OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4722

Core Mathematics 2

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

1 Find the binomial expansion of $(3x - 2)^4$. [4]

2 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

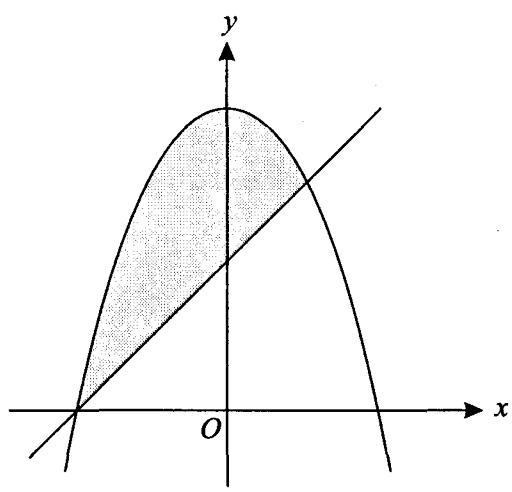
$$u_1 = 2 \qquad \text{and} \qquad u_{n+1} = 1 - u_n \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{100} u_n$. [3]

3 The gradient of a curve is given by $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$, and the curve passes through the point (4, 5). Find the equation of the curve. [6]

4



The diagram shows the curve $y = 4 - x^2$ and the line $y = x + 2$.

(i) Find the x-coordinates of the points of intersection of the curve and the line. [2]

(ii) Use integration to find the area of the shaded region bounded by the line and the curve. [6]

5 Solve each of the following equations, for $0° \leqslant x \leqslant 180°$.

(i) $2\sin^2 x = 1 + \cos x$. [4]

(ii) $\sin 2x = -\cos 2x$. [4]

6 (i) John aims to pay a certain amount of money each month into a pension fund. He plans to pay £100 in the first month, and then to increase the amount paid by £5 each month, i.e. paying £105 in the second month, £110 in the third month, etc.

If John continues making payments according to this plan for 240 months, calculate

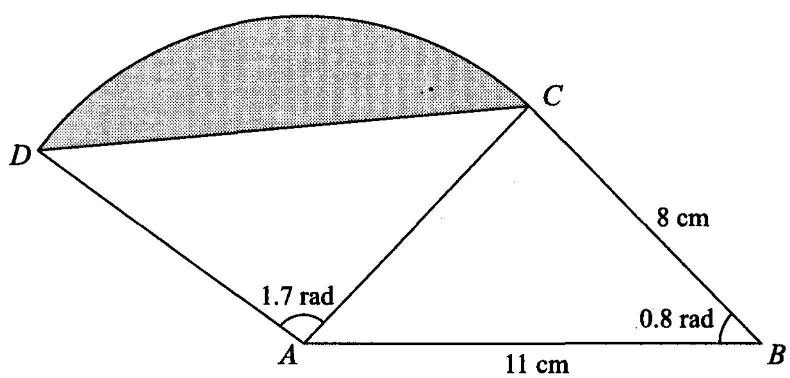
(a) how much he will pay in the final month, [2]

(b) how much he will pay altogether over the whole period. [2]

(ii) Rachel also plans to pay money monthly into a pension fund over a period of 240 months, starting with £100 in the first month. Her monthly payments will form a geometric progression, and she will pay £1500 in the final month.

Calculate how much Rachel will pay altogether over the whole period. [5]

7



The diagram shows a triangle ABC, and a sector ACD of a circle with centre A. It is given that $AB = 11$ cm, $BC = 8$ cm, angle $ABC = 0.8$ radians and angle $DAC = 1.7$ radians. The shaded segment is bounded by the line DC and the arc DC.

(i) Show that the length of AC is 7.90 cm, correct to 3 significant figures. [3]

(ii) Find the area of the shaded segment. [3]

(iii) Find the perimeter of the shaded segment. [4]

8 The cubic polynomial $2x^3 + ax^2 + bx - 10$ is denoted by $f(x)$. It is given that, when $f(x)$ is divided by $(x - 2)$, the remainder is 12. It is also given that $(x + 1)$ is a factor of $f(x)$.

(i) Find the values of a and b. [6]

(ii) Divide $f(x)$ by $(x + 2)$ to find the quotient and the remainder. [5]

[Question 9 is printed overleaf.]

9 (i) Sketch the curve $y = \left(\frac{1}{2}\right)^x$, and state the coordinates of any point where the curve crosses an axis. [3]

(ii) Use the trapezium rule, with 4 strips of width 0.5, to estimate the area of the region bounded by the curve $y = \left(\frac{1}{2}\right)^x$, the axes, and the line $x = 2$. [4]

(iii) The point P on the curve $y = \left(\frac{1}{2}\right)^x$ has y-coordinate equal to $\frac{1}{6}$. Prove that the x-coordinate of P may be written as

$$1 + \frac{\log_{10} 3}{\log_{10} 2}.$$ [4]

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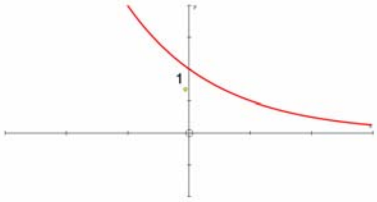
1		$(3x-2)^4 = 81x^4 - 216x^3 + 216x^2 - 96x + 16$	M1 A1 A1 A1	 4 **4**	Attempt binomial expansion, including attempt at coeffs. Obtain one correct, simplified, term Obtain a further two, simplified, terms Obtain a completely correct expansion
2	(i)	$u_2 = -1, u_3 = 2, u_4 = -1$	B1 B1	 2	For correct value –1 for u_2 For correct values for both u_3 and u_4
	(ii)	Sum is $(2+(-1))+(2+(-1))+...+(2+(-1))$ i.e. $50\times(2+(-1)) = 50$	M1 M1 A1	 3 **5**	For correct interpretation of Σ notation For pairing, or $50\times 2 - 50\times 1$ For correct answer 50
3		$y = 4x^{\frac{1}{2}} + c$ Hence $5 = 4\times 4^{\frac{1}{2}} + c \Rightarrow c = -3$ So equation of the curve is $y = 4x^{\frac{1}{2}} - 3$	M1 A1 A1 M1 A1√ A1	 6 **6**	For attempt to integrate For integral of the form $kx^{\frac{1}{2}}$ For $4x^{\frac{1}{2}}$, with or without +*c* For relevant use of (4, 5) to evaluate *c* For correct value –3 (or follow through on integral of form $kx^{\frac{1}{2}}$) For correct statement of the equation in full (aef)
4	(i)	Intersect where $x^2 + x - 2 = 0 \Rightarrow x = -2, 1$	M1 A1	 2	For finding *x* at both intersections For both values correct
	(ii)	Area under curve is $\left[4x - \frac{1}{3}x^3\right]_{-2}^{1}$ i.e. $\left(4-\frac{1}{3}\right) - \left(-8+\frac{8}{3}\right) = 9$ Area of triangle is 4½ Hence shaded area is 9 – 4½ = 4½ **OR** Area under curve is $\int_{-2}^{1}(2-x-x^2)\,dx$ $= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1}$ $= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$ $= 4\frac{1}{2}$	M1 M1 A1 M1 A1 A1 M1 M1 A1 M1 A1 A1	 6 **8**	For integration attempt with any one term correct For use of limits – subtraction and correct order For correct area of 9 Attempt area of triangle (½*bh* or integration) Obtain area of triangle as 4 ½ Obtain correct final area of 4 ½ Attempt subtraction – either order For integration attempt with any one term correct Obtain $\pm\left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]$ For use of limits – subtraction and correct order Obtain ± 4 ½ - consistent with their order of subtraction Obtain 4 ½ only, following correct method only
5	(i)	$\sin^2 x = 1 - \cos^2 x \Rightarrow 2\cos^2 x + \cos x - 1 = 0$ Hence $(2\cos x - 1)(\cos x + 1) = 0$ $\cos x = \frac{1}{2} \Rightarrow x = 60^\circ$ $\cos x = -1 \Rightarrow x = 180^\circ$	M1 M1 A1 A1	 4	For transforming to a quadratic in cos *x* For solution of a quadratic in cos *x* For correct answer 60° For correct answer 180° [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
	(ii)	$\tan 2x = -1 \Rightarrow 2x = 135$ or 315 Hence $x = 67.5^\circ$ or 157.5°	M1 M1 A1 A1	 4	For transforming to an equation of form tan2*x* = *k* For correct solution method, i.e. inverse tan followed by division by 2 For correct value 67.5 For correct value 157.5

		OR $\sin^2 2x = \cos^2 2x$ $2\sin^2 2x = 1 \quad 2\cos^2 2x = 1$ $\sin 2x = \pm\frac{1}{2}\sqrt{2} \quad \cos 2x = \pm\frac{1}{2}\sqrt{2}$ Hence $x = 67.5^\circ$ or 157.5°	M1 M1 A1 A1	**8**	Obtain linear equation in cos $2x$ or sin $2x$ Use correct solution method For correct value 67.5 For correct value 157.5 [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
6	(i)	(a) $100 + 239 \times 5 = £1295$	M1 A1	2	For relevant use of $a + (n-1)d$ For correct value 1295
		(b) $\frac{1}{2} \times 240 \times (100 + 1295) = £167400$	M1 A1	2	For relevant use of $\frac{1}{2}n(a+l)$ or equivalent For correct value 167400
	(ii)	$100r^{239} = 1500 \Rightarrow r = 1.01139...$ Hence total is $\frac{100(1.01139^{240} - 1)}{1.01139 - 1} = £124359$	B1 M1 A1 M1 A1	5 **9**	For correct statement of $100r^{239} = 1500$ Attempt to find r For correct value 1.01 For relevant use of GP sum formula For correct value 124359 (3 s.f. or better)
7	(i)	$AC^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \times \cos 0.8$ $= 62.3796...$ Hence AC=7.90 cm	M1 A1 A1	3	Attempt to use the cosine formula Correct unsimplified expression Show the given answer correctly
	(ii)	Area of sector = $\frac{1}{2} \times 7.90^2 \times 1.7 = 53.0$ Area of triangle = $\frac{1}{2} \times 7.90^2 \times \sin 1.7 = 30.9$ Hence shaded area = 22.1 cm^2	M1 M1 A1	3	Attempt area of sector using $(\frac{1}{2})r^2\theta$ Attempt area of ΔACD, using $(\frac{1}{2})r^2 \sin\theta$, or equiv Obtain 22.1
	(iii)	(arc) $DC = 7.90 \times 1.7 = 13.4$ (line) $DC^2 = 7.90^2 + 7.90^2 - 2 \times 7.90 \times 7.90 \times \cos 1.7$ $DC = 11.9$ Hence perimeter = 25.3cm	M1 A1 M1 A1	4 **10**	Use $r\theta$ to attempt arc length Obtain 13.4 Attempt length of line DC using cosine rule or equiv. Obtain 25.3
8	(i)	$f(2) = 12 \Rightarrow 4a + 2b = 6$ $f(-1) = 0 \Rightarrow a - b = 12$ Hence $a = 5$, $b = -7$	M1 A1 M1 A1 M1 A1	6	For equating f(2) to 12 For correct equation $4a + 2b = 6$ For equating f(–1) to 0 For correct equation $a - b = 12$ For attempt to find a and b For both values correct
	(ii)	Quotient is $2x^2 + x - 9$ Remainder is 8	B1 M1 A1 M1 A1	5 **11**	For correct lead term of $2x^2$ For complete division attempt or equiv For completely correct quotient For attempt at remainder – either division or f(–2) For correct remainder

9	(i)		M1 A1 B1	3	Attempt sketch of any exponential graph, in at least first quadrant Correct graph – must be in both quadrants For identification of (0, 1)
	(ii)	$A \approx \frac{1}{2} \times 0.5 \times \left\{1 + 2\left(0.5^{\frac{1}{2}} + 0.5 + 0.5^{\frac{3}{2}}\right) + 0.5^2\right\}$ ≈ 1.09	B1 M1 A1 A1	4	State, or imply, at least three correct y-values For correct use of trapezium rule, inc correct h For correct unsimplified expression For the correct value 1.09, or better
	(iii)	$\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow x\log_{10}\frac{1}{2} = \log_{10}\frac{1}{6}$ $x = \frac{\log_{10}\frac{1}{6}}{\log_{10}\frac{1}{2}} = \frac{-\log_{10}6}{-\log_{10}2}$ Hence $= \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1	4	For equation $\left(\frac{1}{2}\right)^x = \frac{1}{6}$ and attempt at logs Obtain $x\log\left(\frac{1}{2}\right) = \log\left(\frac{1}{6}\right)$, or equivalent For use of log 6 = log 2 + log 3 For showing the given answer correctly
		OR $\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow 2^x = 6$ $\Rightarrow x\log_{10}2 = \log_{10}6$ $x = \frac{\log_{10}6}{\log_{10}2}$ $= \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1		For equation $2^x = 6$ and attempt at logs Obtain x log 2 = log 6, or equivalent For use of log 6 = log 2 + log 3 For showing the given answer correctly
		OR $\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow 2^x = 6$ $2^{x-1} = 3$ $(x-1)\log_{10}2 = \log_{10}3$ Hence $x = 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1		Attempt to rearrange equation to $2^n = 3$ Obtain $2^{x-1} = 3$ For attempt at logs For showing the given answer correctly
		OR $x = \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= \frac{\log_{10}6}{\log_{10}2}$ $x\log_{10}2 = \log_{10}6$ $\log_{10}2^x = \log_{10}6$ $2^x = 6$ $\left(\frac{1}{2}\right)^x = \frac{1}{6}$	M1 A1 M1 A1	**11**	Use log 2 + log 3 = log 6 Obtain $x$log 2 = log 6 Attempt to remove logarithms Show $\left(\frac{1}{2}\right)^x = \frac{1}{6}$ correctly