OCR

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4721/01**

MATHEMATICS

Core Mathematics 1

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

[Turn over

1 Express $\frac{4}{3-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers. [3]

2 (i) Write down the equation of the circle with centre (0, 0) and radius 7. [1]

(ii) A circle with centre (3, 5) has equation $x^2 + y^2 - 6x - 10y - 30 = 0$. Find the radius of the circle. [2]

3 Given that $3x^2 + bx + 10 = a(x+3)^2 + c$ for all values of x, find the values of the constants a, b and c. [4]

4 Solve the equations

(i) $10^p = 0.1$, [1]

(ii) $(25k^2)^{\frac{1}{2}} = 15$, [3]

(iii) $t^{-\frac{1}{3}} = \frac{1}{2}$. [2]

5 (i) Sketch the curve $y = x^3 + 2$. [2]

(ii) Sketch the curve $y = 2\sqrt{x}$. [2]

(iii) Describe a transformation that transforms the curve $y = 2\sqrt{x}$ to the curve $y = 3\sqrt{x}$. [3]

6 (i) Solve the equation $x^2 + 8x + 10 = 0$, giving your answers in simplified surd form. [3]

(ii) Sketch the curve $y = x^2 + 8x + 10$, giving the coordinates of the point where the curve crosses the y-axis. [3]

(iii) Solve the inequality $x^2 + 8x + 10 \geqslant 0$. [2]

7 (i) Find the gradient of the line l which has equation $x + 2y = 4$. [1]

(ii) Find the equation of the line parallel to l which passes through the point (6, 5), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [3]

(iii) Solve the simultaneous equations

$$y = x^2 + x + 1 \qquad \text{and} \qquad x + 2y = 4.$$ [4]

8 (i) Find the coordinates of the stationary points on the curve $y = x^3 + x^2 - x + 3$. [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iii) For what values of x does $x^3 + x^2 - x + 3$ decrease as x increases? [2]

9 The points A and B have coordinates $(-5, -2)$ and $(3, 1)$ respectively.

(i) Find the equation of the line AB, giving your answer in the form $ax + by + c = 0$. [3]

(ii) Find the coordinates of the mid-point of AB. [2]

The point C has coordinates $(-3, 4)$.

(iii) Calculate the length of AC, giving your answer in simplified surd form. [3]

(iv) Determine whether the line AC is perpendicular to the line BC, showing all your working. [4]

10 Given that $f(x) = 8x^3 + \frac{1}{x^3}$,

(i) find $f''(x)$, [5]

(ii) solve the equation $f(x) = -9$. [5]

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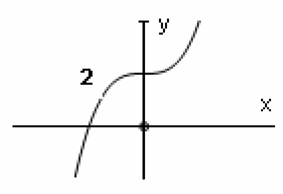
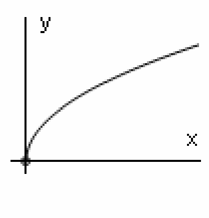
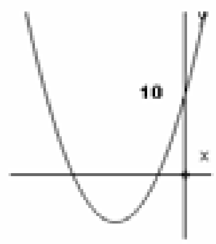
1	$\frac{4(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$ $= \frac{12+4\sqrt{7}}{9-7}$ $= 6 + 2\sqrt{7}$	M1 B1 A1 3 **3**	Multiply top and bottom by conjugate 9 ± 7 soi in denominator $6 + 2\sqrt{7}$
2(i) (ii)	$x^2 + y^2 = 49$ $x^2 + y^2 - 6x - 10y - 30 = 0$ $(x-3)^2 - 9 + (y-5)^2 - 25 - 30 = 0$ $(x-3)^2 + (y-5)^2 = 64$ $r^2 = 64$ $r = 8$	B1 1 M1 A1 2 **3**	$x^2 + y^2 = 49$ 3^2 5^2 30 with consistent signs soi 8 cao
3	$a(x+3)^2 + c = 3x^2 + bx + 10$ $3(x^2 + 6x + 9) + c = 3x^2 + bx + 10$ $3x^2 + 18x + 27 + c = 3x^2 + bx + 10$ $c = -17$	B1 B1 M1 A1 4 **4**	$a = 3$ soi $b = 18$ soi $c = 10 - 9a$ or $c = 10 - \frac{b^2}{12}$ $c = -17$
4(i) (ii) (iii)	$p = -1$ $\sqrt{25k^2} = 15$ $25k^2 = 225$ $k^2 = 9$ $k = \pm 3$ $\sqrt[3]{t} = 2$ $t = 8$	B1 1 M1 A1 A1 3 M1 A1 2 **6**	$p = -1$ Attempt to square 15 or attempt to square root $25k^2$ $k = 3$ $k = -3$ $\frac{1}{t^{\frac{1}{3}}} = \frac{1}{2}$ or $t^{\frac{1}{3}} = 2$ soi $t = 8$

5(i)		B1 B1 2	+ve cubic +ve or -ve cubic with point of inflection at (0, 2) and no max/min points
(ii)		B1 B1 2	curve with correct curvature in +ve quadrant only completely correct curve
(iii)	Stretch scale factor 1.5 parallel to y-axis	B1 B1 B1 3 **7**	stretch factor 1.5 parallel to y-axis or in y-direction
6(i)	EITHER $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-8\pm\sqrt{64-40}}{2}$ $x=\frac{-8\pm\sqrt{24}}{2}$ $x=\frac{-8\pm 2\sqrt{6}}{2}$ $x=-4\pm\sqrt{6}$ OR $(x+4)^2-16+10=0$ $(x+4)^2=6$ $x+4=\pm\sqrt{6}$ M1 A1 $x=\pm\sqrt{6}-4$ A1	M1 A1 A1 3	Correct method to solve quadratic $x=\frac{-8\pm\sqrt{24}}{2}$ $x=-4\pm\sqrt{6}$
(ii)		B1 B1 B1 3	+ve parabola parabola cutting *y*-axis at (0, 10) where (0, 10) is not min/max point parabola with 2 negative roots
(iii)	$x\le-\sqrt{6}-4, x\ge\sqrt{6}-4$	M1 A1 ft 2 **8**	$x\le$ lower root $x\ge$ higher root (allow $<$, $>$) Fully correct answer, ft from roots found in (i)

7(i)	Gradient = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$
(ii)	$y-5=-\frac{1}{2}(x-6)$	M1	Equation of straight line through (6, 5) with any non-zero numerical gradient
		B1 ft	Uses gradient found in (i) in their equation of line
	$2y-10=-x+6$ $x+2y-16=0$	A1 3	Correct answer in correct form (integer coefficients)
(iii)	EITHER $\frac{4-x}{2}=x^2+x+1$	*M1	Substitute to find an equation in x (or y)
	$4-x=2x^2+2x+2$ $2x^2+3x-2=0$ $(2x-1)(x+2)=0$	DM1	Correct method to solve quadratic
	$x=\frac{1}{2}, x=-2$	A1	$x=\frac{1}{2}, x=-2$
	$y=\frac{7}{4}, y=3$	A1 4	$y=\frac{7}{4}, y=3$ **SR** one correct (x,y) pair **www** **B1**
	OR $y=(4-2y)^2+(4-2y)+1$ *M $y=16-16y+4y^2+4-2y+1$ $0=21-19y+4y^2$ $0=(4y-7)(y-3)$ DM1 $y=\frac{7}{4}, y=3$ A1 $x=\frac{1}{2}, x=-2$ A1	8	

8(i)	$\frac{dy}{dx} = 3x^2 + 2x - 1$	*M1 A1	Attempt to differentiate (at least one correct term) 3 correct terms
	At stationary points, $3x^2 + 2x - 1 = 0$	M1	Use of $\frac{dy}{dx} = 0$
	$(3x-1)(x+1) = 0$	DM1	Correct method to solve 3 term quadratic
	$x = \frac{1}{3},\ x = -1$	A1	$x = \frac{1}{3},\ x = -1$
	$y = \frac{76}{27},\ y = 4$	A1 6	$y = \frac{76}{27},\ 4$
			SR one correct (x,y) pair **www** **B1**
(ii)	$\frac{d^2y}{dx^2} = 6x + 2$	M1	Looks at sign of $\frac{d^2y}{dx^2}$ for at least one of their x-values or other correct method
	$x = \frac{1}{3},\ \frac{d^2y}{dx^2} > 0$	A1	$x = \frac{1}{3}$, minimum point CWO
	$x = -1,\ \frac{d^2y}{dx^2} < 0$	A1 3	$x = -1$, maximum point CWO
(iii)	$-1 < x < \frac{1}{3}$	M1	Any inequality (or inequalities) involving both their x values from part (i)
		A1 2	Correct inequality (allow < or ≤)
		11	

9(i)	Gradient of AB = $\frac{-2-1}{-5-3}$ $= \frac{3}{8}$	B1	$\frac{3}{8}$ oe
	$y-1=\frac{3}{8}(x-3)$	M1	Equation of line through either A or B, any non-zero numerical gradient
	$8y-8=3x-9$ $3x-8y-1=0$	A1 3	Correct equation in correct form
(ii)	$\left(\frac{-5+3}{2},\frac{-2+1}{2}\right)$	M1	Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
	$=(-1,\ -\frac{1}{2})$	A1 2	$(-1,\ -\frac{1}{2})$
(iii)	$AC=\sqrt{(-5+3)^2+(-2-4)^2}$	M1	Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
	$=\sqrt{2^2+6^2}$ $=\sqrt{40}$	A1	$\sqrt{40}$
	$=2\sqrt{10}$	A1 3	Correctly simplified surd
(iv)	Gradient of AC $=\frac{-2-4}{-5+3}=3$	B1	3 oe
	Gradient of BC $=\frac{4-1}{-3-3}=-\frac{1}{2}$	B1	$-\frac{1}{2}$ oe
	$3\times-\frac{1}{2}\neq-1$ so lines are not	M1	Attempts to check $m_1 \times m_2$
	perpendicular	A1 4	Correct conclusion **www**
		12	

10(i)	$24x^2 - 3x^{-4}$	B1 B1 B1	$24x^2$ kx^{-4} $-3x^{-4}$
	$48x + 12x^{-5}$	M1 A1 5	Attempt to differentiate their (i) Fully correct
(ii)	$8x^3 + \frac{1}{x^3} = -9$ $8x^6 + 1 = -9x^3$ $8x^6 + 9x^3 + 1 = 0$	*M1	Use a substitution to obtain a 3-term quadratic
	Let $y = x^3$ $8y^2 + 9y + 1 = 0$ $(8y+1)(y+1) = 0$ $y = -\frac{1}{8}, y = -1$ $x = -\frac{1}{2}, x = -1$	DM1 A1 M1 A1 5	Correct method to solve quadratic $-\frac{1}{8}$, -1 Attempt to cube root at least one of their y-values $-\frac{1}{2}$, -1 **SR** one correct x value **www** **B1** **SR for trial and improvement:** x = -1 B1 $x = -\frac{1}{2}$ B2
		10	Justification that there are no further solutions B2