OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS — 2640

Mechanics 4

Friday **28 MAY 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Two flywheels P and Q are rotating, in opposite directions, about the same fixed axis. The angular speed of P is 25 rad s^{-1} and the angular speed of Q is 30 rad s^{-1}. The flywheels lock together, and after this they both rotate with angular speed 10 rad s^{-1} in the direction in which P was originally rotating. The moment of inertia of P about the axis is 0.64 kg m^2. Find the moment of inertia of Q about the axis. [4]

2 A uniform rectangular lamina has mass m and sides of length $3a$ and $4a$, and rotates freely about a fixed horizontal axis. The axis is perpendicular to the lamina and passes through a corner. The lamina makes small oscillations in its own plane, as a compound pendulum.

(i) Find the moment of inertia of the lamina about the axis. [3]

(ii) Find the approximate period of the small oscillations. [3]

3 The region between the curve $y = x\sqrt{(3-x)}$ and the x-axis for $0 \leqslant x \leqslant 3$ is rotated through 2π radians about the x-axis to form a uniform solid of revolution. Find the x-coordinate of the centre of mass of this solid. [6]

4 A uniform solid sphere, of mass 14 kg and radius 0.25 m, is rotating about a fixed axis which is a diameter of the sphere. A couple of constant moment 4.2 N m about the axis, acting in the direction of rotation, is applied to the sphere.

(i) Find the angular acceleration of the sphere. [3]

During a time interval of 30 seconds the sphere rotates through 7500 radians.

(ii) Find the angular speed of the sphere at the start of the time interval. [2]

(iii) Find the angular speed of the sphere at the end of the time interval. [2]

(iv) Find the work done by the couple during the time interval. [2]

5 Two aircraft A and B are flying horizontally at the same height. A has constant velocity 240 m s^{-1} in the direction with bearing 025°, and B has constant velocity 185 m s^{-1} in the direction with bearing 310°.

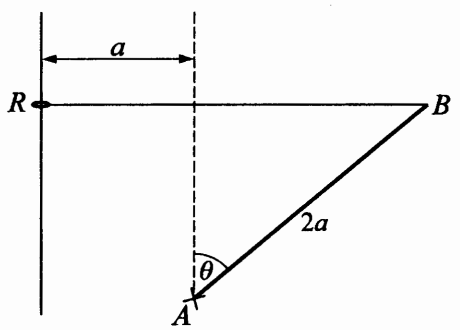
(i) Find the magnitude and direction of the velocity of A relative to B. [5]

Initially A is 4500 m due west of B. For the instant during the subsequent motion when A and B are closest together, find

(ii) the distance between A and B, [3]

(iii) the bearing of A from B. [2]

6



A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed horizontal axis through A. A light elastic string has natural length a and modulus of elasticity mg; one end is attached to B and the other end is attached to a light ring R which can slide along a smooth vertical wire. The wire is in the same vertical plane as AB, and is at a distance a from A. The rod AB makes an angle θ with the upward vertical, where $0 < \theta < \frac{1}{2}\pi$ (see diagram).

(i) Give a reason why the string RB is always horizontal. [1]

(ii) By considering potential energy, find the value of θ for which the system is in equilibrium. [6]

(iii) Determine whether this position of equilibrium is stable or unstable. [4]

7 A uniform rod AB has mass m and length $2a$. The point P on the rod is such that $AP = \frac{2}{3}a$.

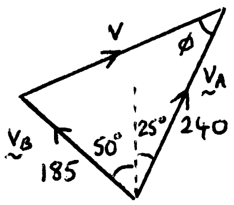
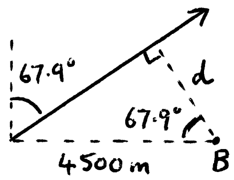
(i) Prove by integration that the moment of inertia of the rod about an axis through P perpendicular to AB is $\frac{4}{9}ma^2$. [4]

The axis through P is fixed and horizontal, and the rod can rotate without resistance in a vertical plane about this axis. The rod is released from rest in a horizontal position. Find, in terms of m and g,

(ii) the force acting on the rod at P immediately after the release of the rod, [5]

(iii) the force acting on the rod at P at an instant in the subsequent motion when B is vertically below P. [5]

1	$0.64 \times 25 - I \times 30 = (0.64 + I) \times 10$ $I = 0.24\ \text{kg m}^2$	M1 A1A1 A1 **4**	Use of angular momentum For LHS and RHS
2 (i)	$I = \frac{4}{3}m(\frac{3}{2}a)^2 + \frac{4}{3}m(2a)^2$ $= \frac{25}{3}ma^2$	B1 M1 A1 **3**	For either term Use of perpendicular axes rule
	OR $I = \frac{1}{3}m\{(\frac{3}{2}a)^2 + (2a)^2\} + m(\frac{5}{2}a)^2$ B1 M1 $= \frac{25}{3}ma^2$ A1		For $\frac{1}{3}m\{(\frac{3}{2}a)^2 + (2a)^2\}$ Use of parallel axes rule
(ii)	Period is $2\pi\sqrt{\frac{I}{mgh}}$ $= 2\pi\sqrt{\frac{\frac{25}{3}ma^2}{mg\frac{5}{2}a}}$ $= 2\pi\sqrt{\frac{10a}{3g}}$	M1 A1 ft A1 **3**	or $(-)\ mgh\sin\theta = I\ddot{\theta}$ or $-mg(\frac{5}{2}a)\theta \approx \frac{25}{3}ma^2\ddot{\theta}$
3	$\int \pi xy^2\,dx = \int_0^3 \pi x^3(3-x)\,dx$ $= \pi\left[\frac{3}{4}x^4 - \frac{1}{5}x^5\right]_0^3 \quad (= 12.15\pi)$ $\int \pi y^2\,dx = \int_0^3 \pi x^2(3-x)\,dx$ $= \pi\left[x^3 - \frac{1}{4}x^4\right]_0^3 \quad (= 6.75\pi)$ $\bar{x} = \frac{12.15\pi}{6.75\pi}$ $= 1.8$	M1 A1 M1 A1 M1 A1 **6**	*(π may be omitted throughout)* *Dependent on previous M1M1*
4 (i)	$I = \frac{2}{5} \times 14 \times 0.25^2 \quad (= 0.35)$ $4.2 = I\alpha$ $\alpha = 12\ \text{rad s}^{-2}$	B1 M1 A1 **3**	
(ii)	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$; $\quad 7500 = \omega_1 \times 30 + \frac{1}{2} \times 12 \times 30^2$ $\omega_1 = 70\ \text{rad s}^{-1}$	M1 A1 ft **2**	Ft $250 - 15\alpha$
(iii)	$\omega_2 = \omega_1 + \alpha t$; $\quad \omega_2 = 70 + 12 \times 30$ $\omega_2 = 430\ \text{rad s}^{-1}$	M1 A1 ft **2**	Ft $250 + 15\alpha$
(iv)	Work done is $L\theta = 4.2 \times 7500$ $= 31500$ J	M1 A1 **2**	Or $\frac{1}{2}I(\omega_2^2 - \omega_1^2)$

5 (i)		B1	Velocity triangle *If wrong triangle used, then* *B0 M1A0 M1A0 for equivalent work* *Full ft in (ii) and (iii)*
	$v^2 = 240^2 + 185^2 - 2 \times 240 \times 185 \cos 75$	M1	
	$v = 262 \text{ m s}^{-1}$	A1	
	$\dfrac{\sin\phi}{185} = \dfrac{\sin 75}{262.4}$	M1	
	$\phi = 42.9°$ Bearing is $25 + \phi = 067.9°$	A1 **5**	Accept 68°, 67.8° *Allow other clearly stated descriptions of direction*
	OR $\mathbf{v} = \begin{pmatrix} 240\sin 25 \\ 240\cos 25 \end{pmatrix} - \begin{pmatrix} 185\sin 310 \\ 185\cos 310 \end{pmatrix}$ M1 $= \begin{pmatrix} 243 \\ 98.6 \end{pmatrix}$ A1 M1 $v = \sqrt{243^2 + 98.6^2} = 262$ A1 Bearing is $\tan^{-1}\dfrac{243}{98.6} = 67.9°$ A1		Finding magnitude *or* angle
(ii)	As viewed from *B* 	M1	Relative displacement diagram
	$d = 4500\cos 67.9$ $= 1690$ m	M1 A1 ft **3**	Ft from bearing in (i)
	OR $\overrightarrow{BA} = \begin{pmatrix} 243t - 4500 \\ 98.6t \end{pmatrix}$ M1 $BA^2 = (243t - 4500)^2 + (98.6t)^2$ is minimum when $t = \dfrac{4500 \times 243}{243^2 + 98.6^2}$ $(= 15.9)$ M1 Then $BA = 1690$ A1 ft		Ft from relative velocity in (i)
(iii)	Bearing is $270 + 67.9$ $= 338°$	M1 A1 ft **2**	Ft from bearing in (i) *SR* B1 ft for 158°
	OR $\overrightarrow{BA} = \begin{pmatrix} -636 \\ 1570 \end{pmatrix}$ Bearing is $360 - \tan^{-1}\dfrac{636}{1570}$ M1 $= 338°$ A1 ft		Ft from relative velocity in (i)

6 (i)	e.g. R has no weight / mass There is no friction There is no vertical force at R String has minimum length / elastic energy	B1 **1**	For any one contributory reason
(ii)	$V = \frac{1}{2}\left(\frac{mg}{a}\right)(2a\sin\theta)^2 + mga\cos\theta$ $= mga(2\sin^2\theta + \cos\theta)$ $\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(4\sin\theta\cos\theta - \sin\theta)$ For equilibrium, $mga\sin\theta(4\cos\theta - 1) = 0$ $\theta = 1.32$ (*or* $75.5°$)	B2 B1 ft B1 ft M1 A1 **6**	Give B1 for correct EE or PE Diffn of $\sin^2\theta$ (or $\cos^2\theta$) Diffn of $\cos\theta$ (or $\sin\theta$) Allow $\cos^{-1}\frac{1}{4}$, $76°$ *Ignore* $\theta = 0$ *if stated* *Marks can be awarded in (iii) if not earned in (ii)* *SR If done by taking moments, M1A1 for* $\frac{mg(2a\sin\theta)}{a}(2a\cos\theta) = mg(a\sin\theta)$ *A1 for* $\theta = 1.32$ *(Max 3 out of 6)* *These marks may only* ***replace*** *all other marks in (ii)*
(iii)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga\cos\theta(4\cos\theta - 1) - 4mga\sin^2\theta$ When $\cos\theta = \frac{1}{4}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -4mga\sin^2\theta$ $\left(= -\frac{15}{4}mga\right) < 0$ Equilibrium is unstable	B2 ft M1 A1 **4**	Give B1 if just one error *Only B1 ft if work is simpler* Fully correct working only
	OR When $\theta < 1.32$, $\frac{\mathrm{d}V}{\mathrm{d}\theta} > 0$ B1 ft When $\theta > 1.32$, $\frac{\mathrm{d}V}{\mathrm{d}\theta} < 0$ B1 ft V has a maximum M1 Equilibrium is unstable A1		Fully correct working, and convincing demonstration of signs above

7 (i)	$I = \sum (\rho\,\delta x)\,x^2$	M1	
	$= \int_{-\frac{2}{3}a}^{\frac{4}{3}a} \frac{m}{2a} x^2\,\mathrm{d}x$	A1	
	$= \left[\frac{mx^3}{6a}\right]_{-\frac{2}{3}a}^{\frac{4}{3}a} = \frac{32}{81}ma^2 + \frac{4}{81}ma^2$	M1	*Dependent on previous M1*
	$= \frac{4}{9}ma^2$	A1 (ag) **4**	
	OR $I_G = \sum (\rho\,\delta x)\,x^2 = \int_{-a}^{a} \frac{m}{2a} x^2\,\mathrm{d}x$ M1A1		Or $2\int_0^a \frac{m}{2a} x^2\,\mathrm{d}x$
	$I = \frac{1}{6}ma^2 + \frac{1}{6}ma^2 + m(\frac{1}{3}a)^2$ M1		*Dependent on previous M1*
	$= \frac{4}{9}ma^2$ A1 (ag)		
(ii)	$mg(\frac{1}{3}a) = (\frac{4}{9}ma^2)\alpha$	M1	Use of $L = I\alpha$
	$\alpha = \frac{3g}{4a}$	A1	
	$mg - R = m(\frac{1}{3}a)\alpha$	M1	For force $= m(\frac{1}{3}a)\alpha$
	$mg - R = \frac{1}{4}mg$	A1 ft	
	$R = \frac{3}{4}mg$ vertically upwards	A1 **5**	Direction must be indicated
(iii)	$\frac{1}{2}(\frac{4}{9}ma^2)\omega^2 = mg(\frac{1}{3}a)$	M1	Use of $\frac{1}{2}I\omega^2$
	$\omega^2 = \frac{3g}{2a}$	A1	
	$S - mg = m(\frac{1}{3}a)\omega^2$	M1	For force $= m(\frac{1}{3}a)\omega^2$
	$S - mg = \frac{1}{2}mg$	A1 ft	
	$S = \frac{3}{2}mg$ vertically upwards	A1 **5**	Direction must be indicated
	Alternative for (ii) and (iii)		(θ measured from horizontal X is radial component Y is transverse component)
	$\ddot{\theta} = \frac{3g\cos\theta}{4a}$ M1A1		
	$\dot{\theta}^2 = \frac{3g\sin\theta}{2a}$ M1A1		
	$mg\cos\theta - Y = m(\frac{1}{3}a)\frac{3g\cos\theta}{4a}$ M1A1 ft		
	$Y = \frac{3}{4}mg\cos\theta$		
	$X - mg\sin\theta = m(\frac{1}{3}a)\frac{3g\sin\theta}{2a}$ M1A1 ft		
	$X = \frac{3}{2}mg\sin\theta$		
	When $\theta = 0$, $X = 0$, $Y = \frac{3}{4}mg$ A1		
	When $\theta = \frac{1}{2}\pi$, $X = \frac{3}{2}mg$, $Y = 0$ A1		