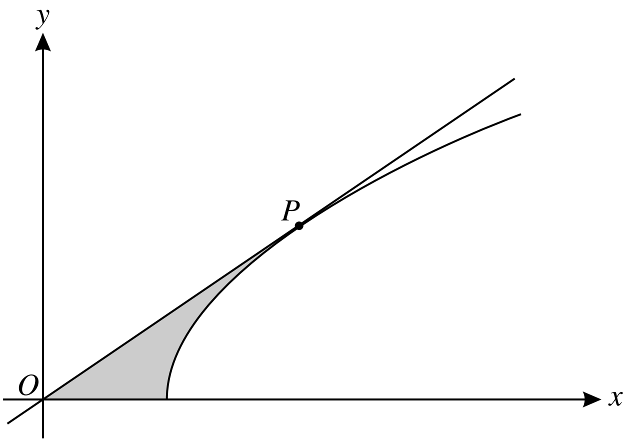
(iii) Use the iterative formula

$$x_{n+1} = 2 + \sqrt[4]{x_n + 16},$$

with a suitable starting value, to find the larger root correct to 3 decimal places. **[4]**

5 The equation of a curve is $y = x^2\ln(4x-3)$. Find the exact value of $\frac{d^2y}{dx^2}$ at the point on the curve for which $x = 2$. **[8]**

6



The diagram shows the curve with equation $y = \sqrt{3x - 5}$. The tangent to the curve at the point P passes through the origin. The shaded region is bounded by the curve, the x-axis and the line OP. Show that the x-coordinate of P is $\frac{10}{3}$ and hence find the exact area of the shaded region. **[9]**

7 The functions f, g and h are defined for all real values of x by

$$\mathrm{f}(x) = |x|, \qquad \mathrm{g}(x) = 3x + 5 \qquad \text{and} \qquad \mathrm{h}(x) = \mathrm{gg}(x).$$

(i) Solve the equation $\mathrm{g}(x + 2) = \mathrm{f}(-12)$. **[3]**

(ii) Find $\mathrm{h}^{-1}(x)$. **[3]**

(iii) Determine the values of x for which

$$x + \mathrm{f}(x) = 0.$$ **[2]**

8 An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400\mathrm{e}^{-0.014t}.$$

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

(i) Find the rate at which the mass of Substance 1 is decreasing when $t = 10$, giving your answer in grams per hour correct to 2 significant figures. **[3]**

(ii) Show that T is the root of an equation of the form $\mathrm{e}^{kt} = c$, where the values of the constants k and c are to be stated. **[5]**

(iii) Hence find the value of T correct to 3 significant figures. **[2]**

[Question 9 is printed overleaf.]

9 **(i)** Prove that $\dfrac{\sin(\theta - \alpha) + 3\sin\theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3\cos\theta + \cos(\theta + \alpha)} \equiv \tan\theta$ for all values of α. **[5]**

(ii) Find the exact value of $\dfrac{4\sin 149^\circ + 12\sin 150^\circ + 4\sin 151^\circ}{3\cos 149^\circ + 9\cos 150^\circ + 3\cos 151^\circ}$. **[3]**

(iii) It is given that k is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of k, the equation

$$\frac{\sin(6\theta - 15^\circ) + 3\sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3\cos 6\theta + \cos(6\theta + 15^\circ)} = k.$$ **[4]**

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1	**(i)** Obtain integral of form ke^{2x+1}	M1		any non-zero constant *k* different from 6; using substitution $u = 2x+1$ to obtain ke^u earns M1 (but answer to be in terms of x)
	Obtain correct $3e^{2x+1}$	A1		or equiv such as $\frac{6}{2}e^{2x+1}$
	(ii) Obtain integral of form $k_1 \ln(2x+1)$	M1		any non-zero constant k_1; allow if brackets absent; $k_1 \ln u$ (after sub'n) earns M1
	Obtain correct $5\ln(2x+1)$	A1		or equiv such as $\frac{10}{2}\ln(2x+1)$; condone brackets rather than modulus signs but brackets or modulus signs must be present (so that $5\ln 2x+1$ earns A0)
	Include … + *c* at least once	B1	**5**	anywhere in the whole of question 1; this mark available even if no marks awarded for integration
			[5]	
2	Apply one of the transformations correctly to their equation	B1		
	Obtain correct $-3\ln x + \ln 4$	B1		or equiv
	Show at least one logarithm property	M1		correctly applied to their equation of resulting curve (even if errors have been made earlier)
	Obtain $y = \ln(4x^{-3})$	A1	**4**	or equiv of required form; $\ln 4x^{-3}$ earns A1; correct answer only earns 4/4; condone absence of $y =$
			[4]	
3	**(a)** State $14\sin\alpha\cos\alpha = 3\sin\alpha$	B1		or unsimplified equiv such as $7(2\sin\alpha\cos\alpha) = 3\sin\alpha$
	Attempt to find value of $\cos\alpha$	M1		by valid process; may be implied
	Obtain $\frac{3}{14}$	A1	**3**	exact answer required; ignore subsequent work to find angle
	(b) Attempt use of identity for $\cos 2\beta$	M1		of form $\pm 2\cos^2\beta \pm 1$; initial use of $\cos^2\beta - \sin^2\beta$ needs attempt to express $\sin^2\beta$ in terms of $\cos^2\beta$ to earn M1
	Obtain $6\cos^2\beta + 19\cos\beta + 10$	A1		or unsimplified equiv or equiv involving $\sec\beta$
	Attempt solution of 3-term quadratic eqn	M1		for $\cos\beta$ or (after adjustment) for $\sec\beta$
	Use $\sec\beta = \dfrac{1}{\cos\beta}$ at some stage	M1		or equiv
	Obtain $-\frac{3}{2}$	A1	**5**	or equiv; and (finally) no other answer
			[8]	

4 **(i)**	Draw sketch of $y=(x-2)^4$	*B1		touching positive x-axis and extending at least as far as the y-axis; no need for 2 or 16 to be marked; ignore wrong intercepts
	Draw straight line with positive gradient	*B1		at least in first quadrant and reaching positive y-axis; assess the two graphs independently of each other
	Indicate two roots	B1	**3**	AG; dep *B *B and two correct graphs which meet on the y-axis; indicated in words or by marks on sketch
	[SC: Draw sketch of $y=(x-2)^4-x-16$ and indicate the two roots : B1 (i.e. max 1 mark)]			
(ii)	State 0 or $x=0$	B1	**1**	not merely for coordinates (0, 16)
(iii)	Obtain correct first iterate	B1		to at least 3 dp; any starting value (> –16)
	Show correct iteration process	M1		producing at least 3 iterates in all; may be implied by plausible converging values
	Obtain at least 3 correct iterates	A1		allowing recovery after error; iterates given to only 3 d.p. acceptable; values may be rounded or truncated
	Obtain 4.118	A1	**4**	answer required to exactly 3 dp; A0 here if number of iterates is not enough to justify 4.118; attempt consisting of answer only earns 0/4

$$
\begin{aligned}
[\,0 &\to 4 \to 4.114743 \to 4.117769 \to 4.117849\,; \\
1 &\to 4.030543 \to 4.115549 \to 4.117790 \to 4.117849\,; \\
2 &\to 4.059767 \to 4.116321 \to 4.117811 \to 4.117850\,; \\
3 &\to 4.087798 \to 4.117060 \to 4.117830 \to 4.117850\,; \\
4 &\to 4.114743 \to 4.117769 \to 4.117849 \to 4.117851\,; \\
5 &\to 4.140695 \to 4.118452 \to 4.117867 \to 4.117851\,]
\end{aligned}
$$

8

5	Attempt use of product rule	*M1		to produce $k_1 x\ln(4x-3)+\dfrac{k_2x^2}{4x-3}$ form
	Obtain $2x\ln(4x-3)$	A1		
	Obtain $\ldots+\dfrac{4x^2}{4x-3}$	A1		or equiv
	Attempt second use of product rule	*M1		
	Attempt use of quotient (or product) rule	*M1		allow numerator the wrong way round
	Obtain $2\ln(4x-3)+\dfrac{8x}{4x-3}+\dfrac{8x(4x-3)-16x^2}{(4x-3)^2}$	A1		or equiv
	Substitute 2 into attempt at second deriv	M1		dep *M *M *M
	Obtain $2\ln 5+\frac{96}{25}$	A1	**8**	or exact equiv consisting of two terms

8

6 Method 1: (Differentiation; assume value $\frac{10}{3}$; eqn of tangent; through origin)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Attempt to find equation of tangent at P and attempt to show tangent passing through origin	M1	assuming value $\frac{10}{3}$; or equiv
Obtain $y=\frac{3}{2\sqrt{5}}x$ and confirm that tangent passes through O	A1	AG; necessary detail needed

Method 2: (Differentiation; equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv; solve for x)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv and attempt solution	M1	
Obtain $\frac{\sqrt{3x-5}}{x}=\frac{3}{2}(3x-5)^{-\frac{1}{2}}$ and solve to obtain $\frac{10}{3}$ only	A1	

Method 3: (Differentiation; find x from $y=\mathrm{f}'(x)\,x$ and $y=\sqrt{3x-5}$)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
State $y=\frac{3}{2}(3x-5)^{-\frac{1}{2}}x$, $y=\sqrt{3x-5}$, eliminate y and attempt solution	M1	condone this attempt at 'eqn of tangent'
Obtain $\frac{10}{3}$ only	A1	

Method 4: (No differentiation; general line through origin to meet curve at one point only)

Eliminate y from equations $y=kx$ and $y=\sqrt{3x-5}$ and attempt formation of quadratic eqn	M1	
Obtain $k^2x^2-3x+5=0$	A1	or equiv
Equate discriminant to zero to find k	M1	
Obtain $k=\frac{3}{2\sqrt{5}}$ or equiv and confirm $x=\frac{10}{3}$	A1	

Method 5: (No differentiation; use coords of P to find eqn of OP; confirm meets curve once)

Use coordinates $(\frac{10}{3},\sqrt{5})$ to obtain $y=\frac{3\sqrt{5}}{10}x$ or equiv as equation of OP	B1	
Eliminate y from this eqn and eqn of curve and attempt quadratic eqn	M1	should be $9x^2-60x+100=0$ or equiv
Attempt solution or attempt discriminant	M1	
Confirm $\frac{10}{3}$ only or discriminant = 0	A1	

Either:

	Integrate to obtain $k(3x-5)^{\frac{3}{2}}$	*M1		any constant k
	Obtain correct $\frac{2}{9}(3x-5)^{\frac{3}{2}}$	A1		
	Apply limits $\frac{5}{3}$ and $\frac{10}{3}$	M1		dep *M; the right way round
	Make sound attempt at triangle area and calculate (triangle area) minus (their area under curve)	M1		or equiv
	Obtain $\frac{10}{6}\sqrt{5}-\frac{10}{9}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	**9**	or exact equiv involving single term
	Or:			
	Arrange to $x = \ldots$ and integrate to obtain $k_1y^3+k_2y$ form	*M1		
	Obtain $\frac{1}{9}y^3+\frac{5}{3}y$	A1		
	Apply limits 0 and $\sqrt{5}$	M1		dep *M; the right way round
	Make sound attempt at triangle area and calculate (their area from integration) minus (triangle area)	M1		
	Obtain $\frac{20}{9}\sqrt{5}-\frac{5}{3}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	(**9**)	or exact equiv involving single term
			9	

7 (i)	Either: Attempt solution of at least one linear eq'n of form $ax+b=12$	M1		
	Obtain $\frac{1}{3}$	A2	**3**	and (finally) no other answer
	Or: Attempt solution of 3-term quadratic eq'n obtained by squaring attempt at $\mathrm{g}(x+2)$ on LHS and squaring 12 or –12 on RHS	M1		
	Obtain $\frac{1}{3}$	A2	(**3**)	and (finally) no other answer
(ii)	Either: Obtain $3(3x+5)+5$ for h	B1		
	Attempt to find inverse function	M1		of function of form $ax+b$
	Obtain $\frac{1}{9}(x-20)$	A1	**3**	or equiv in terms of x
	Or: State or imply g^{-1} is $\frac{1}{3}(x-5)$	B1		
	Attempt composition of g^{-1} with g^{-1}	M1		
	Obtain $\frac{1}{9}(x-5)-\frac{5}{3}$	A1	(**3**)	or more simplified equiv in terms of x
(iii)	State $x \le 0$	B2	**2**	give B1 for answer $x<0$
			8	

8	**(i)**	Differentiate to obtain form $ke^{-0.014t}$	M1		any constant k different from 400
		Obtain $5.6e^{-0.014t}$ or $-5.6e^{-0.014t}$	A1		or (unsimplified) equiv
		Obtain 4.9 or –4.9 or 4.87 or –4.87	A1	**3**	but not greater accuracy; allow if final statement seems contradictory; answer only earns 0/3 – differentiation is needed
	(ii)	Either: State or imply $M_2 = 75e^{kt}$	B1		or equiv
		Attempt to find formula for M_2	M1		
		Obtain $M_2 = 75e^{0.047t}$	A1		or equiv such as $75e^{(\frac{1}{10}\ln\frac{8}{5})t}$
		Equate masses and attempt rearrangement	M1		as far as equation with e appearing once
		Obtain $e^{0.061t} = \frac{16}{3}$	A1	**5**	or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
		Or: State or imply $M_2 = 75 \times r^{0.1t}$	B1		for positive value r
		Obtain $75 \times 1.6^{0.1t}$	B1		
		Attempt to find M_2 in terms of e	M1		
		Equate masses and attempt rearrangement	M1		
		Obtain $e^{0.061t} = \frac{16}{3}$	A1	**5**	or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
	(iii)	Attempt solution involving logarithm of any equation of form $e^{mt} = c_1$	M1		whether the conclusion of part ii or not
		Obtain 27.4	A1	**2**	or greater accuracy 27.4422…; correct answer only earns both marks
				10	

9	(i)	Use at least one identity correctly	B1		angle-sum or angle-difference identity
		Attempt use of relevant identities in single rational expression	M1		not earned if identities used in expression where step equiv to $\frac{A+B+C}{D+E+F}=\frac{A}{D}+\frac{B}{E}+\frac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos\theta\cos\alpha-\sin\theta\sin\alpha+3\cos\theta+\cos\theta\cos\alpha+\sin\theta\sin\alpha$)
		Obtain $\frac{2\sin\theta\cos\alpha+3\sin\theta}{2\cos\theta\cos\alpha+3\cos\theta}$	A1		or equiv but with the other two terms from each of num'r and den'r absent
		Attempt factorisation of num'r and den'r	M1		
		Obtain $\frac{\sin\theta}{\cos\theta}$ and hence $\tan\theta$	A1	5	AG; necessary detail needed
	(ii)	State or imply form $k\tan150°$	M1		obtained without any wrong method seen
		State or imply $\frac{4}{3}\tan150°$	A1		or equiv such as $\frac{12\sin150°}{9\cos150°}$
		Obtain $-\frac{4}{9}\sqrt{3}$	A1	3	or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3
	(iii)	State or imply $\tan6\theta=k$	B1		
		State $\frac{1}{6}\tan^{-1}k$	B1		
		Attempt second value of θ	M1		using $6\theta=\tan^{-1}k+$ (multiple of 180)
		Obtain $\frac{1}{6}\tan^{-1}k+30°$	A1	4	and no other value
				12	