OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4722

Core Mathematics 2

Monday **10 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Simplify $(3 + 2x)^3 - (3 - 2x)^3$. [5]

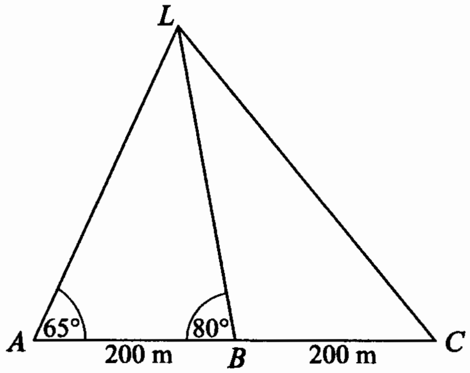
2 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2 \quad \text{and} \quad u_{n+1} = \frac{1}{1 - u_n} \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3, u_4 and u_5. [3]

(ii) Deduce the value of u_{200}, showing your reasoning. [4]

3

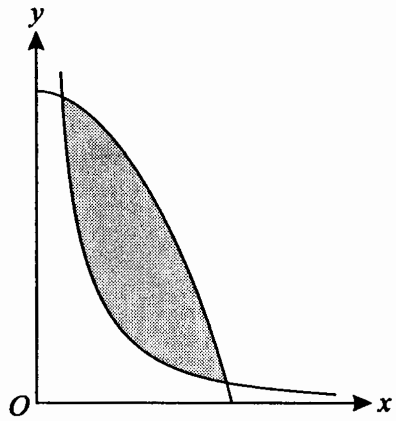


A landmark L is observed by a surveyor from three points A, B and C on a straight horizontal road, where $AB = BC = 200$ m. Angles LAB and LBA are $65°$ and $80°$ respectively (see diagram). Calculate

(i) the shortest distance from L to the road, [4]

(ii) the distance LC. [3]

4



The diagram shows a sketch of parts of the curves $y = \frac{16}{x^2}$ and $y = 17 - x^2$.

(i) Verify that these curves intersect at the points $(1, 16)$ and $(4, 1)$. [1]

(ii) Calculate the exact area of the shaded region between the curves. [7]

5 **(i)** Prove that the equation

$$\sin\theta\tan\theta = \cos\theta + 1$$

can be expressed in the form

$$2\cos^2\theta + \cos\theta - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\sin\theta\tan\theta = \cos\theta + 1,$$

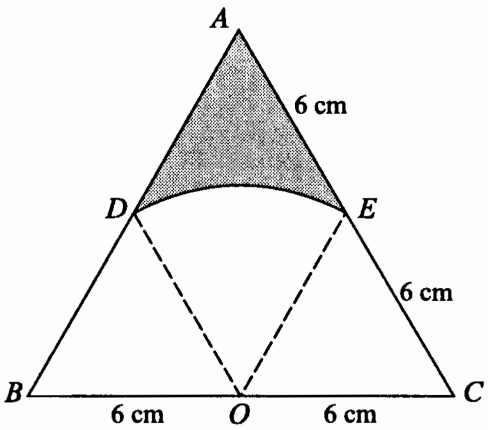
giving all values of θ between $0°$ and $360°$. [5]

6 **(a)** Find $\int x(x^2 + 2)\,dx$. [3]

(b) **(i)** Find $\int \frac{1}{\sqrt{x}}\,dx$. [3]

(ii) The gradient of a curve is given by $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$. Find the equation of the curve, given that it passes through the point (4, 0). [3]

7



The diagram shows an equilateral triangle ABC with sides of length 12 cm. The mid-point of BC is O, and a circular arc with centre O joins D and E, the mid-points of AB and AC.

(i) Find the length of the arc DE, and show that the area of the sector ODE is $6\pi\,\text{cm}^2$. [4]

(ii) Find the exact area of the shaded region. [4]

[Questions 8 and 9 are printed overleaf.]

8 **(i)** On a single diagram, sketch the curves with the following equations. In each case state the coordinates of any points of intersection with the axes.

(a) $y = a^x$, where a is a constant such that $a > 1$. [2]

(b) $y = 2b^x$, where b is a constant such that $0 < b < 1$. [2]

(ii) The curves in part **(i)** intersect at the point P. Prove that the x-coordinate of P is

$$\frac{1}{\log_2 a - \log_2 b}.$$ [5]

9 A geometric progression has first term a, where $a \neq 0$, and common ratio r, where $r \neq 1$. The difference between the fourth term and the first term is equal to four times the difference between the third term and the second term.

(i) Show that $r^3 - 4r^2 + 4r - 1 = 0$. [2]

(ii) Show that $r - 1$ is a factor of $r^3 - 4r^2 + 4r - 1$. Hence factorise $r^3 - 4r^2 + 4r - 1$. [3]

(iii) Hence find the two possible values for the ratio of the geometric progression. Give your answers in an exact form. [2]

(iv) For the value of r for which the progression is convergent, prove that the sum to infinity is $\frac{1}{2}a(1 + \sqrt{5})$. [4]

1	$(3+2x)^3 = 27+54x+36x^2+8x^3$	M1		For recognisable binomial expansion attempt
		A1		For any two terms correct, possibly unsimplified
		A1		For all four terms correct and simplified
	$(3-2x)^3 = 27-54x+36x^2-8x^3$	B1√		For changing the appropriate signs
	Hence $(3+2x)^3-(3-2x)^3 = 108x+16x^3$	A1	**5**	For answer $108x+16x^3$ or $4x(27+4x^2)$
			[5]	
2	**(i)** $u_2=-1,\ u_3=\frac{1}{2},\ u_4=2,\ u_5=-1$	B1		For correct value -1 for u_2
		B1√		For correct u_3 from their u_2
		B1√	**3**	For correct u_4 and u_5 from their u_3 and u_4
	(ii) $u_1, u_4, u_7,$ etc all have the value 2	B1		For recognising the repeating property
	Hence $u_{199}=2$, giving $u_{200}=-1$	M1		For division by 3, or equivalent
		A1		For correctly linking relevant term to a term already found
		A1	**4**	For the correct answer -1
	(SR - Answer only is B1)			
			[7]	
3	**(i)** $\frac{LB}{\sin 65°}=\frac{200}{\sin 35°}$ OR $\frac{LA}{\sin 80°}=\frac{200}{\sin 35°}$	M1		For correct use of the sine rule in ΔLAB (could be in **ii**)
	$\Rightarrow LB = 316.0198\ldots$ $\Rightarrow LA = 343.39\ldots$	A1		For correct value of (or explicit expression for) LB or LA
	Hence $p = LB\sin 80° = 311$ m $p = LA\sin 65 = 311$ m	M1		For calculation of perpendicular distance
		A1	**4**	For correct distance (rounding to) 311
	(ii) $LC^2 = 200^2+316^2-2\times 200\times 316\times\cos 100°$	M1		For use of cosine rule in ΔLBC or LAC
	$\left(or\ LC^2 = 400^2+343^2-2\times 400\times 343\times\cos 65°\right)$	A1√		For correct unsimplified numerical expression for LC^2 following their LA or LB
	Hence $LC = 402$ m	A1	**3**	For correct distance (rounding to) 402
			[7]	
4	**(i)** $\frac{16}{1^2}=16$ and $16=17-1^2$ stated			
	$1=\frac{16}{4^2}$ and $1=17-4^2$ stated	B1	**1**	For complete verification for both points
	(ii) Area is $\int_1^4\left(17-x^2-\frac{16}{x^2}\right)dx$	M1		For appropriate subtraction (at any stage) – correct order
	$=\left[17x-\frac{1}{3}x^3+\frac{16}{x}\right]_1^4$	*M1		For integration attempt with any one term OK
		A1		For $17x-\frac{1}{3}x^3$ completely correct
		M1		For correct form kx^{-1} for third term
		A1		For correct k, for their stage of working
	$=68-\frac{64}{3}+4-17+\frac{1}{3}-16=18$	M1dep*M		For use of limits – correct order
		A1	**7**	For correct answer 18
			[8]	

5	**(i)**	$\sin\theta\tan\theta = \sin\theta \times \frac{\sin\theta}{\cos\theta} = \frac{1-\cos^2\theta}{\cos\theta}$	M1		For use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$
			M1		For use of $\cos^2\theta + \sin^2\theta = 1$
		Hence $1-\cos^2\theta = \cos\theta(\cos\theta+1)$, i.e. $2\cos^2\theta + \cos\theta - 1 = 0$, or equiv	A1	**3**	For showing given equation correctly
	(ii)	$(2\cos\theta - 1)(\cos\theta + 1) = 0$	M1		For solution of quadratic equation in $\cos\theta$
		Hence $\cos\theta = \frac{1}{2}$ or -1	A1		For both values of $\cos\theta$ correct
		So $\theta = 60°,\ 300°,\ 180°$	A1		For correct answer $60°$
			A1		For correct answer $180°$
			A1√	**5**	For a correct non-principal-value answer, following their value of $\cos\theta$ (excluding $\cos\theta$ = -1, 0, 1) **and** no other values for θ.
				8	
6	**(a)**	$\int (x^3 + 2x)\,dx = \frac{1}{4}x^4 + x^2 + c$	M1		For expanding and integration attempt
			A1		For $\frac{1}{4}x^4 + x^2$ correct
			B1	**3**	For addition of an arbitrary constant (this mark can be given in **(b)(i)** if not earned here), and no dx in either
	(b) (i)	$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}} + c$	B1		For use of $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$
			M1		For integral of the form $kx^{\frac{1}{2}}$
			A1	**3**	For correct term $2x^{\frac{1}{2}}$
	(ii)	$0 = 2\sqrt{4} + c \Rightarrow c = -4$	M1		For use of $x = 4,\ y = 0$ to evaluate c
			A1t		For correct c from their answer in **(b)(i)**
		Hence curve is $y = 2x^{\frac{1}{2}} - 4$	A1t	**3**	For equation of the curve correctly stated
				9	
7	**(i)**	Length of OD is 6 cm	B1		For stating or using the correct value of r
		Angle DOE is $\frac{1}{3}\pi$ / 1.047^c / $60°$ / $^1/_6$ of circle	B1		For stating or using the correct angle
		Hence arc length DE is 2π cm (allow 6.28 cm)	B1		For correct use of $s = r\theta$ or equiv in degrees
		Area is $\frac{1}{2} \times 6^2 \times \frac{1}{3}\pi = 6\pi$ cm^2 $\left(\text{or } \frac{60}{360} \times \pi \times 6^2\right)$	B1	**4**	For obtaining the given answer 6π correctly
	(ii)	Area of small triangle is $\frac{1}{2} \times 6^2 \times \frac{1}{2}\sqrt{3} = 9\sqrt{3}$	*M1		For use of $\Delta = \frac{1}{2}ab\sin C$, or equivalent
			A1		For correct value $9\sqrt{3}$, or equiv
		Area of segment is $6\pi - 9\sqrt{3}$	M1dep*M		For relevant use of (sector – triangle)
		Hence shaded area is $(18\sqrt{3} - 6\pi)$ cm^2	A1	**4**	For correct answer $18\sqrt{3} - 6\pi$, or exact equiv
					Scheme for alternative approaches:
			*M1		Attempt area of big triangle / rhombus / segment, using $\Delta = \frac{1}{2}ab\sin C$, or equivalent
			A1		Correct area
			M1dep*M		Relevant subtraction
			A1		For correct answer $18\sqrt{3} - 6\pi$
				8	

8	**(i) (a)** Sketch showing exponential growth	M1		For correct shape in at least 1st quadrant
	Intersection with y-axis is $(0,1)$	A1	**2**	For 1st and 2nd quadrants, and y-coordinate 1 stated
	(b) Sketch showing exponential decay	M1		For correct shape in at least 1st quadrant
	Intersection with y-axis is $(0,2)$	A1	**2**	For 1st and 2nd quadrants, and y-coordinate 2 stated
	(ii) $a^x = 2b^x$	B1		For stating the equation in x
	Hence $x\log_2 a = \log_2 2 + x\log_2 b$	M1		For taking logs (any base)
		M1		For use of one log law
		M1		For use of a second log law
	i.e. $x = \dfrac{1}{\log_2 a - \log_2 b}$	A1	**5**	For showing the given answer correctly
			9	
9	**(i)** $ar^3 - a = 4(ar^2 - ar)$	M1		For using ar^{n-1} to form an equation
	Hence $r^3 - 4r^2 + 4r - 1 = 0$	A1	**2**	For showing the given equation correctly
	(ii) $1-4+4-1=0$	B1		For correct substitution of $r=1$, or state no remainder
	Factors are $(r-1)(r^2-3r+1)$	M1		For attempted division, or equivalent
		A1	**3**	For correct factor r^2-3r+1
	(iii) $r = \dfrac{3\pm\sqrt{5}}{2}$	M1		For solving the relevant quadratic equation
		A1	**2**	For correct roots in exact form
	(iv) The relevant value of r is $\dfrac{3-\sqrt{5}}{2}$ (or decimal equiv)	B1		For selecting the appropriate value of r
	Hence $S_\infty = \dfrac{a}{1-\frac{1}{2}\left(3-\sqrt{5}\right)}$	M1		For relevant use of $\dfrac{a}{1-r}$
	$= \dfrac{2a}{-1+\sqrt{5}} = \dfrac{2a\left(-1-\sqrt{5}\right)}{\left(-1+\sqrt{5}\right)\left(-1-\sqrt{5}\right)}$	M1		For correct process for rationalising, using two term surd expression
	$= \frac{1}{2}a\left(1+\sqrt{5}\right)$	A1	**4**	For showing the given answer correctly
			11	

Final mark scheme

Nikki Adams 16/01/2005