

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Core Mathematics 1

QUESTION PAPER

4721

Candidates answer on the Printed Answer Book

OCR Supplied Materials:

- Printed Answer Book 4721
- List of Formulae (MF1)

Other Materials Required:

None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

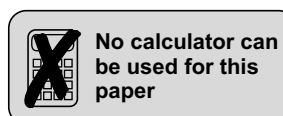
These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** If you need more space for an answer use a 4-page answer book; label your answer clearly. Write your Centre Number and Candidate Number on the 4-page answer book and attach it securely to the Printed Answer Book.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

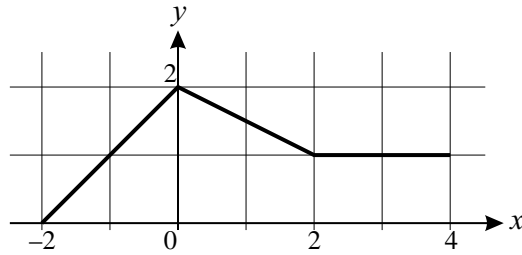
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.



1 Express $x^2 - 12x + 1$ in the form $(x - p)^2 + q$. [3]

2



The graph of $y = f(x)$ for $-2 \leq x \leq 4$ is shown above.

(i) Sketch the graph of $y = 2f(x)$ for $-2 \leq x \leq 4$ on the axes provided. [2]

(ii) Describe the transformation which transforms the graph of $y = f(x)$ to the graph of $y = f(x - 1)$. [2]

3 Find the equation of the normal to the curve $y = x^3 - 4x^2 + 7$ at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [7]

4 Solve the equations

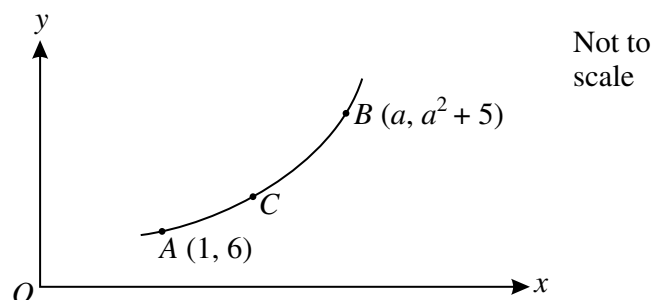
(i) $3^m = 81$, [1]

(ii) $(36p^4)^{\frac{1}{2}} = 24$, [3]

(iii) $5^n \times 5^{n+4} = 25$. [3]

5 Solve the equation $x - 8\sqrt{x} + 13 = 0$, giving your answers in the form $p \pm q\sqrt{r}$, where p , q and r are integers. [7]

6



The diagram shows part of the curve $y = x^2 + 5$. The point A has coordinates $(1, 6)$. The point B has coordinates $(a, a^2 + 5)$, where a is a constant greater than 1. The point C is on the curve between A and B .

(i) Find by differentiation the value of the gradient of the curve at the point A . [2]

(ii) The line segment joining the points A and B has gradient 2.3. Find the value of a . [4]

(iii) State a possible value for the gradient of the line segment joining the points A and C . [1]

7

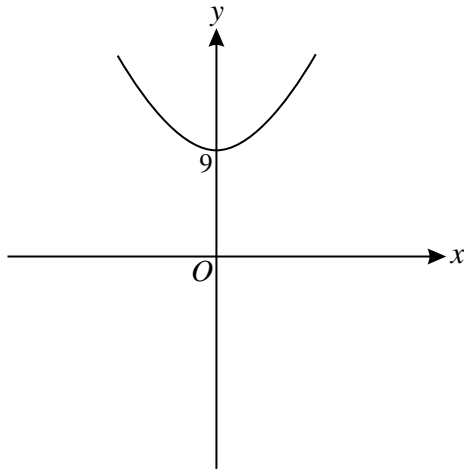


Fig. 1

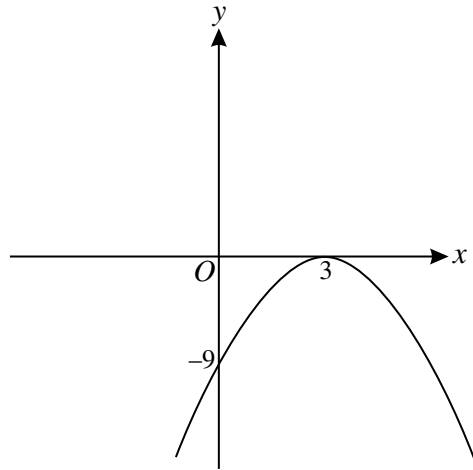


Fig. 2

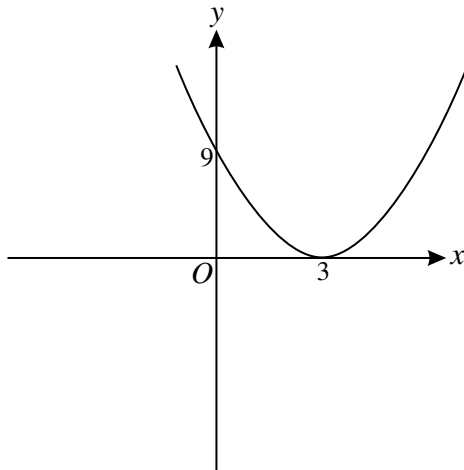


Fig. 3

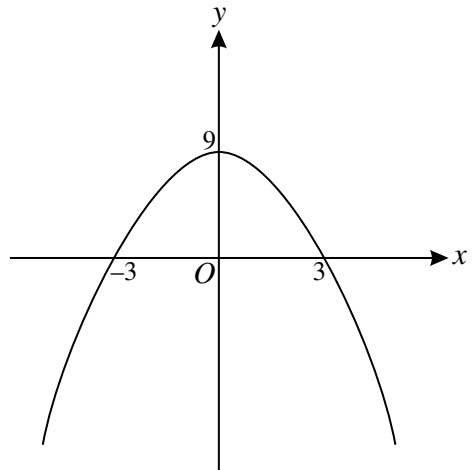


Fig. 4

(i) Each diagram shows a quadratic curve. State which diagram corresponds to the curve

(a) $y = (3 - x)^2$, [1]

(b) $y = x^2 + 9$, [1]

(c) $y = (3 - x)(x + 3)$. [1]

(ii) Give the equation of the curve which does not correspond to any of the equations in part (i). [2]

8 A circle has equation $x^2 + y^2 + 6x - 4y - 4 = 0$.

(i) Find the centre and radius of the circle. [3]

(ii) Find the coordinates of the points where the circle meets the line with equation $y = 3x + 4$. [6]

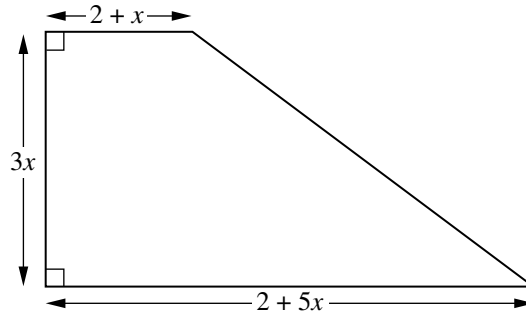
9 Given that $f(x) = \frac{1}{x} - \sqrt{x} + 3$,

(i) find $f'(x)$, [3]

(ii) find $f''(4)$. [5]

10 The quadratic equation $kx^2 - 30x + 25k = 0$ has equal roots. Find the possible values of k . [4]

11 A lawn is to be made in the shape shown below. The units are metres.



(i) The perimeter of the lawn is P m. Find P in terms of x . [2]

(ii) Show that the area, A m², of the lawn is given by $A = 9x^2 + 6x$. [2]

The perimeter of the lawn must be at least 39 m and the area of the lawn must be less than 99 m².

(iii) By writing down and solving appropriate inequalities, determine the set of possible values of x . [7]

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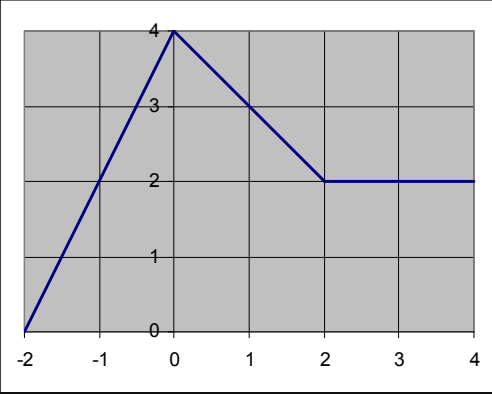
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4721 Core Mathematics 1

1	$[(x-6)^2 - 36] + 1$ $= (x-6)^2 - 35$	B1	$(x-6)^2$
		M1	$q = 1 - (\text{their } p)^2$
		A1	$q = -35$
			3
2	(i)		
		B1	For $x < 0$, straight line joining $(-2, 0)$ and $(0, 4)$
		B1	2 For $x > 0$, line joining $(0, 4)$ to $(2, 2)$ and horizontal line joining $(2, 2)$ and $(4, 2)$
	(ii)		
	Translation 1 unit right parallel to x axis	B1	
		B1	2 Allow: 1 unit right, 1 along the x axis, 1 in direction , allow vector notation e.g. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, 1 unit horizontally
			4
3	$\frac{dy}{dx} = 3x^2 - 8x$	M1	Attempt to differentiate (one of $3x^2, -8x$)
		A1	Correct derivative
	When $x = 2$, $\frac{dy}{dx} = -4$	M1	Substitutes $x = 2$ into their $\frac{dy}{dx}$
		A1	
	\therefore Gradient of normal to curve = $\frac{1}{4}$	B1 ft	Must be numerical $= -1 \div \text{their } m$
	$y + 1 = \frac{1}{4}(x - 2)$	M1	Correct equation of straight line through $(2, -1)$, any non-zero numerical gradient
	$x - 4y - 6 = 0$	A1	7 Correct equation in required form
			7

4	(i)	$m = 4$	B1	1	May be embedded
	(ii)	$6p^2 = 24$ $p^2 = 4$ $p = 2$ or $p = -2$	M1 A1 A1	3	$(\pm)6p^2 = 24$ or $36p^4 = 576$
	(iii)	$5^{2n+4} = 25$ $\therefore 2n+4 = 2$ $n = -1$	M1 M1 A1	3 7	Addition of indices as powers of 5 Equate powers of 5 or 25
5		$k = \sqrt{x}$ $k^2 - 8k + 13 = 0$ $k - 4 = \pm\sqrt{3}$ or $k = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 13}}{2}$ $k = 4 \pm \sqrt{3}$ $\therefore x = (4 + \sqrt{3})^2$ or $x = (4 - \sqrt{3})^2$ $x = 19 \pm 8\sqrt{3}$ or $19 \pm 4\sqrt{12}$	M1* M1 dep A1 A1 M1 M1 A1	7 7	Use a substitution to obtain a quadratic (may be implied by squaring or rooting later) or factorise into 2 brackets each containing \sqrt{x} Correct method to solve resulting quadratic $k = 4 \pm \sqrt{3}$ or $k = \frac{8 \pm \sqrt{12}}{2}$ or $k = 4 \pm \frac{\sqrt{12}}{2}$ Recognise the need to square to obtain x Correct method for squaring $a + \sqrt{b}$ (3 or 4 term expansion)
6	(i)	$\frac{dy}{dx} = 2x$ When $x = 1$, $\frac{dy}{dx} = 2$	B1* B1 dep	2	
	(ii)	$\frac{a^2 + 5 - 6}{a - 1} = 2.3$ $a^2 - 2.3a + 1.3 = 0$ $(a - 1.3)(a - 1) = 0$ $a = 1.3$	M1 A1 M1 A1	4	uses $\frac{y_2 - y_1}{x_2 - x_1}$ correct expression correct method to solve a quadratic or correct factorisation and cancelling to get $a + 1 = 2.3$ 1.3 only

Alternative method:			
Equation of straight line through (1,6) with $m = 2.3$ found then			
$a^2 + 5 = 2.3a + "c"$ seen M1			
with $c = 3.7$ A1			
then as main scheme			
	(iii)	A value between 2 and 2.3	B1 1 7 2 < value < 2.3 (strict inequality signs)
7	(i)	(a) Fig 3 (b) Fig 1 (c) Fig 4	B1 B1 B1 3
	(ii)	$-(x-3)^2$	M1 Quadratic expression with correct x^2 term and correct y -intercept and/or roots for their unmatched diagram (e.g. negative quadratic with y -intercept of -9 or root of 3 for Fig 2)
		$y = -(x-3)^2$	A1 2 5 Completely correct equation for Fig 2
8	(i)	Centre $(-3, 2)$ $(x+3)^2 - 9 + (y-2)^2 - 4 - 4 = 0$ $r^2 = 17$ $r = \sqrt{17}$	B1 M1 Correct method to find r^2 A1 3 Correct radius
	(ii)	$x^2 + (3x+4)^2 + 6x - 4(3x+4) - 4 = 0$	M1* substitute for x/y or attempt to get an equation in 1 variable only
			A1 correct unsimplified expression
		$10x^2 + 18x - 4 = 0$ $(5x-1)(x+2) = 0$ $x = \frac{1}{5}$ or $x = -2$	A1 obtain correct 3 term quadratic M1 correct method to solve their quadratic dep A1
		$y = \frac{23}{5}$ or $y = -2$	A1 6 SR If A0 A0, one correct pair of values, spotted or from correct factorisation www B1
			9
9	(i)	$f'(x) = -x^{-2} - \frac{1}{2}x^{-\frac{1}{2}}$	M1 Attempt to differentiate
			A1 $-x^{-2}$ or $-\frac{1}{2}kx^{-\frac{1}{2}}$ www
			A1 3 Fully correct expression

(ii)	$f''(x) = 2x^{-3} + \frac{1}{4}x^{-\frac{3}{2}}$	M1	Attempt to differentiate their $f'(x)$
		A1 ft	One correctly differentiated term
		A1	Fully correct expression www in either part of the question
	$f''(4) = \frac{2}{4^3} + \frac{1}{4} \cdot \frac{1}{8}$ $= \frac{1}{16}$	M1	Substitution of $x = 4$ into their $f''(x)$
		A1	oe single fraction www in either part of the question
10	$(-30)^2 - 4 \times k \times 25k = 0$	M1	Attempts $b^2 - 4ac$ involving k
	$900 - 100k^2 = 0$	M1	States their discriminant = 0
	$k = 3$	B1	
	or $k = -3$	B1	
		4	
		4	
11	(i) $P = 2 + x + 3x + 2 + 5x + 5x$ $= 14x + 4$	M1	Adds lengths of all 4 edges with attempt to use Pythagoras to find the missing length
		A1	2
			May be left unsimplified
	(ii) Area of rectangle = $3x(2 + x) = 6x + 3x^2$	M1	Correct method – splitting or formula for area of trapezium
	Area of triangle = $\frac{1}{2}(3x)(4x) = 6x^2$		
	Total area = $9x^2 + 6x$	A1	2
			Convincing working leading to given expression AG
	(iii) $14x + 4 \geq 39$	B1 ft	ft on their expression for P from (i) unless restarted in (iii). (Allow $>$)
	$\frac{5}{2}$	B1	o.e. (e.g. $\frac{35}{14}$) soi by subsequent working
	$9x^2 + 6x < 99$	B1	
	$3x^2 + 2x - 33 < 0$		
	$(3x + 11)(x - 3) < 0$	M1	Allow \leq
	$\left(-\frac{11}{3} < \right)x < 3$		Correct method to find critical values
		B1	$x < 3$ identified
		M1	root from linear $< x <$ upper root from quadratic
	$\therefore \frac{5}{2} \leq x < 3$	A1	7
			11
			Fully correct including inequality signs or exact equivalent in words cwo
Total			72