OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4722

Core Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4722
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

1 The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + ax^2 - ax - 14$, where a is a constant.

(i) Given that $(x - 2)$ is a factor of $f(x)$, find the value of a. **[3]**

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

2 **(i)** Use the trapezium rule, with 3 strips each of width 3, to estimate the area of the region bounded by the curve $y = \sqrt[3]{7 + x}$, the x-axis, and the lines $x = 1$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate of the area. **[1]**

3 **(i)** Find and simplify the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{10}$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of x^3 in the expansion of $(3 + 4x + 2x^2)(1 + \frac{1}{2}x)^{10}$. **[3]**

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5n + 1$.

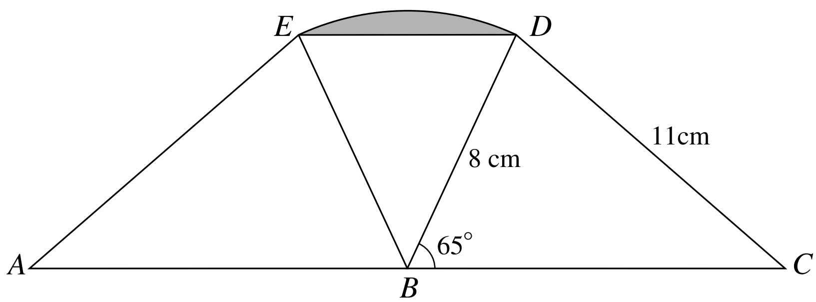
(i) State the values of u_1, u_2 and u_3. **[1]**

(ii) Evaluate $\sum_{n=1}^{40} u_n$. **[3]**

Another sequence $w_1, w_2, w_3, \ldots$ is defined by $w_1 = 2$ and $w_{n+1} = 5w_n + 1$.

(iii) Find the value of p such that $u_p = w_3$. **[3]**

5



The diagram shows two congruent triangles, BCD and BAE, where ABC is a straight line. In triangle BCD, $BD = 8$ cm, $CD = 11$ cm and angle $CBD = 65°$. The points E and D are joined by an arc of a circle with centre B and radius 8 cm.

(i) Find angle BCD. **[2]**

(ii) **(a)** Show that angle EBD is 0.873 radians, correct to 3 significant figures. **[2]**

(b) Hence find the area of the shaded segment bounded by the chord ED and the arc ED, giving your answer correct to 3 significant figures. **[4]**

6 **(a)** Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. **[4]**

(b) Find $\int (2 - 6\sqrt{y})\,dy$. **[3]**

(c) Evaluate $\int_1^{\infty} \frac{8}{x^3}\,dx$. **[4]**

7 **(i)** Show that $\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. **[2]**

(ii) Hence solve the equation

$$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x,$$

for $0° \leqslant x \leqslant 360°$. **[6]**

8 **(a)** Use logarithms to solve the equation $5^{3w-1} = 4^{250}$, giving the value of w correct to 3 significant figures. **[5]**

(b) Given that $\log_x(5y + 1) - \log_x 3 = 4$, express y in terms of x. **[4]**

9 A geometric progression has first term a and common ratio r, and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

(i) Show that $r^3 - 2r + 1 = 0$. **[3]**

(ii) Given that the geometric progression converges, find the exact value of r. **[5]**

(iii) Given also that the sum to infinity of this geometric progression is $3 + \sqrt{5}$, find the value of the integer a. **[4]**

1 (i)	$f(2) = 8 + 4a - 2a - 14$ $2a - 6 = 0$ $a = 3$	**M1***		Attempt f(2) or equiv, including inspection / long division / coefficient matching
		M1d*		Equate attempt at f(2), or attempt at remainder, to 0 and attempt to solve
		A1	**3**	Obtain $a = 3$
(ii)	$f(-1) = -1 + 3 + 3 - 14$ $= -9$	**M1**		Attempt f(-1) or equiv, including inspection / long division / coefficient matching
		A1 ft	**2**	Obtain -9 (or $2a - 15$, following their a)
			5	
2 (i)	area $\approx \frac{1}{2} \times 3 \times \left(\sqrt[3]{8} + 2\left(\sqrt[3]{11} + \sqrt[3]{14}\right) + \sqrt[3]{17}\right)$	**B1**		State or imply at least 3 of the 4 correct y-coords , and no others
		M1		Use correct trapezium rule, any h, to find area between $x = 1$ and $x = 10$
	≈ 20.8			
		M1		Correct h (soi) for their y-values – must be at equal intervals
		A1	**4**	Obtain 20.8 (allow 20.7)
(ii)	use more strips / narrower strips	**B1**	**1**	Any mention of increasing n or decreasing h
			5	
3 (i)	$(1 + \frac{1}{2}x)^{10} = 1 + 5x + 11.25x^2 + 15x^3$	**B1**		Obtain $1 + 5x$
		M1		Attempt at least the third (or fourth) term of the binomial expansion, including coeffs
		A1		Obtain $11.25x^2$
		A1		Obtain $15x^3$
			4	
(ii)	coeff of $x^3 = (3 \times 15) + (4 \times 11.25) + (2 \times 5)$ $= 100$	**M1**		Attempt at least one relevant term, with or without powers of x
		A1 ft		Obtain correct (unsimplified) terms (not necessarily summed) – either coefficients or still with powers of x involved
		A1	**3**	Obtain 100
			7	

4 (i)	$u_1 = 6$, $u_2 = 11$, $u_3 = 16$	**B1**	**1**	State 6, 11, 16
(ii)	$S_{40} = {}^{40}/_2\,(2 \times 6 + 39 \times 5)$ $= 4140$	**M1**		Show intention to sum the first 40 terms of a sequence
		M1		Attempt sum of their AP from (i), with $n = 40$, a = their u_1 and d = their $u_2 - u_1$
		A1	**3**	Obtain 4140
(iii)	$w_3 = 56$ $5p + 1 = 56$ or $6 + (p - 1) \times 5 = 56$	**B1**		State or imply $w_3 = 56$
	$p = 11$	**M1**		Attempt to solve $u_p = k$
		A1	**3**	Obtain $p = 11$
			7	
5 (i)	$\dfrac{\sin\theta}{8} = \dfrac{\sin 65}{11}$	**M1**		Attempt use of correct sine rule
	$\theta = 41.2^{\circ}$	**A1**	**2**	Obtain 41.2°, or better
(ii) a	$180 - (2 \times 65) = 50^{\circ}$ or $65 \times {}^{\pi}/_{180} = 1.134$ $50 \times {}^{\pi}/_{180} = 0.873$ **A.G.** $\pi - (2 \times 1.134) = 0.873$	**M1**		Use conversion factor of ${}^{\pi}/_{180}$
		A1	**2**	Show 0.873 radians convincingly (**AG)**
(ii) b	area sector = $\frac{1}{2} \times 8^2 \times 0.873 = 27.9$ area triangle = $\frac{1}{2} \times 8^2 \times \sin 0.873 = 24.5$	**M1**		Attempt area of sector, using $(\frac{1}{2})\, r^2\theta$
	area segment = $27.9 - 24.5$ $= 3.41$	**M1**		Attempt area of triangle using $(\frac{1}{2})\, r^2\sin\theta$
		M1		Subtract area of triangle from area of sector
		A1	**4**	Obtain 3.41or 3.42
			8	

6 a	$\int_3^5 (x^2+4x)\,dx = \left[\frac{1}{3}x^3 + 2x^2\right]_3^5$	M1		Attempt integration
	$= (^{125}/_3 + 50) - (9 + 18)$	A1		Obtain $^1/_3x^3 + 2x^2$
	$= 64\ ^2/_3$	M1		Use limits $x = 3, 5$ – correct order & subtraction
		A1	4	Obtain 64 $^2/_3$ or any exact equiv
b	$\int (2 - 6\sqrt{y})\,dy = 2y - 4y^{\frac{3}{2}} + c$	B1		State $2y$
		M1		Obtain $ky^{\frac{3}{2}}$
		A1	3	Obtain $-4y^{\frac{3}{2}}$ (condone absence of + c)
c	$\int_1^\infty 8x^{-3}\,dx = \left[\frac{-4}{x^2}\right]_1^\infty$	B1		State or imply $\frac{1}{x^3} = x^{-3}$
	$= (0) - (-4)$	M1		Attempt integration of kx^n
	$= 4$	A1		Obtain correct $-4x^{-2}$ $(+c)$
		A1 ft	4	Obtain 4 (or $-k$ following their kx^{-2})
			11	
7 (i)	$\frac{\sin^2 x - \cos^2 x}{1-\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$	M1		Use either $\sin^2 x + \cos^2 x = 1$, or $\tan x = {}^{\sin x}/_{\cos x}$
	$= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$			
	$= \tan^2 x - 1$	A1	2	Use other identity to obtain given answer convincingly.
(ii)	$\tan^2 x - 1 = 5 - \tan x$	B1		State correct equation
	$\tan^2 x + \tan x - 6 = 0$ $(\tan x - 2)(\tan x + 3) = 0$	M1		Attempt to solve three term quadratic in $\tan x$
	$\tan x = 2$, $\tan x = -3$ $x = 63.4^\circ, 243^\circ$ $x = 108^\circ, 288^\circ$	A1		Obtain 2 and -3 as roots of their quadratic
		M1		Attempt to solve $\tan x = k$ (at least one root)
		A1ft		Obtain at least 2 correct roots
		A1	6	Obtain all 4 correct roots
			8	

8 a	$\log 5^{3w-1} = \log 4^{250}$	**M1***		Introduce logarithms throughout
	$(3w-1)\log 5 = 250 \log 4$	**M1***		Use $\log a^b = b \log a$ at least once
	$3w - 1 = \frac{250 \log 4}{\log 5}$	**A1**		Obtain $(3w-1)\log 5 = 250 \log 4$ or equiv
	$w = 72.1$	**M1d***		Attempt solution of linear equation
		A1	**5**	Obtain 72.1, or better
b	$\log_x \frac{5y+1}{3} = 4$	**M1**		Use $\log a - \log b = \log {}^a/_b$ or equiv
	$\frac{5y+1}{3} = x^4$	**M1**		Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$
	$5y + 1 = 3x^4$	**M1**		Attempt to make y the subject of $f(y) = x^4$
	$y = \frac{3x^4 - 1}{5}$	**A1**	**4**	Obtain $y = \frac{3x^4-1}{5}$, or equiv
			9	
9 (i)	$ar = a + d, \quad ar^3 = a + 2d$ $2ar - ar^3 = a$ $ar^3 - 2ar + a = 0$ $r^3 - 2r + 1 = 0$ **A.G.**	**M1**		Attempt to link terms of AP and GP, implicitly or explicitly.
		M1		Attempt to eliminate d, implicitly or explicitly, to show given equation.
		A1	**3**	Show $r^3 - 2r + 1 = 0$ convincingly
(ii)	$f(r) = (r-1)(r^2 + r - 1)$	**B1**		Identify $(r-1)$ as factor or $r = 1$ as root
		M1*		Attempt to find quadratic factor
	$r = \frac{-1 \pm \sqrt{5}}{2}$	**A1**		Obtain $r^2 + r - 1$
	Hence $r = \frac{-1+\sqrt{5}}{2}$	**M1d***		Attempt to solve quadratic
		A1	**5**	Obtain $r = \frac{-1+\sqrt{5}}{2}$ only
(iii)	$\frac{a}{1-r} = 3 + \sqrt{5}$	**M1**		Equate S_∞ to $3 + \sqrt{5}$
	$a = (\frac{3}{2} - \frac{\sqrt{5}}{2})(3 + \sqrt{5})$	**A1**		Obtain $\frac{a}{1 - \left(\frac{-1+\sqrt{5}}{2}\right)} = 3 + \sqrt{5}$
	$a = {}^9/_2 - {}^5/_2$ $a = 2$	**M1**		Attempt to find a
		A1	**4**	Obtain $a = 2$
			12	