OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2642

Probability and Statistics 2

Tuesday **12 JUNE 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

 [Turn over

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1 The percentage, $X\%$, of a certain metal in an alloy can be modelled as having a normal distribution with mean 70 and standard deviation σ. It is given that $P(X > 77) = 0.0808$. Find the value of σ. [4]

2 In a large school, 5% of the pupils have gained a Silver Award. A random sample of 60 pupils is obtained, and the random variable R is the number of these pupils who have gained a Silver Award.

(i) State, with a reason, whether a Poisson approximation to the distribution of R would be appropriate. [3]

(ii) Describe briefly how, in practice, the random sample of 60 pupils might have been obtained. [2]

3 A fair six-sided die is thrown 540 times. Use a suitable approximation to calculate the probability that at least 80 sixes are obtained. [6]

4 An employee is accused by his employer of being late for work too often. The employee claims that, on average, he is late on no more than one day in ten. The employer finds that, over a random sample of 20 days, the employee is late on r days. The employer carries out a significance test, at the 5% level, to decide whether, on average, the employee is late on more than one day in ten.

(i) State suitable null and alternative hypotheses for the test. [2]

(ii) Find the set of values of r for which the null hypothesis would be rejected, and state the conclusion of the test in the case $r = 4$. [5]

(iii) Given that, in fact, the probability that the employee is late for work on a randomly chosen day is 0.2, find the probability of making a Type II error in the test. [2]

5 A company manufactures ropes for use by climbers. The company claims that the ropes have a mean breaking strength of 13 000 newtons. The standard deviation of the breaking strength is known to be 200 newtons. The breaking strengths of a sample of 50 ropes are measured, in order to test whether the company's claim is justified.

(i) State why, in this context, a one-tail test is more appropriate than a two-tail test. [1]

(ii) The sample is found to have a mean breaking strength of 12 950 newtons. Carry out a significance test, at the 5% level, to decide whether the company's claim is justified. [6]

(iii) Name a theorem used in carrying out your test, and state why it is needed. [2]

6 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} k(x-2)^2 & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) the value of k, [3]

(ii) $P(X < 2.5)$, [3]

(iii) $E(X)$, [4]

(iv) the value m such that $P(X < m) = 0.5$. [3]

7 Over a long period it has been found that the number of typing errors made by a secretary is, on average, 6 per hour.

(i) State an assumption which you need to make in order to model the number of typing errors in a randomly chosen hour by a Poisson distribution. [1]

Assuming that this model is valid, and that the relevant periods of time are randomly chosen,

(ii) calculate the probability that, in one hour, the secretary makes more than 5 typing errors, [2]

(iii) calculate the probability that, in ten minutes, the secretary makes exactly two typing errors, [3]

(iv) use a suitable approximation to calculate the probability that, in four hours, the secretary makes more than 30 typing errors, [5]

(v) find the longest period of time, in minutes, for which the probability that the secretary makes no typing errors is greater than 0.9. [3]

1 $X \sim \mathrm{N}\left(70,\ \sigma^2\right)$

$\mathrm{p}(X > 77) = 0.0808 \quad \Rightarrow \quad \Phi\left(\dfrac{77-70}{\sigma}\right) = 0.9192 \quad \Rightarrow \quad \dfrac{7}{\sigma} = 1.400 \quad \Rightarrow \quad \boldsymbol{\sigma = 5}$ [4]

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2 $R \approx B\left(60,\ 0.05\right)$ (only approximate since p not quite constant - finite population)

$n > 50$ and $np = 3 < 5$ so a Poisson distribution ($\lambda = 3$) is a good approximation. [3]

Describe one way of obtaining a random sample. [2]

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3 No. of sixes $X \sim \mathrm{B}\left(540,\ \frac{1}{6}\right) \approx \mathrm{N}\left(90,\ 75\right)$

$$\mathrm{p}(X \geq 80) = 1 - \Phi\left(\frac{79.5-90}{\sqrt{75}}\right) = 1 - \Phi(-1.212) = 1 - 0.1127 = \mathbf{0.887} \quad (3\ s.f.)$$ [6]

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4 $\mathrm{H}_0:\ p = \frac{1}{10}$ $\qquad \mathrm{H}_1:\ p > \frac{1}{10}$ [2]

working on H_0 ...... no. of lates $X \sim B\left(20,\ \frac{1}{10}\right)$

$$\left.\begin{aligned} \mathrm{p}(X \geq 4) &= 1 - 0.8670 = 13.3\% \\ \mathrm{p}(X \geq 5) &= 1 - 0.9568 = 4.32\% \end{aligned}\right\} \text{ Reject } \mathrm{H}_0 \text{ if } r \geq 5$$

$\underline{r = 4}$ insufficient evidence to cast doubt on the employee's assertion. [5]

$p = \frac{1}{5}$ $\quad \mathrm{p}\left(\text{Type II error}\right) = \mathrm{p}\left(X \leq 4 \,\middle|\, X \sim \mathrm{B}\left(20,\ \frac{1}{5}\right)\right) = 0.6296 = \mathbf{0.630} \quad (3\ s.f.)$ [2]

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5 What matters is whether the breaking strength is 13 kN <u>or more</u>, so a one-tail test is more appropriate. [1]

$$H_0 : \mu = 13000 \qquad H_1 : \mu < 13000$$

$$Z = \frac{\overline{X} - 13000}{\left(200/\sqrt{50}\right)} \sim N(0, 1) \qquad \text{Reject } H_0 \text{ if } Z < -1.645$$

[6]

For given sample …. $z = \dfrac{12950 - 13000}{\left(200/\sqrt{50}\right)} = -1.767... < -1.645$

So sufficient evidence on which to reject the company's claim.

[2]

The **Central Limit Theorem** is needed to ensure the normality of the test statistic.

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6 $1 = k\int_2^3 (x-2)^2 \, dx = k\left[\frac{1}{3}(x-2)^3\right]_2^3 = k\left(\frac{1}{3} - 0\right) = \frac{1}{3}k \qquad \therefore \boldsymbol{k = 3}$ [3]

$p(X < 2.5) = \int_2^{2.5} 3(x-2)^2 \, dx = \left[(x-2)^3\right]_2^{2.5} = \mathbf{\frac{1}{8}}$ [3]

$E[X] = \int_2^3 3x(x-2)^2 \, dx = \int_2^3 \left(3x^3 - 12x^2 + 12x\right) dx = \left[\frac{3}{4}x^4 - 4x^3 + 6x^2\right]_2^3 = \mathbf{\frac{11}{4}}$ [4]

$\int_2^m 3(x-2)^2 \, dx = 0.5 \qquad (m-2)^3 = 0.5 \qquad \boldsymbol{m = 2 + \frac{1}{\sqrt[3]{2}}} \ (= 2.79 \quad 3\,s.f.)$

[3]

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7 name **one** assumption ….. independent, uniform, etc. [1]

$Po(6) \qquad p(X > 5) = 1 - p(X \le 5) = 1 - 0.4457 = \mathbf{0.554}$ [2]

$Po(1) \qquad p(2) = e^{-1}\dfrac{1^2}{2!} = \mathbf{0.184}$

[3]

no. of errors in 4 hours … $X \sim Po(24) \approx N(24, 24)$

$$p(X > 30) = 1 - \Phi\left(\frac{30.5 - 24}{\sqrt{24}}\right) = 1 - \Phi(1.327) = 1 - 0.9077 = \mathbf{0.0923}$$ [5]

in an interval of t minutes …. $X \sim Po\left(t/10\right)$

$$p(0 \text{ errors}) > 0.9$$
$$e^{-t/10} > 0.9$$
$$t < -10\ln 0.9 = 1.05$$

longest period of time is 1.05 minutes

[3]