Friday 13 January 2012 – Morning

AS GCE MATHEMATICS

4721 Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer book 4721
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

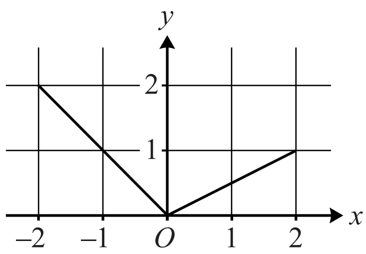
INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

No calculator can be used for this paper

1 Express $\dfrac{15+\sqrt{3}}{3-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers. **[4]**

2



The graph of $y = \mathrm{f}(x)$ for $-2 \leqslant x \leqslant 2$ is shown above.

(i) Sketch the graph of $y = \mathrm{f}(-x)$ for $-2 \leqslant x \leqslant 2$. **[2]**

(ii) Sketch the graph of $y = \mathrm{f}(x) + 2$ for $-2 \leqslant x \leqslant 2$. **[2]**

3 Given that

$$5x^2 + px - 8 = q(x-1)^2 + r$$

for all values of x, find the values of the constants p, q and r. **[4]**

4 Evaluate

(i) 3^{-2}, **[1]**

(ii) $16^{\frac{3}{4}}$, **[2]**

(iii) $\dfrac{\sqrt{200}}{\sqrt{8}}$. **[2]**

5 Find the real roots of the equation $\dfrac{3}{y^4} - \dfrac{10}{y^2} - 8 = 0$. **[5]**

6 Given that $\mathrm{f}(x) = \dfrac{4}{x} - 3x + 2$,

(i) find $\mathrm{f}'(x)$, **[3]**

(ii) find $\mathrm{f}''\left(\frac{1}{2}\right)$. **[4]**

7 A curve has equation $y = (x+2)(x^2 - 3x + 5)$.

(i) Find the coordinates of the minimum point, justifying that it is a minimum. **[8]**

(ii) Calculate the discriminant of $x^2 - 3x + 5$. **[2]**

(iii) Explain why $(x+2)(x^2 - 3x + 5)$ is always positive for $x > -2$. **[2]**

8 The line l has gradient -2 and passes through the point $A(3, 5)$. B is a point on the line l such that the distance AB is $6\sqrt{5}$. Find the coordinates of each of the possible points B. **[6]**

9 **(i)** Sketch the curve $y = 12 - x - x^2$, giving the coordinates of all intercepts with the axes. **[5]**

(ii) Solve the inequality $12 - x - x^2 > 0$. **[2]**

(iii) Find the coordinates of the points of intersection of the curve $y = 12 - x - x^2$ and the line $3x + y = 4$. **[5]**

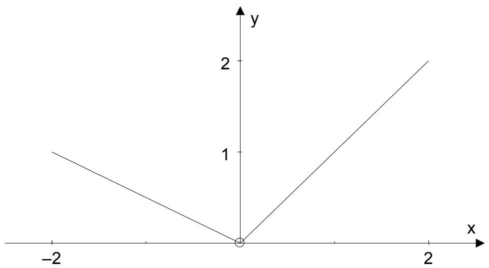
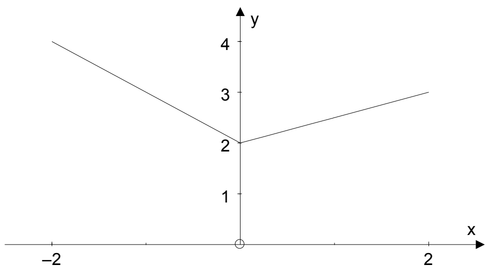
10 A circle has centre $C(-2, 4)$ and radius 5.

(i) Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

(ii) Show that the tangent to the circle at the point $P(-5, 8)$ has equation $3x - 4y + 47 = 0$. **[5]**

(iii) Verify that the point $T(3, 14)$ lies on this tangent. **[1]**

(iv) Find the area of the triangle CPT. **[4]**

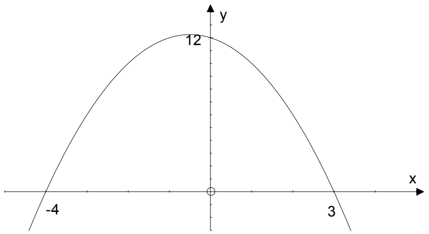
Question			Answer	Marks	Guidance	
1			$\frac{15+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$	M1	Multiply top and bottom by $\pm(3+\sqrt{3})$	**SC** If A0A0A0 scored, both parts correct but unsimplified **B1** i.e. $\frac{45+15\sqrt{3}+3\sqrt{3}+3}{9+3\sqrt{3}-3\sqrt{3}-3}$ o.e.
			$=\frac{48+18\sqrt{3}}{9-3}$	A1	Numerator correct and simplified	<u>Alternative method:</u> Equates expression to $a+b\sqrt{3}$ and forms simultaneous equations in a and b **M1**
				A1	Denominator correct and simplified to 6	Correct method to solve simultaneous equations **M1**
			$=8+3\sqrt{3}$	A1	cao	a = 8 found **A1**
				[4]		b = 3 found **A1**
2	(i)			M1	Reflection of given graph in either axis	
				A1	Correct reflection in y-axis	Clear intention to show (-2, 1), (0,0), (2,2) by numbers, dashes or co-ordinates **A0** If significantly short or long
				[2]		
2	(ii)			M1	Translation of **given** graph vertically (up or down)	
				A1	Correct translation of two units vertically	Clear intention to show (-2, 4), (0,2), (2,3) by numbers, dashes or co-ordinates **A0** If significantly short or long
				[2]		

Question			Answer	Marks	Guidance	
3			$5x^2 + px - 8 = 5(x-1)^2 + r$ $= 5(x^2 - 2x + 1) + r$ $= 5x^2 - 10x + 5 + r$ $p = -10$ $r = -13$	B1 B1 M1 A1 **[4]**	$q = 5$ (may be embedded on RHS) $p = -10$ $-8 = \pm q + r$ or $\frac{-p^2}{20} - 8 = r$ $r = -13$	Allow from $p = 10$
4	(i)		$\frac{1}{9}$	B1 **[1]**		
4	(ii)		$\left(\sqrt[4]{16}\right)^3$ $= 8$	M1 A1 **[2]**	Interprets the power $\frac{3}{4}$ correctly ± 8 is **A0**	$(\sqrt[4]{16})^3$ or $(\sqrt[4]{16^3})$ or $\left(16^{\frac{1}{4}}\right)^3$ or $\left(16^3\right)^{\frac{1}{4}}$
4	(iii)		$5\sqrt{8} \div \sqrt{8}$ $= 5$	M1 A1 **[2]**	$\sqrt{100}\ \sqrt{2} \div \sqrt{4}\ \sqrt{2}$ or $\sqrt{\frac{200}{8}}$ or $\sqrt{25}\ \sqrt{8} \div \sqrt{8}$ or $\sqrt{1600} \div 8$ soi Condone ± 5	

Question			Answer	Marks	Guidance	
5			$k=\frac{1}{y^2}$	M1*	Use a correct substitution or pair of substitutions to obtain a quadratic or factorise into 2 brackets each containing $\frac{1}{y^2}$	**No marks** if straight to quadratic formula to get $y=$ "$-\frac{2}{3}$", $y=$ "4" unless correct substitution applied later i.e. reciprocal and square root
			$3k^2-10k-8=0$ $(3k+2)(k-4)=0$	M1dep	Correct method to solve a quadratic	
			$k=-\frac{2}{3}$ or $k=4$	A1	$k=4$ from correct method. If other root stated it must be correct.	**No marks** if quadratic found from incorrect substitution
			$y^2=-\frac{3}{2}$ or $y^2=\frac{1}{4}$	M1	Attempt to reciprocal and square root to obtain y (either term)	**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
			$y=\pm\frac{1}{2}$	A1	No other roots given. Must be from $k=4$ from correct method.	
				[5]		
			Alternative method below: $3-10y^2-8y^4=0$ $k=y^2$ $8k^2+10k-3=0$ M1* $(4k-1)(2k+3)=0$ M1 dep			
			$k=\frac{1}{4}$ or $k=-\frac{3}{2}$ A1		$k=\frac{1}{4}$ from correct method. If other root stated it must be correct.	
			$y=\pm\frac{1}{2}$ M1 A1			

Question			Answer	Marks	Guidance	
6	(i)		$f'(x) = -4x^{-2} - 3$	M1 A1 A1 **[3]**	Attempt to differentiate $-4x^{-2}$ Fully correct derivative (no "+ *c*")	kx^{-2} or -3 correctly obtained
6	(ii)		$f''(x) = 8x^{-3}$ $f''\left(\frac{1}{2}\right) = \frac{8}{\left(\frac{1}{2}\right)^3}$ $= 64$	M1* A1 M1dep A1 **[4]**	Attempts to differentiate their **(i)** Correct derivative Substitutes $x = \frac{1}{2}$ correctly into their $f''(x)$ e.g. $8\left(\frac{1}{2}\right)^{-3}$ (allow "invisible brackets") **www**	Must involve reducing power of an *x* term by 1 $f''(x)$ must involve *x*.
7	(i)		$x^3 - 3x^2 + 5x + 2x^2 - 6x + 10$ $= x^3 - x^2 - x + 10$ $\frac{dy}{dx} = 3x^2 - 2x - 1$ $(3x + 1)(x - 1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $\frac{d^2y}{dx^2} = 6x - 2$, $x = 1$ gives +ve (4) Min point at $x = 1$ $y = 9$ found	M1 M1 M1* M1 A1 M1dep A1 A1 **[8]**	Attempt to multiply out brackets Attempt to differentiate their cubic Sets their $\frac{dy}{dx} = 0$ Correct method to solve quadratic Correct *x* values of turning points found **www** Valid method to establish which is min point with a conclusion Correct conclusion for $x = 1$ found from correct factorisation (even if other root incorrect) **www** for (1, 9) given as minimum point (ignore other point here)	<u>Alternative for product rule</u> Attempt to use product rule **M1** Expand brackets of both parts **M1** Then as main scheme Any extra values for turning points loses all three **A** marks (eg by sketching positive cubic, second diff method for either of their *x* values, *y* co-ords etc.) If constant incorrect in initial expansion, max **5/8**

Question			Answer	Marks	Guidance	
7	(ii)		$(-3)^2 - 4 \times 1 \times 5$ $= -11$	M1 A1 **[2]**	Uses $b^2 - 4ac$	$\sqrt{b^2 - 4ac}$ is **M0**
7	(iii)			B2 **[2]**	Fully correct argument - no extra incorrect statements e.g. 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ and stating it's a positive cubic 2) Sketch of positive cubic with one root at (–2, 0) and a min point at (1, 9) (f/t positive $y(1)$ from (i))	Award **B1** for either of: 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ 2) Sketch of positive cubic with one root at (–2, 0) and a min point with y coordinate positive or 0
8			B lies on l so has coordinates $(x, 11 - 2x)$ $(x-3)^2 + (11-2x-5)^2 = (6\sqrt{5})^2$ $5x^2 - 30x - 135 = 0$ $5(x+3)(x-9) = 0$ $x = -3,\ x = 9$ $y = 17,\ y = -7$	M1 M1 M1* M1dep A1 A1 **[6]**	Attempt to find equation of l with gradient -2 $(x-3)^2 + (y - 5)^2 = (6\sqrt{5})^2$ o.e. seen Attempts to solve the equations simultaneously to get a quadratic Correct method to solve their quadratic Both x values Both y values	e.g. by substitution as shown **SC** If A0 A0, one correct pair of values from correct factorisation **www B1**
			Alternative method: Use of (1, 2, $\sqrt{5}$) triangle with –ve gradient M1 Scaling to $6\sqrt{5}$ M1 (3, 5) + (6, -12) M1 (9, -7) A1 (3, 5) - (6, -12) M1 (-3, 17) A1		**SC** Spotted solutions Each correct pair **www B1** (May also earn first two Ms as in main scheme)* **-1** for one or two extra incorrect solutions **-2** for three or more extra incorrect solutions Checks solutions and justifies only two solutions **B2** *** NB –** First M1 may also be awarded for establishing gradient between (3,5) and their solution(s) is – 2	

Question			Answer	Marks	Guidance	
9	(i)		$(x-3)(x+4)=0$ $x=3$ or $x=-4$  	M1 A1 B1 B1 B1 **[5]**	Correct method to find roots Correct roots Negative quadratic curve y intercept (0, 12) Good curve, with correct roots 3 and –4 indicated and max point in 2nd quadrant	 i.e. max at (0, 12) **B0** Curve must go below x-axis for final mark
9	(ii)		$-4 < x < 3$	M1 A1 **[2]**	Correct method to solve quadratic inequality Allow ≤ for the method mark but not the accuracy mark	their lower root < x < their higher root Allow "$x > -4, x < 3$" Allow "$x > -4$ and $x < 3$" Do not allow "$x > -4$ **or** $x < 3$"
9	(iii)		$y = 4 - 3x$ $12 - x - x^2 = 4 - 3x$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2)=0$ $x=4$ or $x=-2$ $y=-8$ or $y=10$	 M1 A1 M1 A1 A1 **[5]**	 substitute for x/y or attempt to get an equation in 1 variable only obtain correct 3 term quadratic correct method to solve 3 term quadratic	e.g. for first mark $3x + 12 - x - x^2 = 4$, or $y = 12 - \left(\frac{4-y}{3}\right) - \left(\frac{4-y}{3}\right)^2$ (this leads to $y^2 - 2y - 80 = 0$). Condone poor algebra for this mark. **SC** If **A0 A0**, give **B1** for one correct pair of values spotted or from correct factorisation **www**

Question			Answer	Marks	Guidance	
10	(i)		$(x+2)^2+(y-4)^2=25$ $x^2+4x+4+y^2-8y+16-25=0$ $x^2+y^2+4x-8y-5=0$	M1 M1 A1 **[3]**	$(x+2)^2$ and $(y-4)^2$ seen (or implied by x^2+4x+y^2-8y) $(x\pm 2)^2+(y\pm 4)^2 = 25$ Correct equation in correct form (terms can be in any order but must have "=0")	<u>Alternative markscheme for *f, g, c* method</u>: x^2+4x+y^2-8y **B1** $c = 2^2 + (\pm 4)^2 - 25$ **M1** Correct equation in correct form **A1**
10	(ii)		gradient of radius = $\frac{8-4}{-5+2}$ $= -\frac{4}{3}$ gradient of tangent = $\frac{3}{4}$ $y-8=\frac{3}{4}(x+5)$ $3x-4y+47=0$	M1 A1 B1FT M1 A1 **[5]**	uses $\frac{y_2-y_1}{x_2-x_1}$ (3/4 substitutions correct) Allow $\frac{4}{-3}$ correct equation of straight line through (– 5, 8), any non-zero gradient Shows rearrangement to given equation **AG** **CWO** throughout for A1	
			<u>Alternative by rearrangement</u> Gradient of radius = $\frac{8-4}{-5+2}$ = $\frac{-4}{3}$ **M1* A1** Attempts to rearrange equation of line to find gradient of line = $\frac{3}{4}$ **M1dep** Multiply gradients to get -1 **B1** Check (-5, 8) lies on line **B1 (dep on both M1s)**		<u>Alternative for equating given line to circle</u> Substitute for *x*/*y* or attempt to get an equation in 1 variable only **M1** $k(x^2 + 10x + 25) = 0$ or $k(y^2 - 16y + 64) = 0$ **A1** Correct method to solve quadratic **M1** x = -5, y = 8 found **A1** States one root implies tangent **B1**	<u>Alternative markscheme for implicit differentiation</u>: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term **A1ft** $2x+2y\frac{dy}{dx}+4-8\frac{dy}{dx}=0$ ft from their equation in (i) **A1** Substitution of (-5, 8) to obtain $\frac{3}{4}$ then final 2 marks as main scheme

Question			Answer	Marks	Guidance	
10	(iii)		(3 × 3) – (4 × 14) + 47 = 0	B1 **[1]**	Sufficient correct working to verify statement e.g. verifying co-ordinate as shown	Alt: showing line joining (-5, 8) to (3, 14) has same gradient etc.
10	(iv)		$\sqrt{(3-{}^{-}5)^2+(14-8)^2}$ $=10$ Area of triangle $=\frac{1}{2}\times 10\times 5$ $=25$	M1 A1 M1 A1 **[4]**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ for *TP* Must use their *TP* and their *CP*	Alternative method: Attempt to find area of enclosing rectangle and subtract areas of other three triangles **M1*** Correct use area of triangle formula **M1 dep** All four values correct **A1** Final answer correct **A1** (Use the same principle for any enclosing shape)