OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2639

Mechanics 3

Tuesday **10 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

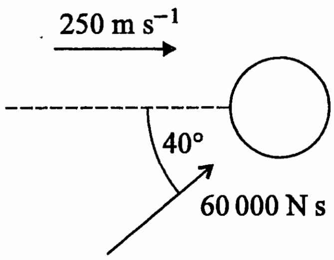
INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\ \mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

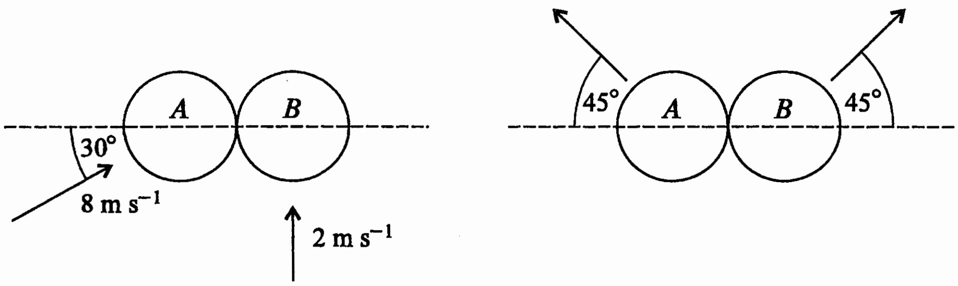
1 

A spacecraft of mass 300 kg is moving at 250 m s^{-1} in a straight line. An asteroid hits the spacecraft, and applies to the spacecraft an impulse of magnitude 60 000 N s in a direction making an angle of 40° with the initial direction of motion of the spacecraft (see diagram). Find the angle between the final velocity and the initial velocity of the spacecraft. [5]

2 A simple pendulum consists of a particle connected to a fixed point by a light inextensible string of length 0.8 m. The pendulum makes small oscillations in a vertical plane. Air resistance may be neglected.

(i) Prove that the motion is approximately simple harmonic, and find the period of oscillation. [5]

(ii) The pendulum is released from rest with the string making an angle of 0.05 radians with the vertical. Find the angle between the string and the vertical 0.64 seconds after it is released. [3]

3 

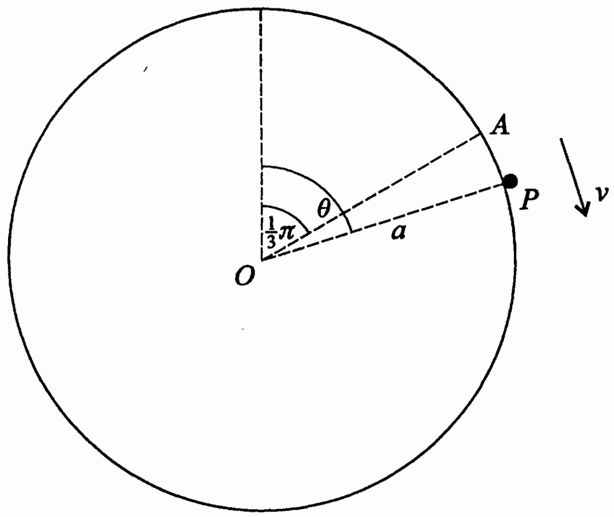
Before collision　　　　After collision

Two uniform smooth spheres A and B, of equal radius, are moving on a horizontal surface when they collide. Immediately before the collision, A has velocity 8 m s^{-1} at 30° to the line of centres, and B has velocity 2 m s^{-1} perpendicular to the line of centres. Immediately after the collision, A and B move in perpendicular directions, each making an angle of 45° with the line of centres (see diagram).

(i) Find the coefficient of restitution between the two spheres. [5]

(ii) Given that the mass of B is 5 kg, find the mass of A. [3]

4



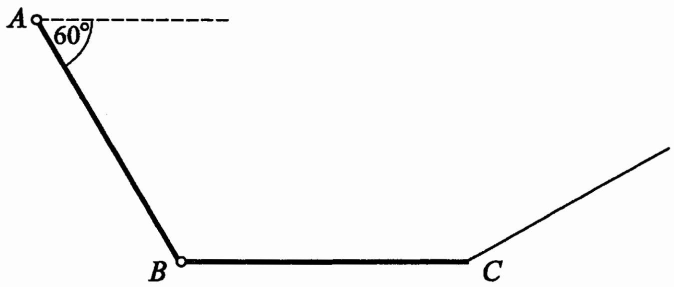
A fixed sphere, with centre O and radius a, has a smooth surface. A particle P of mass m is released from rest at a point A on the surface of the sphere, where OA makes an angle of $\frac{1}{3}\pi$ with the upward vertical. At an instant during the subsequent motion, P is still on the surface of the sphere, OP makes an angle θ with the upward vertical, and the speed of P is v (see diagram). Air resistance may be neglected.

(i) Show that $v^2 = ag(1 - 2\cos\theta)$. [2]

(ii) Find, in terms of m, g and θ, the normal reaction between P and the surface. [4]

(iii) Find the value of $\cos\theta$ when P leaves the surface of the sphere. [2]

5



Two uniform rods AB and BC, each of length 1.2 m and mass 8 kg, are freely jointed to each other at B, and AB is freely hinged to a fixed point at A. They are held in equilibrium, with AB at an angle of 60° to the horizontal, and BC horizontal, by a light string attached to C (see diagram).

(i) Find the vertical and horizontal components of the force acting on AB at B. [6]

(ii) Find the angle which the string makes with the horizontal. [3]

[Questions 6 and 7 are printed overleaf.]

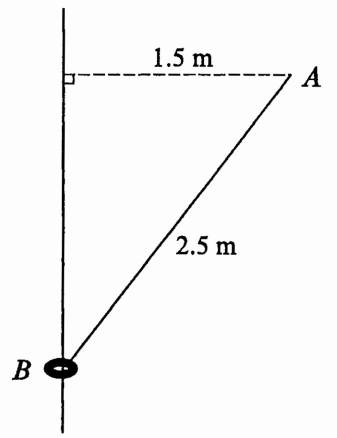
6 A sack of ballast of mass 15 kg is dropped from a descending balloon. At time t seconds after it is dropped, the downwards velocity of the sack is $v\,\mathrm{m\,s^{-1}}$. The only forces acting on the sack are its weight and air resistance $4.2v$ newtons.

(i) Show that $\frac{dv}{dt} = 0.28(35 - v)$. [3]

(ii) Given that the initial velocity of the sack is $8\,\mathrm{m\,s^{-1}}$ downwards, find its speed 5 seconds after it is dropped. [7]

7 A light elastic string has natural length 1.8 m and modulus of elasticity 12.6 N.

(i) The string is stretched to a length of 2.5 m. Find the tension in the string and the elastic potential energy stored in the string. [3]

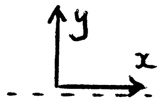
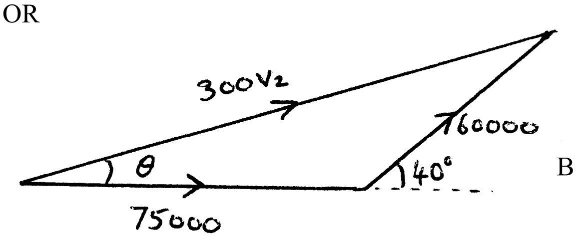
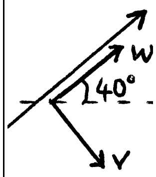


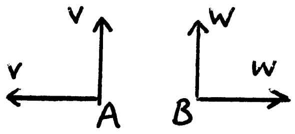
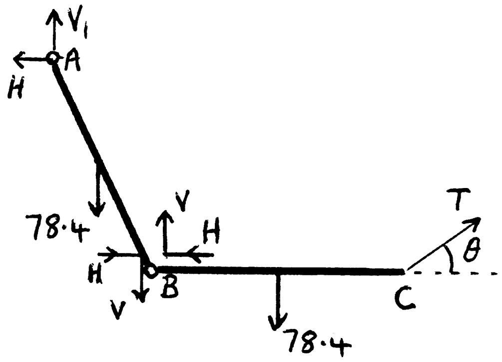
This string is now used to connect a small bead B to a fixed point A. The bead slides without resistance on a fixed vertical wire, and the point A is 1.5 m from the wire. The bead is in equilibrium when the length of the string is 2.5 m (see diagram).

(ii) Show that the mass of the bead is 0.4 kg. [3]

The bead is now pulled down the wire to a position where the length of the string is 3 m, and it is released from rest in this position.

(iii) Use conservation of energy to find the speed of the bead when it passes through the equilibrium position. [6]

1	Final velocity  $60\,000\sin 40 = 300y$ $y = 128.6$	B1	
	$60\,000\cos 40 = 300(x-250)$ $x = 403.2$	M1A1	
	$\theta = \tan^{-1}\frac{y}{x} = 17.7°$	DM1A1 **5**	Accept 162°
	OR  B1		Correct momentum (or velocity) triangle
	By cos rule, $300v_2 = 127\,000$ $(v_2 = 423)$ M1A1 By sine rule, $\theta = 17.7°$ DM1A1		Accept 162°
	OR $v = 250\sin 40$ $(= 160.7)$ B1  $60\,000 = 300(w - 250\cos 40)$ M1A1 $w = 391.5$ $\theta = 40 - \tan^{-1}\frac{v}{w} = 17.7°$ DM1A1		Accept 162°
2 (i)	Transverse acceleration is $0.8\ddot{\theta}$	B1	Accept *r*, *l* as alternatives to 0.8
	Hence $-mg\sin\theta = m(0.8\ddot{\theta})$	M1	
	OR $\frac{1}{2}m(0.8\dot{\theta})^2 - mg(0.8\cos\theta) = K$ B1		Energy equation
	$m(0.8)^2\dot{\theta}\ddot{\theta} + mg(0.8\sin\theta)\dot{\theta} = 0$ M1		Differentiating energy equation
	$\ddot{\theta} = -\frac{g}{0.8}\sin\theta$		
	Since θ is small, $\sin\theta \approx \theta$	M1	
	$\ddot{\theta} \approx -\frac{g}{0.8}\theta = -12.25\theta$	A1	
	Approx SHM with period $\frac{2\pi}{\sqrt{12.25}} = \frac{4}{7}\pi = 1.80$ s	B1 **5**	Accept direct use of $2\pi\sqrt{\frac{0.8}{9.8}}$ Accept $\frac{4}{7}\pi$ and 1.8
(ii)	$\theta = 0.05\cos 3.5t$ When $t = 0.64$, $\theta = -0.0310$	M1A1 ft	Accept $x = 0.04\cos 3.5t$
	String makes angle 0.0310 rad with the vertical	A1 cao **3**	Accept −0.031

3 (i)	Final velocities  $v = 8\sin 30 \quad (= 4)$ $w = 2$ $w + v = e(8\cos 30)$ $e = 0.866$	B1 B1 M1 A1 A1 cao **5**	 Restitution along line of centres Accept 0.867 and √3/2
(ii)	$-mv + 5w = m(8\cos 30)$ $m = 0.915$ kg	M1 A1 A1 cao **3**	Conservation of momentum
4 (i)	By conservation of energy, $\frac{1}{2}mv^2 = mg(a\cos\frac{1}{3}\pi - a\cos\theta)$ $v^2 = ag(2\cos\frac{1}{3}\pi - 2\cos\theta)$ $v^2 = ag(1 - 2\cos\theta)$	M1 A1 (ag) **2**	
(ii)	$mg\cos\theta - R = m\frac{v^2}{a}$ $mg\cos\theta - R = mg(1 - 2\cos\theta)$ $R = mg(3\cos\theta - 1)$	B1 M1A1 A1 **4**	Radial Acceleration is $\frac{v^2}{a}$
(iii)	Leaves surface when $R = 0$ $\cos\theta = \frac{1}{3}$	M1 A1 **2**	
5 (i)	 Moments about C for BC $8 \times 9.8 \times 0.6 - V \times 1.2 = 0$ $V = 39.2$ N Moments about A for AB $H \times 1.2\sin 60 - V \times 1.2\cos 60 - 8 \times 9.8 \times 0.6\cos 60 = 0$ $H = 45.3$ N	M1 A1 M1 A1 M1 A1 cao **6**	Or other method for finding V Moments equation involving H Complete method for finding H
(ii)	$T\sin\theta + V = 78.4$ $T\sin\theta = 39.2$ $T\cos\theta = H = 45.3$ $\tan\theta = \frac{39.2}{45.3}$ $\theta = 40.9°$	M1 M1 A1 cao **3**	Finding vertical component of T

6 (i)	$15\times 9.8-4.2v=15\dfrac{dv}{dt}$ $\dfrac{dv}{dt}=9.8-0.28v$ $\dfrac{dv}{dt}=0.28(35-v)$	M1A1 A1 (ag) **3**	
(ii)	$\int\dfrac{1}{35-v}\,dv=\int 0.28\,dt$	M1	Or $\dfrac{dv}{dt}+0.28v=9.8$ and use of IF $e^{0.28t}$ or $v=CF+PI$
	$-\ln(35-v)=0.28t+C$	A2	Give A1* if one minor error Or $e^{0.28t}v=35e^{0.28t}+A$ or $v=Ae^{-0.28t}+35$
	$v=8$ when $t=0 \Rightarrow C=-\ln 27$	M1	Using initial condition to find constant
	$-\ln(35-v)=0.28t-\ln 27$	A1 ft	depA1* (or as above with $A=-27$)
	When $t=5$, $\ln(35-v)=\ln 27-1.4$ $v=28.3\text{ m s}^{-1}$	M1 A1 cao **7**	Solving to obtain a value of v by using exponentiation
7 (i)	Tension is $\dfrac{12.6}{1.8}\times 0.7=4.9$ N	M1 A1	Use of extension 0.7
	Energy is $\frac{1}{2}\times\dfrac{12.6}{1.8}\times 0.7^2=1.715$ J	A1 **3**	Accept 1.71 and 1.72
(ii)	$T\cos\theta=mg$	M1	
	where $\sin\theta=\dfrac{1.5}{2.5}=0.6$, $\cos\theta=0.8$	B1	
	$4.9\times 0.8=m\times 9.8$ $m=0.4$ kg	A1 (ag) **3**	
(iii)	Gain in PE is $0.4\times 9.8\left(\sqrt{3^2-1.5^2}-\sqrt{2.5^2-1.5^2}\right)$ $=0.4\times 9.8(2.598-2)$ $=2.344$	M1 A1	Calculation of vertical distance
	Loss of EE is $\frac{1}{2}\times\dfrac{12.6}{1.8}\times 1.2^2-1.715\quad(=3.325)$	M1 A1 ft	Calculation of EE in starting position ft incorrect energy from part (i)
	$\frac{1}{2}\times 0.4v^2+2.344=3.325$	M1	Equation involving KE, PE, EE
	$v=2.21\text{ m s}^{-1}$	A1 cao **6**	