OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT

4721/01

MATHEMATICS

Core Mathematics 1

TUESDAY 16 JANUARY 2007

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

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1 Express $\dfrac{5}{2-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

2 Evaluate

(i) 6^0, [1]

(ii) $2^{-1} \times 32^{\frac{4}{5}}$. [3]

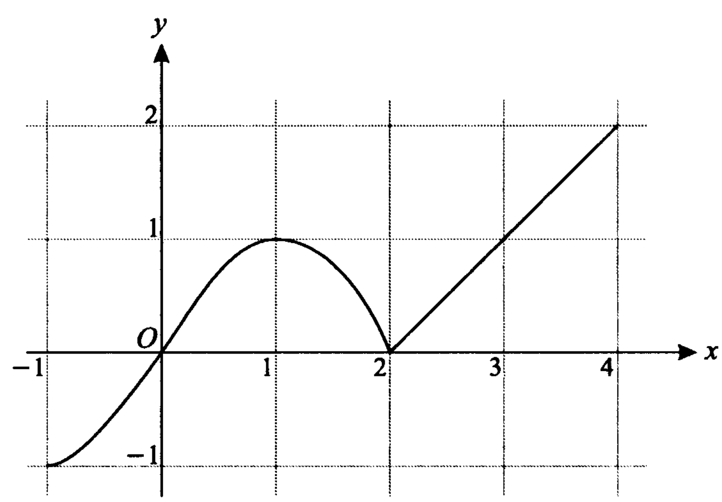
3 Solve the inequalities

(i) $3(x-5) \leqslant 24$, [2]

(ii) $5x^2 - 2 > 78$. [3]

4 Solve the equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$. [5]

5



The graph of $y = \mathrm{f}(x)$ for $-1 \leqslant x \leqslant 4$ is shown above.

(i) Sketch the graph of $y = -\mathrm{f}(x)$ for $-1 \leqslant x \leqslant 4$. [2]

(ii) The point $P(1, 1)$ on $y = \mathrm{f}(x)$ is transformed to the point Q on $y = 3\mathrm{f}(x)$. State the coordinates of Q. [2]

(iii) Describe the transformation which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{f}(x+2)$. [2]

6 **(i)** Express $2x^2 - 24x + 80$ in the form $a(x-b)^2 + c$. [4]

(ii) State the equation of the line of symmetry of the curve $y = 2x^2 - 24x + 80$. [1]

(iii) State the equation of the tangent to the curve $y = 2x^2 - 24x + 80$ at its minimum point. [1]

7 Find $\frac{dy}{dx}$ in each of the following cases.

(i) $y = 5x + 3$ [1]

(ii) $y = \frac{2}{x^2}$ [3]

(iii) $y = (2x + 1)(5x - 7)$ [4]

8 **(i)** Find the coordinates of the stationary points of the curve $y = 27 + 9x - 3x^2 - x^3$. [6]

(ii) Determine, in each case, whether the stationary point is a maximum or minimum point. [3]

(iii) Hence state the set of values of x for which $27 + 9x - 3x^2 - x^3$ is an increasing function. [2]

9 A is the point $(2, 7)$ and B is the point $(-1, -2)$.

(i) Find the equation of the line through A parallel to the line $y = 4x - 5$, giving your answer in the form $y = mx + c$. [3]

(ii) Calculate the length of AB, giving your answer in simplified surd form. [3]

(iii) Find the equation of the line which passes through the mid-point of AB and which is perpendicular to AB. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

10 A circle has equation $x^2 + y^2 + 2x - 4y - 8 = 0$.

(i) Find the centre and radius of the circle. [3]

(ii) The circle passes through the point $(-3, k)$, where $k < 0$. Find the value of k. [3]

(iii) Find the coordinates of the points where the circle meets the line with equation $x + y = 6$. [6]

1	$\frac{5}{2-\sqrt{3}}$ x $\frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{5(2+\sqrt{3})}{4-3}$ $= 10+5\sqrt{3}$	M1 A1 A1 3 **3**	Multiply top and bottom by $\pm(2+\sqrt{3})$ $(2+\sqrt{3})(2-\sqrt{3})=1$ (may be implied) $10+5\sqrt{3}$
2(i) (ii)	1 $\frac{1}{2}\times 2^4$ $= 8$	B1 1 M1 M1 A1 3 **4**	 $2^{-1}=\frac{1}{2}$ **<u>or</u>** $32^{\frac{1}{5}}=2$ **<u>or</u>** $2^5=32$ soi $32^{\frac{4}{5}}=2^4$ or 16 seen or implied 8
3(i) (ii)	$3x-15\le 24$ $3x\le 39$ $x\le 13$ **or** $x-5\le 8$ M1 $x\le 13$ A1 $5x^2>80$ $x^2>16$ $x>4$ or $x<-4$	M1 A1 2 M1 B1 A1 3 **5**	Attempt to simplify expression by multiplying out brackets $x\le 13$ Attempt to simplify expression by dividing through by 3 Attempt to rearrange inequality or equation to combine the constant terms $x>4$ fully correct, not wrapped, not 'and' **SR** B1 for $x\ge 4$, $x\le -4$

4	Let $y = x^{\frac{1}{3}}$ $y^2 + 3y - 10 = 0$ $(y-2)(y+5) = 0$ $y = 2, y = -5$ $x = 2^3, x = (-5)^3$ $x = 8, x = -125$	*M1 DM1 A1 DM1 A1 ft 5 **5**	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct attempt to solve quadratic Both values correct Attempt cube Both answers correctly followed through **SR** B2 $x = 8$ from T & I
5 (i)		M1 A1 2	Reflection in either axis Correct reflection in x axis
(ii)	(1, 3)	B1 B1 2	Correct x coordinate Correct y coordinate **SR** B1 for (3, 1)
(iii)	Translation 2 units in negative x direction	B1 B1 2 **6**	
6 (i)	$2(x^2 - 12x + 40)$ $= 2[(x-6)^2 - 36 + 40]$ $= 2[(x-6)^2 + 4]$ $= 2(x-6)^2 + 8$	B1 B1 M1 A1 4	$a = 2$ $b = 6$ $80 - 2b^2$ or $40 - b^2$ or $80 - b^2$ or $40 - 2b^2$ (their b) $c = 8$
(ii)	$x = 6$	B1 ft 1	
(iii)	$y = 8$	B1 ft 1 **6**	

7(i)	$\frac{dy}{dx} = 5$	B1 1	
(ii)	$y = 2x^{-2}$	B1	x^{-2} soi
	$\frac{dy}{dx} = -4x^{-3}$	B1	$-4x^{c}$
		B1 3	kx^{-3}
(iii)	$y = 10x^2 - 14x + 5x - 7$ $y = 10x^2 - 9x - 7$	M1 A1	Expand the brackets to give an expression of form $ax^2 + bx + c$ $(a \neq 0, b \neq 0, c \neq 0)$ Completely correct (allow 2 x-terms)
	$\frac{dy}{dx} = 20x - 9$	B1 ft B1 ft 4	1 term correctly differentiated Completely correct (2 terms)
		8	
8 (i)	$\frac{dy}{dx} = 9 - 6x - 3x^2$	*M1 A1	Attempt to differentiate y or $-y$ (at least one correct term) 3 correct terms
	At stationary points, $9 - 6x - 3x^2 = 0$	M1	Use of $\frac{dy}{dx} = 0$ (for y or $-y$)
	$3(3 + x)(1 - x) = 0$ $x = -3$ or $x = 1$	DM1 A1	Correct method to solve 3 term quadratic $x = -3,\ 1$
	$y = 0,\ 32$	A1ft 6	y = 0, 32 (1 correct pair www A1 A0)
(ii)	$\frac{d^2y}{dx^2} = -6x - 6$	M1	Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from $k\frac{dy}{dx}$, or other correct method
	When $x = -3, \frac{d^2y}{dx^2} > 0$	A1	$x = -3$ minimum
	When $x = 1, \frac{d^2y}{dx^2} < 0$	A1 3	$x = 1$ maximum
(iii)	$-3 < x < 1$	M1 A1 2	Uses the x values of both turning points in inequality/inequalities Correct inequality or inequalities. Allow ≤
		11	

9 (i)	Gradient = 4	B1	Gradient of 4 soi
	$y - 7 = 4(x - 2)$	M1	Attempts equation of straight line through (2, 7) with any gradient
	$y = 4x - 1$	A1 3	
(ii)	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(2 - ^-1)^2 + (7 - ^-2)^2}$	M1	Use of correct formula for d or d^2 (3 values correctly substituted)
	$= \sqrt{3^2 + 9^2}$ $= \sqrt{90}$	A1	$\sqrt{3^2 + 9^2}$
	$= 3\sqrt{10}$	A1 3	Correct simplified surd
(iii)	Gradient of AB = 3	B1	
	Gradient of perpendicular line = $-\frac{1}{3}$	B1 ft	**SR** Allow B1 for $-\frac{1}{4}$
	Midpoint of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$	B1	
	$y - \frac{5}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$	M1	Attempts equation of straight line through their midpoint with any non-zero gradient
	$x + 3y - 8 = 0$	A1	$y - \frac{5}{2} = \frac{-1}{3}(x - \frac{1}{2})$
		A1 6	$x + 3y - 8 = 0$
		12	