OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

AS GCE MATHEMATICS

4721 Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer book 4721
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

No calculator can be used for this paper

1 **(i)** Solve the equation $x^2 - 6x - 2 = 0$, giving your answers in simplified surd form. **[3]**

(ii) Find the gradient of the curve $y = x^2 - 6x - 2$ at the point where $x = -5$. **[2]**

2 Solve the equations

(i) $3^n = 1$, **[1]**

(ii) $t^{-3} = 64$, **[2]**

(iii) $\left(8p^6\right)^{\frac{1}{3}} = 8$. **[3]**

3 **(i)** Sketch the curve $y = (1+x)(2-x)(3+x)$, giving the coordinates of all points of intersection with the axes. **[3]**

(ii) Describe the transformation that transforms the curve $y = (1+x)(2-x)(3+x)$ to the curve $y = (1-x)(2+x)(3-x)$. **[2]**

4 **(i)** Solve the simultaneous equations

$$y = 2x^2 - 3x - 5, \qquad 10x + 2y + 11 = 0.$$ **[5]**

(ii) What can you deduce from the answer to part **(i)** about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$? **[1]**

5 **(i)** Simplify $(x+4)(5x-3) - 3(x-2)^2$. **[3]**

(ii) The coefficient of x^2 in the expansion of

$$(x+3)(x+k)(2x-5)$$

is –3. Find the value of the constant k. **[3]**

6 **(i)** The line joining the points (–2, 7) and (–4, p) has gradient 4. Find the value of p. **[3]**

(ii) The line segment joining the points (–2, 7) and (6, q) has mid-point (m, 5). Find m and q. **[3]**

(iii) The line segment joining the points (–2, 7) and (d, 3) has length $2\sqrt{13}$. Find the two possible values of d. **[4]**

7 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases:

(i) $y = \frac{(3x)^2 \times x^4}{x}$, **[3]**

(ii) $y = \sqrt[3]{x}$, **[3]**

(iii) $y = \frac{1}{2x^3}$. **[2]**

8 The quadratic equation $kx^2 + (3k-1)x - 4 = 0$ has no real roots. Find the set of possible values of k. **[7]**

9 A circle with centre C has equation $x^2 + y^2 - 2x + 10y - 19 = 0$.

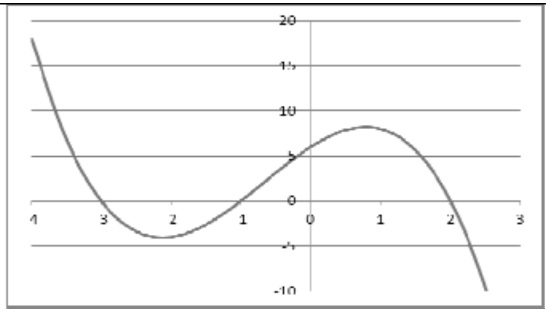
(i) Find the coordinates of C and the radius of the circle. **[3]**

(ii) Verify that the point (7, –2) lies on the circumference of the circle. **[1]**

(iii) Find the equation of the tangent to the circle at the point (7, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[5]**

10 Find the coordinates of the points on the curve $y = \frac{1}{3}x^3 + \frac{9}{x}$ at which the tangent is parallel to the line $y = 8x + 3$. **[10]**

Question			Answer	Marks	Guidance	
1	(i)		$\frac{6\pm\sqrt{(-6)^2-4\times1\times-2}}{2\times1}$	M1	Valid attempt to use quadratic formula	**No marks for attempting to factorise**
			$=\frac{6\pm\sqrt{44}}{2}$	A1		
			$=3\pm\sqrt{11}$	A1	Both roots correct and simplified	
			OR: $(x-3)^2-9-2=0$ $x-3=\pm\sqrt{11}$	M1 A1	Correct method to complete square	Must get to $(x-3)$ and ± stage for the M mark, constants combined correctly gets A1
			$x=3\pm\sqrt{11}$	A1 **[3]**	Rearranged to correct form **cao**	
1	(ii)		$\frac{dy}{dx}=2x-6$	B1		
			$=-16$	B1 **[2]**	**www**	
2	(i)		$n=0$	**B1** **[1]**	Allow 3^0	
2	(ii)		$\frac{1}{t^3}=64$ (or 4^3)	M1	or $t^3=\frac{1}{64}$ or $64t^3=1$ or $\left(\frac{1}{t}\right)^3=64$	Allow embedded
			$t=\frac{1}{4}$	A1 **[2]**	4^{-1} is **A0** $t=\pm\frac{1}{4}$ is **A0**	4^{-1} **www** alone implies **M1 A0**
2	(iii)		$2p^2=8$	M1	or $8p^6=8^3$. Allow $2p^{\frac{6}{3}}=8$ for **M1**	If not 512, evidence of 8 × 8 × 8 needed.
			$p=2$	A1	**www**	**SC** Spotted **B1** for 2, **B1** for -2, **B1** for justifying exactly 2 solutions
			or $p=-2$	A1 **[3]**	**www**	**SC** $8p^2=8$, $p=\pm1$ **B1**

Question			Answer	Marks	Guidance	
3	(i)			B1 B1 B1 [3]	-ve cubic with 3 distinct roots (0, 6) labelled or indicated on y-axis – seen elsewhere not enough (-3, 0), (-1, 0) and (2, 0) labelled or indicated on x-axis and no other x-intercepts.	Must not stop at x-axis. Condone errors in curvature at the extremes unless extra turning point(s)/root(s) clearly implied. **Must have a curve for 2nd and 3rd marks** Do not allow final B1 if shown as repeated root(s)
3	(ii)		Reflection in the y axis	B1 B1 [2]	Not mirrored/flipped etc. or $x = 0$. No/through/along etc. Must be "in". Cannot get 2nd B1 without some indication of a reflection e.g. flip etc. Do not **ISW** if contradictory statement seen	**Alt** Stretch (scale) factor –1 **B1** parallel to the x axis for **B1** **Must be a single transformation for any marks**
4	(i)		$2x^2 - 3x - 5 = \dfrac{-10x-11}{2}$ $4x^2 + 4x + 1 = 0$ $(2x + 1)(2x + 1) = 0$ $x = -\dfrac{1}{2}$ $y = -3$	*M1 A1 DM1 A1 A1 [5]	Substitute for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic – could be a multiple e.g. $2x^2 + 2x + 0.5 = 0$ Correct method to solve resulting 3 term quadratic	or $10x + 2(2x^2 - 3x - 5) + 11 = 0$ If x is eliminated, expect $k(8y^2 + 48y + 72) = 0$ **SC** If DM0 and $x = -\dfrac{1}{2}$ spotted **B1** for x value, **B1** for y value **B1** justifying only one root
4	(ii)		Line is a tangent to the curve	B1√ [1]	**Must be consistent with their answers to their quadratic in (i).** **1 repeated root –** indicates one point. Accept tangent, meet at, intersect, touch etc. but do not accept cross **2 roots –** indicates meet at two points **0 roots –** indicates do not meet. Do not accept "do not cross"	**Follow through from their solution to (i)**

Question			Answer	Marks	Guidance	
5	(i)		$5x^2 + 17x - 12 - 3(x^2 - 4x + 4)$	M1	Attempt to expand both pairs of brackets	
				A1	$5x^2 + 17x - 12$ **and** $x^2 - 4x + 4$ soi ; may be unsimplified, no more than one incorrect term, no "extra" terms at all. No "invisible brackets"	
			$= 2x^2 + 29x - 24$	A1	$2x^2 + 29x - 24$	**ISW** if they then put expression equal to zero and go on to "solve"
				[3]		
5	(ii)		$-5x^2 + 2kx^2 + 6x^2$	M1	Correct method to multiply out 3 brackets or correctly identify all x^2 terms	No more than 8 terms, but ignore sign errors/accuracy of non x^2 terms
				A1	All x^2 terms correct, no extras	
			$k = -2$	A1		
				[3]		

Question			Answer	Marks	Guidance	
6	(i)		$\dfrac{p-7}{-4-^{-}2}$ or $\dfrac{7-p}{-2-^{-}4}$	M1	uses $\dfrac{y_2-y_1}{x_2-x_1}$ (at least 3out of 4 correct)	**Alternative method:** Equation of line through one of the given points with gradient 4 **M1** Substitutes **other point** into their equation **M1**
			$\dfrac{p-7}{-4-^{-}2}=4$ or $\dfrac{7-p}{-2-^{-}4}=4$	A1	Correct, unsimplified **equation**	Obtains $p=-1$ (Accept $y=-1$)**A1**
			$p=-1$	A1 **[3]**		**Note:** Other "informal" methods can score full marks provided **www**
6	(ii)		$\dfrac{^{-}2+6}{2}=m,\quad \dfrac{7+q}{2}=5$	M1	Correct method (may be implied by one correct coordinate)	Use the same marking principle for candidates who add/subtract half the difference to an end point or use similar triangles or other valid "informal" methods.
			$m=2$	A1		
			$q=3$	A1 **[3]**		
6	(iii)		$\sqrt{(-2-d)^2+(7-3)^2}$	*M1	Correct method to find line length/square of line length using Pythagoras' theorem (at least 3out of 4 correct)	
			$d^2+4d+20=52$	B1	$\left(2\sqrt{13}\right)^2=52$ or $2\sqrt{13}=\sqrt{52}$	
			$d^2+4d-32=0$ $(d+8)(d-4)=0$	DM1	Correct method to solve 3 term quadratic, must involve their "52"	**SC: B1** for each value of d found or "spotted" from correct working
			$d=-8$ or 4	A1 **[4]**		**Note:** Other "informal" methods can score full marks provided **www**

Question			Answer	Marks	Guidance	
7	(i)		$y = 9x^5$	M1	Obtain kx^5	If individual terms are differentiated then **M0A0B0**
				A1	Correct expression for y $(9x^5)$	
			$\frac{dy}{dx} = 45x^4$	B1 ft **[3]**	Follow through from their single kx^n, $n \neq 0$. Must be simplified.	$\frac{3x^2 + x^4}{x}$ is **not** a misread **M0A0B0**
7	(ii)		$y = x^{\frac{1}{3}}$	B1	$\sqrt[3]{x} = x^{\frac{1}{3}}$	
				B1	$kx^{-\frac{2}{3}}$	**SC** $\sqrt[3]{x} = x^{-\frac{1}{3}}$ differentiated to $-\frac{1}{3}x^{-\frac{4}{3}}$ **B1**
			$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$	B1 **[3]**	$\frac{1}{3}x^{-\frac{2}{3}}$. Allow $0.\dot{3}$ (not finite)	
7	(iii)		$y = \frac{1}{2}x^{-3}$	M1	kx^{-4} seen	
			$\frac{dy}{dx} = -\frac{3}{2}x^{-4}$	A1 **[2]**		
8			$(3k-1)^2 - 4 \times k \times -4$	*M1	Attempts $b^2 - 4ac$ or an equation or inequality involving b^2 and $4ac$. Must involve k^2 in first term (but no x anywhere). If $b^2 - 4ac$ not stated, must be clear attempt.	Must be working with the discriminant explicitly and not only as part of the quadratic formula. Allow $\sqrt{b^2 - 4ac}$ for first **M1 A1**
			$= 9k^2 + 10k + 1$	A1	Correct discriminant, simplified to 3 terms	
			$9k^2 + 10k + 1 < 0$	M1	States discriminant < 0 or $b^2 < 4ac$.	Can be awarded at any stage. Doesn't need first M1. No square root here.
			$(9k+1)(k+1) < 0$	DM1	Correct method to find roots of a three term quadratic	
			$-1, -\frac{1}{9}$	A1	Both values of k correct	
			$-1 < k < -\frac{1}{9}$	M1	Chooses "inside region" of inequality	Allow correct region for their inequality
				A1 **[7]**	Allow "$k < -\frac{1}{9}$ **and** $k > -1$" etc. must be strict inequalities for A mark	Do not allow "$k < -\frac{1}{9}$ **or** $k > -1$";

Question			Answer	Marks	Guidance	
9	(i)		Centre $(1, -5)$ $(x-1)^2 + (y+5)^2 - 19 - 1 - 25 = 0$ $(x-1)^2 + (y+5)^2 = 45$ Radius $= \sqrt{45}$	B1 M1 A1 **[3]**	Correct centre Correct method to find r^2 Correct radius. **Do not allow if wrong centre used in calculation of radius.**	$r^2 = (\pm 5)^2 + (\pm 1)^2 + 19$ for the M mark **A0** if $\pm\sqrt{45}$
9	(ii)		$7^2 + (-2)^2 - 14 - 20 - 19$ $= 0$	B1 **[1]**	Substitution of coordinates into equation of circle in any form or use of Pythagoras' theorem to calculate the distance of (7, -2) from *C*	**No follow through** for this part as **AG. Must be consistent–** do not allow finding the distance as $\sqrt{45}$ if no/wrong radius found in 9(i).
9	(iii)		gradient of radius $= \frac{-5-(-2)}{1-7}$ or $\frac{-2-(-5)}{7-1}$ $= \frac{1}{2}$ gradient of tangent $= -2$ $y + 2 = -2(x-7)$ $2x + y - 12 = 0$	M1 A1√ B1√ M1 A1 **[5]**	uses $\frac{y_2 - y_1}{x_2 - x_1}$ with **their C** (3/4 correct) Follow through from their C, allow unsimplified single fraction e.g. $\frac{-3}{-6}$ Follow through from their gradient, even if M0 scored. Allow $\frac{-1}{\text{their fraction}}$ **B1** correct equation of straight line through (7, −2), any non-zero numerical gradient oe 3 term equation in correct form i.e. $k(2x + y - 12) = 0$ where k is an integer **cao**	**Follow through from 9(i) until final mark.** If (−1,5) is used for C, then expect Gradient of radius $= \frac{5-(-2)}{-1-7} = -\frac{7}{8}$ Gradient of tangent $= \frac{8}{7}$ <u>Alternative markscheme for implicit differentiation</u>: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x + 2y\frac{dy}{dx} - 2 + 10\frac{dy}{dx} = 0$ **A1** Substitution of (7, −2) to obtain gradient of tangent = −2 Then **M1 A1** as main scheme

Question			Answer	Marks	Guidance	
10			$\frac{dy}{dx} = x^2 - 9x^{-2}$	B1	x^2 from differentiating first term	
				M1	kx^{-2}	
				A1	$-9x^{-2}$ (no + c)	
			Gradient of line = 8	B1		
			$x^2 - 9x^{-2} = 8$	M1	Equate their $\frac{dy}{dx}$ to 8 (or their gradient of line, if clear)	**Note:** If equated to +/-1/8 then M0 but the next M1 and its dependencies are available
			$x^4 - 8x^2 - 9 = 0$ $k^2 - 8k - 9 = 0$	*M1	Use a correct substitution to obtain a 3 term quadratic or factorise into 2 brackets each containing x^2	If no substitution stated and treated as a quadratic (e.g. quadratic formula), no more marks until square rooting seen
			$(k - 9)(k + 1) = 0$	DM1	Correct method to solve 3 term quadratic – **dependent on previous M1**	**SC:** If spotted after first five marks- (3, 12) **B1** (-3, -12) **B1** Justifies exactly two solutions **B3**
			$k = 9$ (don't need $k = -1$)	A1	No extras	
			$x = 3,\ -3$	DM1	Attempt to find x by square rooting – accept one value	
			$y = 12,\ -12$	A1 **[10]**	No extras	

More Additional Guidance for Q10

If curve equated to line and before differentiating:

First four marks **B1 M1 A1 B1** available as main scheme
Then **M0** for equating as this not been explicitly done
Allow the **M1** for the substitution
DM1 for quadratic as main scheme (dependent on a correct substitution)
A0 for the 9 (as follows wrong working)
DM1 for square rooting (dependent on a correct substitution)
A0 for the co-ordinates (as follows wrong working). Max mark **7/10**

Allocation of method mark for solving a quadratic

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times 2}$	earns **M1** (minus sign incorrect at start of formula)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$	earns **M1** (18 for *c* instead of −18)
$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$	**M0** (2 sign errors: initial sign and *c* incorrect)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times -5}$	**M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

← This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.