OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4723

Core Mathematics 3

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

 [Turn over

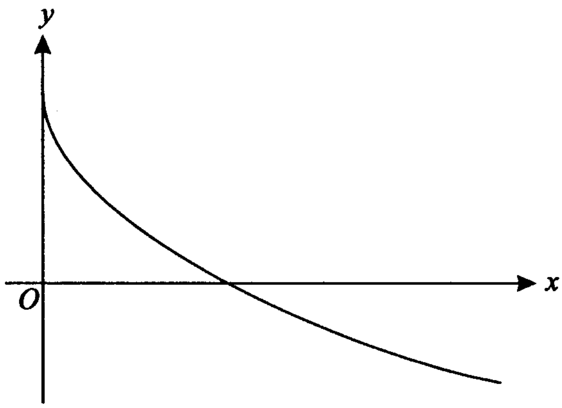
1 Show that $\int_2^8 \frac{3}{x}\,dx = \ln 64$. [4]

2 Solve, for $0° < \theta < 360°$, the equation $\sec^2\theta = 4\tan\theta - 2$. [5]

3 **(a)** Differentiate $x^2(x+1)^6$ with respect to x. [3]

(b) Find the gradient of the curve $y = \frac{x^2+3}{x^2-3}$ at the point where $x = 1$. [3]

4



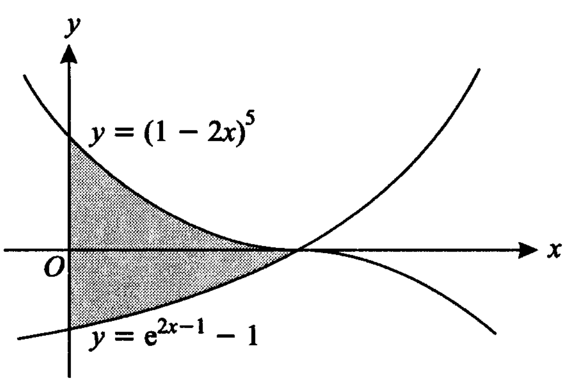
The function f is defined by $f(x) = 2 - \sqrt{x}$ for $x \geqslant 0$. The graph of $y = f(x)$ is shown above.

(i) State the range of f. [1]

(ii) Find the value of $ff(4)$. [2]

(iii) Given that the equation $|f(x)| = k$ has two distinct roots, determine the possible values of the constant k. [2]

5



The diagram shows the curves $y = (1-2x)^5$ and $y = e^{2x-1} - 1$. The curves meet at the point $(\frac{1}{2}, 0)$. Find the exact area of the region (shaded in the diagram) bounded by the y-axis and by part of each curve. [8]

6 **(a)**

t	0	10	20
X	275	440	

The quantity X is increasing exponentially with respect to time t. The table above shows values of X for different values of t. Find the value of X when $t = 20$. [3]

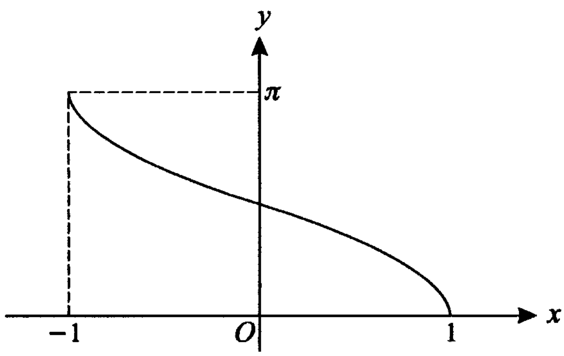
(b) The quantity Y is decreasing exponentially with respect to time t where

$$Y = 80e^{-0.02t}.$$

(i) Find the value of t for which $Y = 20$, giving your answer correct to 2 significant figures. [3]

(ii) Find by differentiation the rate at which Y is decreasing when $t = 30$, giving your answer correct to 2 significant figures. [3]

7



The diagram shows the curve with equation $y = \cos^{-1} x$.

(i) Sketch the curve with equation $y = 3\cos^{-1}(x - 1)$, showing the coordinates of the points where the curve meets the axes. [3]

(ii) By drawing an appropriate straight line on your sketch in part **(i)**, show that the equation $3\cos^{-1}(x - 1) = x$ has exactly one root. [1]

(iii) Show by calculation that the root of the equation $3\cos^{-1}(x - 1) = x$ lies between 1.8 and 1.9. [2]

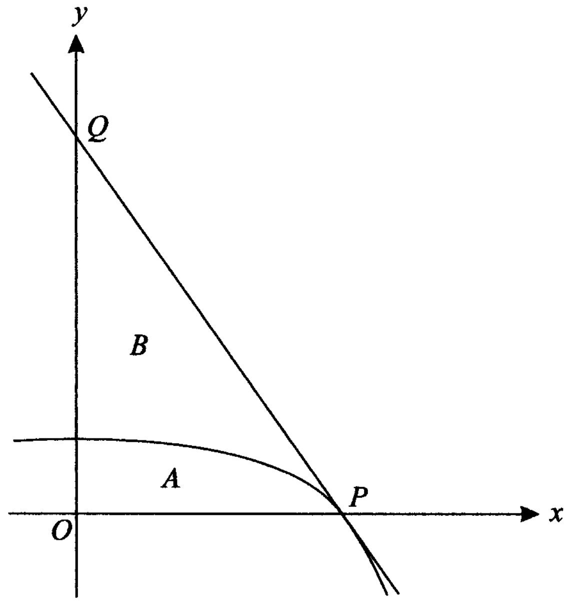
(iv) The sequence defined by

$$x_1 = 2, \qquad x_{n+1} = 1 + \cos\left(\tfrac{1}{3}x_n\right)$$

converges to a number α. Find the value of α correct to 2 decimal places and explain why α is the root of the equation $3\cos^{-1}(x - 1) = x$. [5]

[Questions 8 and 9 are printed overleaf.]

8



The diagram shows part of the curve $y = \ln(5 - x^2)$ which meets the x-axis at the point P with coordinates (2, 0). The tangent to the curve at P meets the y-axis at the point Q. The region A is bounded by the curve and the lines $x = 0$ and $y = 0$. The region B is bounded by the curve and the lines PQ and $x = 0$.

(i) Find the equation of the tangent to the curve at P. [5]

(ii) Use Simpson's Rule with four strips to find an approximation to the area of the region A, giving your answer correct to 3 significant figures. [4]

(iii) Deduce an approximation to the area of the region B. [2]

9 **(i)** By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta. \quad [4]$$

(ii) Determine the greatest possible value of

$$9\sin\left(\tfrac{10}{3}\alpha\right) - 12\sin^3\left(\tfrac{10}{3}\alpha\right),$$

and find the smallest positive value of α (in degrees) for which that greatest value occurs. [3]

(iii) Solve, for $0° < \beta < 90°$, the equation $3\sin 6\beta \operatorname{cosec} 2\beta = 4$. [6]

1		Obtain integral of form $k \ln x$	M1		[any non-zero constant k; or equiv such as $k \ln 3x$]
		Obtain $3 \ln 8 - 3 \ln 2$	A1		[or exact equiv]
		Attempt use of at least one relevant log property	M1		[would be earned by initial $\ln x^3$]
		Obtain $3 \ln 4$ or $\ln 8^3 - \ln 2^3$ and hence $\ln 64$	A1	4	[AG; with no errors]
2		Attempt use of identity linking $\sec^2 \theta$, $\tan^2 \theta$ and 1	M1		[to write eqn in terms of $\tan \theta$]
		Obtain $\tan^2 \theta - 4 \tan \theta + 3 = 0$	A1		[or correct unsimplified equiv]
		Attempt solution of quadratic eqn to find two values of $\tan \theta$	M1		[any 3 term quadratic eqn in $\tan \theta$]
		Obtain at least two correct answers	A1		[after correct solution of eqn]
		Obtain all four of 45, 225, 71.6, 251.6	A1	5	[allow greater accuracy or angles to nearest degree – and no other answers between 0 and 360]
3	(a)	Attempt use of product rule	M1		[involving ... + ...]
		Obtain $2x(x+1)^6$ ...	A1		
		Obtain ... $+ 6x^2(x+1)^5$	A1	3	[or equivs; ignore subsequent attempt at simplification]
	(b)	Attempt use of quotient rule	M1		[or, with adjustment, product rule; allow u / v confusion]
		Obtain $\dfrac{(x^2-3)2x-(x^2+3)2x}{(x^2-3)^2}$	A1		[or equiv]
		Obtain -3	A1	3	[from correct derivative only]
4	(i)	State $y \le 2$	B1	1	[or equiv; allow <; allow any letter or none]
	(ii)	Show correct process for composition of functions	M1		[numerical or algebraic]
		Obtain 0 and hence 2	A1	2	[and no other value]
	(iii)	State a range of values with 2 as one end-point	M1		[continuous set, not just integers]
		State $0 < k \le 2$	A1	2	[with correct < and ≤ now]
5		Obtain integral of form $k(1-2x)^6$	M1		[any non-zero constant k]
		Obtain correct $-\frac{1}{12}(1-2x)^6$	A1		[or unsimplified equiv; allow $+ c$]
		Use limits to obtain $\frac{1}{12}$	A1		[or exact (unsimplified) equiv]
		Obtain integral of form $k e^{2x-1}$	M1		[or equiv; any non-zero constant k]
		Obtain correct $\frac{1}{2}e^{2x-1} - x$	A1		[or equiv; allow $+ c$]
		Use limits to obtain $-\frac{1}{2}e^{-1}$	A1		[or exact (unsimplified) equiv]
		Show correct process for finding required area	M1		[at any stage of solution; if process involves two definite integrals, second must be negative]
		Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	A1	8	[or exact equiv; no $+ c$]

6 (a)	Either: State proportion $\frac{440}{275}$	B1		
	Attempt calculation involving proportion	M1		[involving multn and X value]
	Obtain 704	A1	3	
	Or: Use formula of form $275e^{kt}$ or $275a^t$	M1		[or equiv]
	Obtain $k = 0.047$ or $a = \sqrt[10]{1.6}$	A1		[or equiv]
	Obtain 704	A1	(3)	[allow ±0.5]
(b)(i)	Attempt correct process involving logarithm	M1		[or equiv including systematic trial and improvement attempt]
	Obtain $\ln \frac{20}{80} = -0.02t$	A1		[or equiv]
	Obtain 69	A1	3	[or greater accuracy; scheme for T&I: M1A2]
(ii)	Differentiate to obtain $k\,e^{-0.02t}$	M1		[any constant k different from 80]
	Obtain $-1.6e^{-0.02t}$ (or $1.6e^{-0.02t}$)	A1		[or unsimplified equiv]
	Obtain 0.88	A1	3	[or greater accuracy; allow −0.88]

7 (i)	Sketch curve showing (at least) translation in x direction	M1		[either positive or negative]
	Show correct sketch with one of 2 and 3π indicated	A1		
	... and with other one of 2 and 3π indicated	A1	3	
(ii)	Draw straight line through O with positive gradient	B1	1	[label and explanation not required]
(iii)	Attempt calculations using 1.8 and 1.9	M1		[allow here if degrees used]
	Obtain correct values and indicate change of sign	A1	2	[or equiv; $x = 1.8$: LHS = 1.93, diff = 0.13; $x = 1.9$: LHS = 1.35, diff = -0.55; radians needed now]
(iv)	Obtain correct first iterate 1.79 or 1.78	B1		[or greater accuracy]
	Attempt correct process to produce at least 3 iterates	M1		
	Obtain 1.82	A1		[answer required to exactly 2 d.p.; $2 \to 1.7859 \to 1.8280 \to 1.8200$; SR: answer 1.82 only - B2]
	Attempt rearrangement of $3\cos^{-1}(x-1) = x$ or of $x = 1 + \cos(\frac{1}{3}x)$	M1		[involving at least two steps]
	Obtain required formula or equation respectively	A1	5	

8	(i)	Differentiate to obtain $kx(5-x^2)^{-1}$	M1		[any non-zero constant]
		Obtain correct $-2x(5-x^2)^{-1}$	A1		[or equiv]
		Obtain –4 for value of derivative	A1		
		Attempt equation of straight line through (2, 0) with numerical value of gradient obtained from attempt at derivative	M1		[not for attempt at eqn of normal]
		Obtain $y = -4x + 8$	A1	5	[or equiv]
	(ii)	State or imply $h = \frac{1}{2}$	B1		
		Attempt calculation involving attempts at y values	M1		[addition with each of coefficients 1, 2, 4 occurring at least once]
		Obtain $k(\ln 5 + 4\ln 4.75 + 2\ln 4 + 4\ln 2.75 + \ln 1)$	A1		[or equiv perhaps with decimals; any constant k]
		Obtain 2.44	A1	4	[allow ±0.01]
	(iii)	Attempt difference of two areas	M1		[allow if area of their triangle < area A]
		Obtain 8 – 2.44 and hence 5.56	A1√	2	[following their tangent and area of A providing answer positive]

9	(i)	State $\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	B1		
		Use at least one of $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$	B1		
		Attempt complete process to express in terms of $\sin\theta$	M1		[using correct identities]
		Obtain $3\sin\theta - 4\sin^3\theta$	A1	4	[AG; all correctly obtained]
	(ii)	State 3	B1		
		Obtain expression involving $\sin 10\alpha$	M1		[allow θ / α confusion]
		Obtain 9	A1	3	[and no other value]
	(iii)	Recognise cosec 2β as $\frac{1}{\sin 2\beta}$	B1		[allow θ / β confusion]
		Attempt to express equation in terms of $\sin 2\beta$ only	M1		[or equiv involving $\cos 2\beta$]
		Attempt to find non-zero value of $\sin 2\beta$	M1		[or of $\cos 2\beta$]
		Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$	A1		[or equiv, exact or approx]
		Attempt correct process to find two values of β	M1		[provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$]
		Obtain 20.1, 69.9	A1	6	[and no others between 0 and 90]