RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4723

Core Mathematics 3

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 The function f is defined for all real values of x by

$$f(x) = 10 - (x+3)^2.$$

(i) State the range of f. [1]

(ii) Find the value of ff(-1). [3]

2 Find the exact solutions of the equation $|6x - 1| = |x - 1|$. [4]

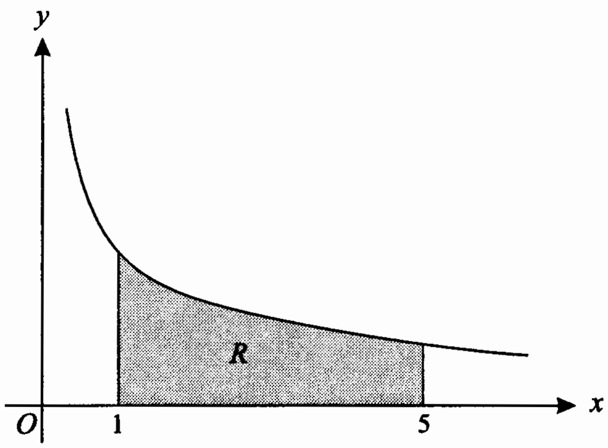
3 The mass, m grams, of a substance at time t years is given by the formula

$$m = 180e^{-0.017t}.$$

(i) Find the value of t for which the mass is 25 grams. [3]

(ii) Find the rate at which the mass is decreasing when $t = 55$. [3]

4 (a)



The diagram shows the curve $y = \dfrac{2}{\sqrt{x}}$. The region R, shaded in the diagram, is bounded by the curve and by the lines $x = 1$, $x = 5$ and $y = 0$. The region R is rotated completely about the x-axis. Find the exact volume of the solid formed. [4]

(b) Use Simpson's rule, with 4 strips, to find an approximate value for

$$\int_1^5 \sqrt{(x^2+1)}\,dx,$$

giving your answer correct to 3 decimal places. [4]

5 (i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $3\sin\theta + 2\cos\theta = \frac{7}{2}$, giving all solutions for which $0° < \theta < 360°$. [5]

6 **(a)** Find the exact value of the x-coordinate of the stationary point of the curve $y = x \ln x$. [4]

(b) The equation of a curve is $y = \dfrac{4x + c}{4x - c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]

7 **(i)** Write down the formula for $\cos 2x$ in terms of $\cos x$. [1]

(ii) Prove the identity $\dfrac{4\cos 2x}{1 + \cos 2x} \equiv 4 - 2\sec^2 x$. [3]

(iii) Solve, for $0 < x < 2\pi$, the equation $\dfrac{4\cos 2x}{1 + \cos 2x} = 3\tan x - 7$. [5]

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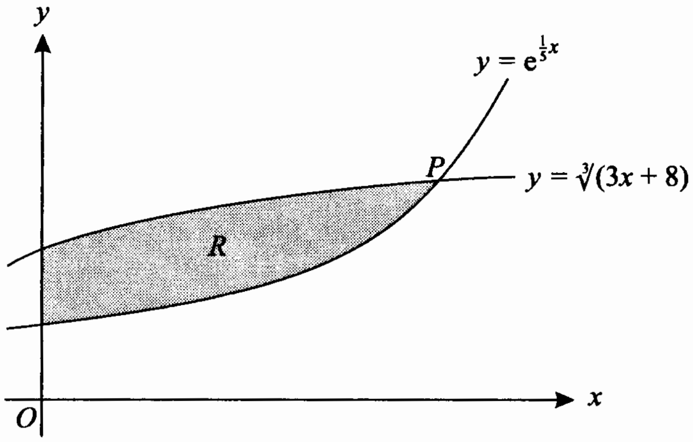


The diagram shows part of each of the curves $y = e^{\frac{1}{5}x}$ and $y = \sqrt[3]{(3x + 8)}$. The curves meet, as shown in the diagram, at the point P. The region R, shaded in the diagram, is bounded by the two curves and by the y-axis.

(i) Show by calculation that the x-coordinate of P lies between 5.2 and 5.3. [3]

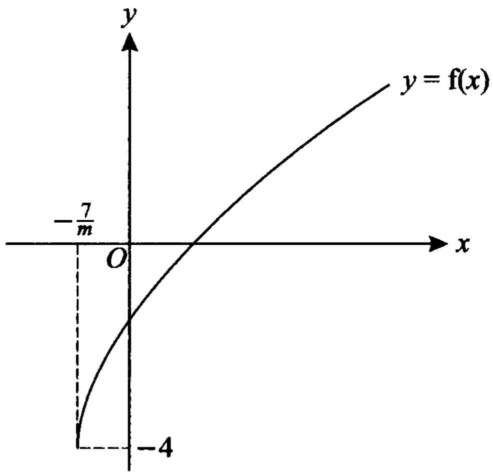
(ii) Show that the x-coordinate of P satisfies the equation $x = \frac{5}{3}\ln(3x + 8)$. [2]

(iii) Use an iterative formula, based on the equation in part **(ii)**, to find the x-coordinate of P correct to 2 decimal places. [3]

(iv) Use integration, and your answer to part **(iii)**, to find an approximate value of the area of the region R. [5]

[Question 9 is printed overleaf.]

9



The function f is defined by $f(x) = \sqrt{(mx + 7)} - 4$, where $x \geqslant -\dfrac{7}{m}$ and m is a positive constant. The diagram shows the curve $y = f(x)$.

(i) A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations. [4]

(ii) Explain how you can tell that f is a one-one function and find an expression for $f^{-1}(x)$. [4]

(iii) It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Explain how it can be deduced that neither curve meets the line $y = x$, and hence determine the set of possible values of m. [5]

1	(i)	State $f(x) \le 10$	B1	1 [Any equiv but must be or imply $\le$]
	(ii)	Attempt correct process for composition of functions	M1	[whether algebraic or numerical]
		Obtain 6 or correct expression for $ff(x)$	A1	
		Obtain -71	A1	3
2		Either Obtain $x = 0$	B1	[ignoring errors in working]
		Form linear equation with signs of $6x$ and x different	M1	[ignoring other sign errors]
		State $6x - 1 = -x + 1$	A1	[or correct equiv with or without brackets]
		Obtain $\frac{2}{7}$ and no other non-zero value	A1	4 [or exact equiv]
	Or	Obtain $36x^2 - 12x + 1 = x^2 - 2x + 1$	B1	[or equiv]
		Attempt to solve quadratic equation	M1	[as far as factorisation or subn into formula]
		Obtain $\frac{2}{7}$ and no other non-zero value	A1	[or exact equiv]
		Obtain 0	B1	(4) [ignoring errors in working]
3	(i)	Attempt solution involving (natural) logarithm	M1	
		Obtain $-0.017t = \ln\frac{25}{180}$	A1	[or equiv]
		Obtain 116	A1	3 [or greater accuracy rounding to 116]
	(ii)	Differentiate to obtain $k\,e^{-0.017t}$	M1	[any constant k different from 180; solution must involve differentiation]
		Obtain correct $-3.06e^{-0.017t}$	A1	[or unsimplified equiv; accept + or –]
		Obtain 1.2	A1	3 [or greater accuracy; accept + or – answer]
4	(a)	State or imply $\int \pi\, y^2\, dx$	B1	
		Integrate to obtain $k \ln x$	M1	[any constant k, involving π or not; or equiv such as $k \ln 4x$]
		Obtain $4\pi \ln x$ or $4 \ln x$	A1	[or equiv]
		Obtain $4\pi \ln 5$	A1	4 [or similarly simplified equiv]

	(b)	Attempt calculation involving attempts at y values	M1	[with each of 1, 4, 2 present at least once as coefficients]
		Attempt $\frac{1}{3}\times 1(y_0+4y_1+2y_2+4y_3+y_4)$	M1	[with attempts at five y values]
		Obtain $\frac{1}{3}(\sqrt{2}+4\sqrt{5}+2\sqrt{10}+4\sqrt{17}+\sqrt{26})$	A1	[or exact equiv or decimal equivs]
		Obtain 12.758	A1	4 [or greater accuracy]
5	(i)	Obtain $R=\sqrt{13}$, or 3.6 or 3.61 or greater accuracy	B1	
		Attempt recognisable process for finding α	M1	[allow sine/cosine muddles]
		Obtain $\alpha = 33.7$	A1	3 [or greater accuracy]
	(ii)	Attempt to find at least one value of $\theta + \alpha$	*M1	
		Obtain value rounding to 76 or 104	A1√	[following their R]
		Subtract their α from at least one value	M1	[dependent on *M]
		Obtain one value rounding to 42 or 43, or to 70	A1	
		Obtain other value 42.4 or 70.2	A1	5 [or greater accuracy; no other answers between 0 and 360; ignore answers outside 0 to 360]
6	(a)	Attempt use of product rule	*M1	
		Obtain $\ln x + 1$	A1	[or unsimplified equiv]
		Equate attempt at first derivative to zero and obtain value involving e	M1	[dependent on *M]
		Obtain e^{-1}	A1	4 [or exact equiv]
	(b)	Attempt use of quotient rule	M1	[or equiv using product rule or …]
		Obtain $\frac{(4x-c)4-4(4x+c)}{(4x-c)^2}$	A1	[or equiv]
		Show that first derivative cannot be zero	A1	3 [AG; derivative must be correct]
7	(i)	State $2\cos^2 x - 1$	B1	1
	(ii)	Attempt to express left hand side in terms of $\cos x$	M1	[using expression of form $a\cos^2 x + b$]
		Identify $\frac{1}{\cos x}$ as $\sec x$	M1	[maybe implied]

		Confirm result	A1	3 [AG; necessary detail required]
	(iii)	Use identity $\sec^2 x = 1 + \tan^2 x$	B1	
		Attempt solution of quadratic equation in tan x	M1	[or equiv]
		Obtain $2\tan^2 x + 3\tan x - 9 = 0$ and hence tan $x = -3, \frac{3}{2}$	A1	
		Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of 0.313π, 1.31π, 0.602π, 1.60π)	A1	[allow answers with only 2 s.f.; allow greater accuracy; allow $0.983 + \pi$, $1.89 + \pi$ allow degrees: 56, 236, 108, 288]
		Obtain all four solutions	A1	5 [now with at least 3 s.f.; must be radians; no other solutions in the range $0 - 2\pi$; ignore solutions outside range $0 - 2\pi$]
8	(i)	Attempt relevant calculations with 5.2 and 5.3	M1	
		Obtain correct values	A1	x y_1 y_2 $y_1 - y_2$ 5.2 2.83 2.87 −0.04 5.3 2.89 2.88 0.006
		Conclude appropriately	A1	3 [AG; comparing y values or noting sign change in difference in y values or equiv]
	(ii)	Equate expressions and attempt rearrangement to $x =$	M1	
		Obtain $x = \frac{5}{3}\ln(3x+8)$	A1	2 [AG; necessary detail required]
	(iii)	Obtain correct first iterate	B1	
		Carry out correct process to find at least two iterates in all	M1	
		Obtain 5.29	A1	3 [must be exactly 2 decimal places; 5.2→5.2687→5.2832→5.2863→5.2869; 5.25→5.2793→5.2855→5.2868→5.2870; 5.3→5.2898→5.2877→5.2872→5.2871]
	(iv)	Obtain integral of form $k(3x+8)^{\frac{4}{3}}$	M1	
		Obtain integral of form $k\,e^{\frac{1}{5}x}$	M1	

		Obtain $\frac{1}{4}(3x+8)^{\frac{4}{3}} - 5e^{\frac{1}{5}x}$	A1	[or equiv]
		Apply limits 0 and their answer to **(iii)**	M1	[applied to difference of two integrals]
		Obtain 3.78	A1	5 [or greater accuracy]
9	(i)	Indicate stretch and (at least one) translation	M1	[… in general terms]
		State translation by 7 units in negative x direction	A1	[or equiv; using correct terminology]
		State stretch in x direction with factor $1/m$	A1	[must follow the translation by 7; or equiv; using correct terminology]
		Indicate translation by 4 units in negative y direction	B1	4 [or equiv; at any stage; the two translations may be combined]
	(ii)	Refer to each y value being image of unique x value	B1	[or equiv]
		Attempt correct process for finding inverse	M1	
		Obtain expression involving $(x+4)^2$ or $(y+4)^2$	M1	
		Obtain $\frac{(x+4)^2 - 7}{m}$	A1	4 [or equiv]
	(iii)	Refer to fact that curves are reflections of each other in line $y = x$	B1	[or equiv]
		Attempt arrangement of either $f(x) = x$ or $f^{-1}(x) = x$	M1	
		Apply discriminant to resulting quadratic equati on	M1	
		Obtain $(m-2)(m-14) < 0$	A1	[or equiv]
		Obtain $2 < m < 14$	A1	5