OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4721

Core Mathematics 1

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This question paper consists of 3 printed pages and 1 blank page.

1 The points $A\,(1, 3)$ and $B\,(4, 21)$ lie on the curve $y = x^2 + x + 1$.

(i) Find the gradient of the line AB. [2]

(ii) Find the gradient of the curve $y = x^2 + x + 1$ at the point where $x = 3$. [2]

2 **(i)** Evaluate $27^{-\frac{2}{3}}$. [2]

(ii) Express $5\sqrt{5}$ in the form 5^n. [1]

(iii) Express $\dfrac{1 - \sqrt{5}}{3 + \sqrt{5}}$ in the form $a + b\sqrt{5}$. [3]

3 **(i)** Express $2x^2 + 12x + 13$ in the form $a(x + b)^2 + c$. [4]

(ii) Solve $2x^2 + 12x + 13 = 0$, giving your answers in simplified surd form. [3]

4 **(i)** By expanding the brackets, show that

$$(x - 4)(x - 3)(x + 1) = x^3 - 6x^2 + 5x + 12.$$ [3]

(ii) Sketch the curve

$$y = x^3 - 6x^2 + 5x + 12,$$

giving the coordinates of the points where the curve meets the axes. Label the curve C_1. [3]

(iii) On the same diagram as in part **(ii)**, sketch the curve

$$y = -x^3 + 6x^2 - 5x - 12.$$

Label this curve C_2. [2]

5 Solve the inequalities

(i) $1 < 4x - 9 < 5$, [3]

(ii) $y^2 \geqslant 4y + 5$. [5]

6 **(i)** Solve the equation $x^4 - 10x^2 + 25 = 0$. [4]

(ii) Given that $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$, find $\dfrac{dy}{dx}$. [2]

(iii) Hence find the number of stationary points on the curve $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$. [2]

7 (i) Solve the simultaneous equations

$$y = x^2 - 5x + 4, \qquad y = x - 1.$$ [4]

(ii) State the number of points of intersection of the curve $y = x^2 - 5x + 4$ and the line $y = x - 1$. [1]

(iii) Find the value of c for which the line $y = x + c$ is a tangent to the curve $y = x^2 - 5x + 4$. [4]

8 A cuboid has a volume of 8 m^3. The base of the cuboid is square with sides of length x metres. The surface area of the cuboid is $A \text{ m}^2$.

(i) Show that $A = 2x^2 + \frac{32}{x}$. [3]

(ii) Find $\frac{\mathrm{d}A}{\mathrm{d}x}$. [3]

(iii) Find the value of x which gives the smallest surface area of the cuboid, justifying your answer. [4]

9 The points A and B have coordinates $(4, -2)$ and $(10, 6)$ respectively. C is the mid-point of AB. Find

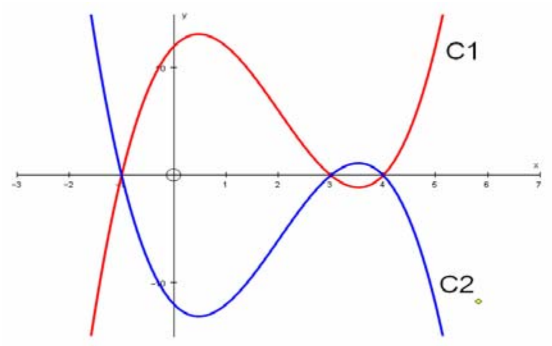
(i) the coordinates of C, [2]

(ii) the length of AC, [2]

(iii) the equation of the circle that has AB as a diameter, [3]

(iv) the equation of the tangent to the circle in part (iii) at the point A, giving your answer in the form $ax + by = c$. [5]

1	(i)	$\frac{21-3}{4-1}=\frac{18}{3}=6$	M1 A1	 2	Uses $\frac{y_2-y_1}{x_2-x_1}$ 6 (not left as $\frac{18}{3}$)
	(ii)	$\frac{dy}{dx}=2x+1$ $2\times3+1=7$	B1 B1	 2	
2	(i)	$27^{-\frac{2}{3}}=\frac{1}{27^{\frac{2}{3}}}=\frac{1}{9}$	M1 A1	 2	$\frac{1}{27^{\frac{2}{3}}}$ or $27^{\frac{2}{3}}=9$ or 3^{-2} soi $\frac{1}{9}$
	(ii)	$5\sqrt{5}=5^{\frac{3}{2}}$	B1	1	
	(iii)	$\frac{1-\sqrt{5}}{3+\sqrt{5}}=\frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$ $=\frac{8-4\sqrt{5}}{4}$ $=2-\sqrt{5}$	M1 B1 A1	 3	Multiply numerator and denominator by conjugate $(\sqrt{5})^2=5$ soi $2-\sqrt{5}$
3	(i)	$2x^2+12x+13=2(x^2+6x)+13$ $=2[(x+3)^2-9]+13$ $=2(x+3)^2-5$	B1 B1 M1 A1	 4	$a=2$ $b=3$ $13-2b^2$ or $13-b^2$ or $\frac{13}{2}-b^2$ (their b) $c=-5$
	(ii)	$2(x+3)^2-5=0$ $(x+3)^2=\frac{5}{2}$ $x=-3\pm\sqrt{\frac{5}{2}}$	M1 A1 A1	 3	Uses correct quadratic formula or completing square method $x=\frac{-12\pm\sqrt{40}}{4}$ or $(x+3)^2=\frac{5}{2}$ $x=-3\pm\sqrt{\frac{5}{2}}$ or $-3\pm\frac{1}{2}\sqrt{10}$

4	(i)	$(x-4)(x-3)(x+1)$ $\equiv (x^2-7x+12)(x+1)$ $\equiv x^3+x^2-7x^2-7x+12x+12$ $\equiv x^3-6x^2+5x+12$	B1 M1 A1	 3	$x^2-7x+12$ or x^2-2x-3 or x^2-3x-4 seen Attempt to multiply a quadratic by a linear factor or attempt to list an 8 term expansion of all 3 brackets $x^3-6x^2+5x+12$ **(AG)** obtained (no wrong working seen)
	(ii) (iii)		B1 B1 B1 M1 A1√	 3 2	+ve cubic with 3 roots (not 3 line segments) (0, 12) labelled or indicated on *y*-axis (– 1, 0), (3,0), (4, 0) labelled or indicated on *x*-axis Reflect *their* (ii) in either *x*- or *y*-axis Reflect *their* (ii) in *x*-axis
5	(i)	$1<4x-9<5$ $10<4x<14$ $2.5<x<3.5$	M1 A1 A1	 3	2 equations or inequalities both dealing with all 3 terms 2.5 and 3.5 seen oe $2.5<x<3.5$ (or '$x>2.5$ <u>and</u> $x<3.5$')
	(ii)	$y^2 \ge 4y+5$ $y^2-4y-5 \ge 0$ $(y-5)(y+1) \ge 0$ $y \le -1,\ y \ge 5$	B1 M1 A1 M1 A1	 5	$y^2-4y-5=0$ soi Correct method to solve quadratic -1, 5 (**SR** If **both** values obtained from trial and improvement, award **B3**) Correct method to solve inequality $y \le -1,\ y \ge 5$

6	(i)	$x^4 - 10x^2 + 25 = 0$ Let $y = x^2$ $y^2 - 10y + 25 = 0$ $(y-5)^2 = 0$ $y = 5$ $x^2 = 5$ $x = \pm\sqrt{5}$	*M1 dep*M1 A1 A1	 4	Use a substitution to obtain a quadratic or $(x^2 - 5)(x^2 - 5) = 0$ Correct method to solve a quadratic 5 (not $x = 5$ with no subsequent working) $x = \pm\sqrt{5}$
	(ii)	$y = \frac{2x^5}{5} - \frac{20x^3}{3} + 50x + 3$ $\frac{dy}{dx} = 2x^4 - 20x^2 + 50$	B1 B1	 2	$2x^4$ or $- 20x^2$ oe seen $2x^4 - 20x^2 + 50$ (integers required)
	(iii)	$2x^4 - 20x^2 + 50 = 0$ $x^4 - 10x^2 + 25 = 0$ which has 2 roots	M1 A1	 2	*their* $\frac{dy}{dx} = 0$ seen (or implied by correct answer) 2 stationary points **www in any part**
7	(i)	$y = x^2 - 5x + 4$ $y = x - 1$ $x^2 - 5x + 4 = x - 1$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x = 1 \quad x = 5$ $y = 0 \quad y = 4$	M1 M1 A1 A1	 4	Substitute to find an equation in x (or y) Correct method to solve quadratic $x = 1, 5$ $y = 0, 4$ (**N.B.** This final A1 may be awarded in part (ii) if y coordinates only seen in part (ii)) **SR** one correct (x,y) pair **www** **B1**
	(ii)	2 points of intersection	B1	1	
	(iii)	EITHER $x^2 - 5x + 4 = x + c$ has 1 solution $x^2 - 6x + (4 - c) = 0$ $b^2 - 4ac = 0$ $36 - 4(4 - c) = 0$ $c = -5$ OR $\frac{dy}{dx} = 1 = 2x - 5$ $x = 3 \quad y = -2$ $-2 = 3 + c$ $c = -5$	M1 M1 A1 A1 M1 A1 A1 A1	 4 4	$x^2 - 5x + 4 = x + c$ has 1 soln seen or implied Discriminant = 0 or $(x - a)^2 = 0$ soi $36 - 4(4 - c) = 0$ or $9 = 4 - c$ $c = -5$ Algebraic expression for gradient of curve = non-zero gradient of line used $2x - 5 = 1$ $x = 3$ $c = -5$ **SR** $c = -5$ without any working **B1**

8	(i)	Height of box = $\frac{8}{x^2}$	*B1		Area of 1 vertical face $= \frac{8}{x^2} \times x$ $= \frac{8}{x}$
		4 vertical faces $= 4 \times \frac{8}{x}$ $= \frac{32}{x}$	*B1		
		Total surface area = $x^2 + x^2 + \frac{32}{x}$	B1 dep on both **		Correct final expression
		$A = 2x^2 + \frac{32}{x}$		3	
	(ii)	$\frac{dA}{dx} = 4x - \frac{32}{x^2}$	B1 B1 B1	3	$4x$ kx^{-2} $-32x^{-2}$
	(iii)	$4x - \frac{32}{x^2} = 0$	M1		$\frac{dA}{dx} = 0$ soi
		$4x^3 = 32$ $x = 2$	A1		$x = 2$
			M1 A1	4	Check for minimum Correctly justified
					SR If $x = 2$ stated **www** but with no evidence of differentiated expression(s) having been used in part (iii) **B1**

9	(i)	$\left(\frac{4+10}{2},\frac{-2+6}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
		(7, 2)	A1	2	(7, 2) (integers required)
	(ii)	$\sqrt{(7-4)^2+(2--2)^2}$	M1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
		$=\sqrt{3^2+4^2}$ $=5$	A1	2	5
	(iii)	$(x-7)^2+(y-2)^2=25$	B1√		$(x-7)^2$ and $(y-2)^2$ used (*their* centre)
			B1√		$r^2=25$ used (*their* r^2)
			B1	3	$(x-7)^2+(y-2)^2=25$ cao
					<u>Expanded form:</u> $-14x$ and $-4y$ used B1√ $r=\sqrt{g^2+f^2-c}$ used B1√ $x^2+y^2-14x-4y+28=0$ B1 cao
					<u>By using ends of diameter:</u> $(x-4)(x-10)+(y+2)(y-6)=0$ Both x brackets correct B1 Both y brackets correct B1 Final equation fully correct B1
	(iv)	Gradient of $AB=\frac{6--2}{10-4}=\frac{4}{3}$	B1		oe
		Gradient of tangent $=-\frac{3}{4}$	B1√		
			M1		Correct equation of straight line through A, any non-zero gradient
		$y--2=-\frac{3}{4}(x-4)$	A1		
		$3x+4y=4$	A1	5	a, b, c need not be integers