OCR

General Certificate of Education
Advanced Subsidiary (AS) and Advanced Level

MATHEMATICS **M3**

Mechanics 3

Additional materials:
Answer paper
Graph paper
List of Formulae

SPECIMEN PAPER

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.

Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.

You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

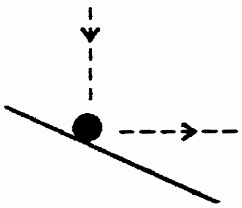
The total number of marks for this paper is 60.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

1



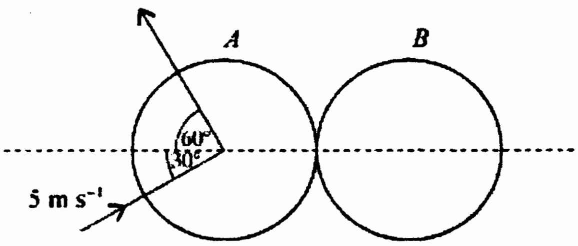
A ball of mass 0.2 kg falls vertically onto a sloping grass bank, and rebounds horizontally (see diagram). Immediately before the bounce the speed of the ball is 8 m s^{-1}, and immediately after the bounce the speed is 3 m s^{-1}. Calculate the magnitude and direction of the impulse on the ball due to the impact. **[4]**

2 A light elastic string of modulus 28 N and natural length 0.8 m has one end attached to a fixed point O. A particle of mass 0.5 kg is attached to the other end.

(i) The particle hangs in equilibrium at the point E. Calculate the distance OE. **[2]**

(ii) The particle is held at O and is released from rest. Calculate the speed of the particle as it passes the point E. **[4]**

3



Two uniform smooth spheres A and B, of equal radius, are free to move on a smooth horizontal table. The mass of B is twice the mass of A. Initially B is at rest and A is moving with speed 5 m s^{-1}. The spheres collide, and immediately before impact the direction of motion of A makes an angle of 30° with the line of centres. After the collision A moves at right angles to its original direction (see diagram). Show that

(i) the speed of A immediately after the collision is $\frac{5}{3}\sqrt{3} \text{ m s}^{-1}$, **[2]**

(ii) the speed of B immediately after the collision is also $\frac{5}{3}\sqrt{3} \text{ m s}^{-1}$, **[3]**

(iii) the collision is perfectly elastic. **[3]**

4 A particle of mass 0.2 kg is connected by two equal light elastic *springs*, each of natural length 0.5 m and modulus of elasticity 5 N, to two points A and B on a smooth horizontal table. The mid-point of AB is O and the length of AB is 1 m. The particle is displaced from O, towards B, through a distance of 0.3 m to the point C and released from rest. In the subsequent motion air resistance may be neglected. After t seconds the displacement of the particle from O is x metres. Show that

$$\frac{d^2x}{dt^2} = -100x.$$ **[3]**

The particle moves a distance 0.1 m from C to D. Find

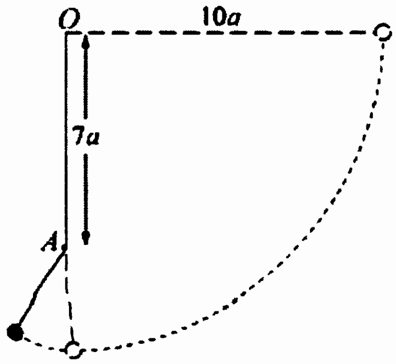
(i) the speed of the particle at D, **[3]**

(ii) the time taken to reach D. **[3]**

5 A particle of mass m is attached to one end of a light inextensible string of length $10a$. The other end of the string is attached to a fixed point O. The particle is released from rest with the string taut and horizontal. Assuming there is no air resistance, find

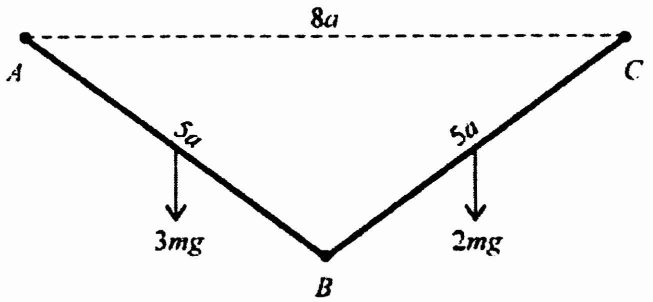
(i) the speed of the particle when the string has turned through 30°, **[2]**

(ii) the tension in the string at this instant. **[3]**



When the string reaches the vertical position, it comes into contact with a small fixed peg A which is a distance $7a$ below O. The particle begins to move in a vertical circle of radius $3a$ with centre A (see diagram). Determine, showing your working, whether the particle describes a complete circle about A. **[5]**

6



Two uniform beams AB and BC, each of length $5a$, have masses $3m$ and $2m$ respectively. The beams are freely jointed to fixed points at A and C, and to each other at B. The points A and C are on the same horizontal level at a distance $8a$ apart, and the beams are in equilibrium with B vertically below the mid-point of AC, as shown in the diagram.

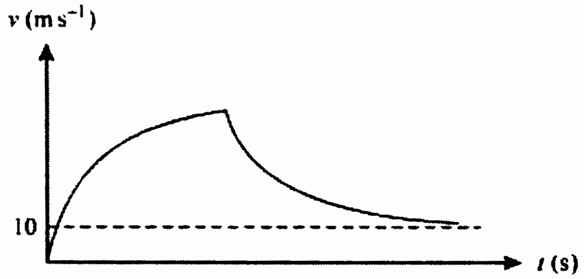
(i) Find the vertical component of the force acting on BC at C, and show that the horizontal component of this force is $\frac{5}{3}mg$. **[6]**

(ii) Find the magnitude and direction of the force acting on AB at B. **[5]**

7 A body falls vertically, the forces acting being gravity and air resistance. The air resistance is proportional to v, where v is the body's speed at time t. The value of v for which the acceleration is zero is known as the 'terminal velocity' for the motion, and is denoted by U. Show that the equation of motion of the body may be expressed as

$$\frac{dv}{dt} = \frac{g}{U}(U - v).$$ **[3]**

A parachutist jumps from a helicopter which is hovering at a height of several hundred metres, and falls vertically. Assume that, before the parachute is opened, the terminal velocity for the motion is $50\,\mathrm{m\,s^{-1}}$. The parachutist opens the parachute 10 s after jumping. Find the speed at which the parachutist is falling just before the parachute opens. **[5]**

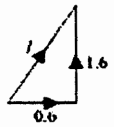


The diagram shows a (t, v) graph for the parachutist's motion, as modelled using the above differential equation.

(i) Explain the significance of the speed of $10\,\mathrm{m\,s^{-1}}$ in relation to the differential equation. **[1]**

(ii) What has been assumed about the opening of the parachute? **[1]**

(iii) Find the deceleration of the parachutist just after the parachute opens. **[2]**

1  Magnitude = $\sqrt{0.6^2 + 1.6^2}$ Direction above horizontal = $\tan^{-1}\left(\frac{1.6}{0.6}\right)$ Impulse is 1.71 N s at 69.4° to horizontal	B1 M1 A1 A1✓	 4	For identifying impulse vector with the change of momentum, by means of a triangle or otherwise For either Pythagoras or trig calculation For magnitude For angle to horizontal, or equivalent
2 (i) $0.5g = \frac{28x}{0.8}$ $x = 0.14 \Rightarrow OE = 0.94$ m	M1 A1	 2	For equilibrium equation and Hooke Correct answer for OE
(ii) Conservation of energy: $\frac{1}{2} \times 0.5v^2 + \frac{28 \times 0.14^2}{2 \times 0.8} = 0.5g \times 0.94$ $0.25v^2 + 0.343 = 4.606$, hence $v = 4.13\,\text{m s}^{-1}$	M1 B1✓ B1✓ A1	 4	For equation involving KE, PE and EE For correct EE term For PE term, correct apart possibly from sign
3 (i) $v_A \cos 30° = 5\cos 60°$ $v_A = \frac{5}{2} \div \left(\frac{1}{2}\sqrt{3}\right) = \frac{5}{3}\sqrt{3}$	M1 A1	 2	Equating components ⊥ line of centres Given answer correctly shown
(ii) $m \times 5\cos 30° = 2mv_B - mv_A \cos 60°$ $2v_B = \frac{5}{2}\sqrt{3} + \frac{5}{6}\sqrt{3} \Rightarrow v_B = \frac{5}{3}\sqrt{3}$	M1 A1 A1	 3	Using momentum ‖ line of centres Correct equation (v_A need not be numerical) Given answer correctly shown
(iii) $v_A \cos 60° + v_B = e \times 5\cos 30°$ $\frac{5}{3}\sqrt{3}\cos 60° + \frac{5}{3}\sqrt{3} = 5e\cos 30°$ Hence $e = 1$, as required	M1 A1 A1	 3	Using restitution ‖ line of centres Correct equation Given result correctly shown
4 Force in each spring is $\frac{5x}{0.5}$ Equation of motion is $10x + 10x = -0.2\ddot{x}$ i.e. $\ddot{x} = -100x$	B1 M1 A1	 3	Correct expression for a *general* position For relevant use of NII Given answer correctly shown
(i) Motion is SHM with amplitude 0.3 m $v_D^2 = 100(0.3^2 - 0.2^2)$ Speed at D is $2.24\,\text{m s}^{-1}$	B1 M1 A1	 3	Allow at any stage in the question
(ii) $0.2 = 0.3\cos(10t_D)$ $t_D = 0.1\cos^{-1}\left(\frac{2}{3}\right)$ Time to reach D is 0.0841 s	B1 M1 A1	 3	For correct SHM equation involving t_D Or equivalent complete solution method

5	(i) $\frac{1}{2}mv^2 = mg \times 10a \sin 30°$ Hence $v = \sqrt{10ga}$	M1 A1	2	For relevant use of conservation of energy
	(ii) $T - mg\cos 60° = m \times \frac{10ga}{10a}$ $T = \frac{3}{2}mg$	M1 A1√ A1	3	3-term NII equation ‖ string Correct unsimplified equation
	Critical case is $T = 0$ at highest point $\frac{1}{2}mv_H^2 = mg \times 4a$ $v_H^2 = 8ga$ $mg + T = \frac{mv^2}{3a}$ Hence it does make a complete circle	B1 M1 A1 M1 A1√	5	May be implied Use of energy to find v_H at highest point Resolving to find T ($=\frac{5}{3}mg$) when $v = v_H$ or to find critical v^2 ($= 3ga$) for $T = 0$ Correct result and reason
6	(i) Moments about A for the system: $3mg \times 2a + 2mg \times 6a = Y_C \times 8a$ $Y_C = \frac{9}{4}mg$ Moments about B for BC: $2mg \times 2a + X_C \times 3a = \frac{9}{4}mg \times 4a$ $X_C = \frac{5}{3}mg$	M1 A1 A1 M1 A1 A1	6	Equation with 3 terms needed For correct unsimplified equation Correct answer for the vertical component Equation with 3 terms needed For correct unsimplified equation Given answer correctly shown
	(ii) $X_B = \frac{5}{3}mg$ $Y_B = \frac{1}{4}mg$ Magnitude $= \sqrt{X_B^2 + Y_B^2}$ Dir above horizontal $= \tan^{-1}\left(\frac{Y_B}{X_B}\right)$ Magnitude $= \frac{1}{12}\sqrt{409}mg \approx 1.69mg$ Dir above horizontal $= \tan^{-1}\left(\frac{3}{20}\right) \approx 8.5°$	B1 B1√ M1 A1√ A1√	5	 Follow the answer for Y_C in **(i)** For numerical Pythagoras or trig calculation Correct exact or approximate value Correct exact or approximate angle
7	Equation of motion is $m\frac{dv}{dt} = mg - kv$ At terminal velocity $mg = kU$ Hence $\frac{dv}{dt} = \frac{g}{U}(U - v)$	B1 B1 B1	3	 Given answer correctly shown
	$\int \frac{1}{50 - v}\,dv = \int \frac{g}{50}\,dt$ $-\ln(50 - v) = 0.196t + c$ $v = 0, t = 0 \Rightarrow -\ln 50 = c$ $-\ln(50 - v_{10}) = 1.96 - \ln 50$ $v_{10} = 50(1 - e^{-1.96}) \approx 43.0 \text{ m s}^{-1}$	M1 A1 M1 A1 A1	5	For separation and attempt at integration For both indefinite integrals correct Evaluation of constant or equiv use of limits Correct equation for v_{10} For correct exact or approximate answer
	(i) 10 m s^{-1} is the terminal velocity (value of U) after the parachute opens	B1	1	
	(ii) The parachute is assumed to open instantaneously	B1	1	
	(iii) $\frac{dv}{dt} = \frac{9.8}{10}(10 - 43.0)$ Hence deceleration is 32.3 m s^{-2}	M1 A1√	2	

