OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4768
Statistics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Coastal wildlife wardens are monitoring populations of herring gulls. Herring gulls usually lay 3 eggs per nest and the wardens wish to model the number of eggs per nest that hatch. They assume that the situation can be modelled by the binomial distribution B(3, p) where p is the probability that an egg hatches. A random sample of 80 nests each containing 3 eggs has been observed with the following results.

Number of eggs hatched	0	1	2	3
Number of nests	7	23	29	21

(i) Initially it is assumed that the value of p is $\frac{1}{2}$. Test at the 5% level of significance whether it is reasonable to suppose that the model applies with $p = \frac{1}{2}$. **[10]**

(ii) The model is refined by estimating p from the data. Find the mean of the observed data and hence an estimate of p. **[2]**

(iii) Using the estimated value of p, the value of the test statistic X^2 turns out to be 2.3857. Is it reasonable to suppose, at the 5% level of significance, that this refined model applies? **[3]**

(iv) Discuss the reasons for the different outcomes of the tests in parts **(i)** and **(iii)**. **[2]**

2 **(a)** A continuous random variable, X, has probability density function

$$f(x) = \begin{cases} \frac{1}{72}(8x - x^2) & 2 \leqslant x \leqslant 8, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find F(x), the cumulative distribution function of X. **[3]**

(ii) Sketch F(x). **[3]**

(iii) The median of X is m. Show that m satisfies the equation $m^3 - 12m^2 + 148 = 0$. Verify that $m \approx 4.42$. **[3]**

(b) The random variable in part **(a)** is thought to model the weights, in kilograms, of lambs at birth. The birth weights, in kilograms, of a random sample of 12 lambs, given in ascending order, are as follows.

3.16 3.62 3.80 3.90 4.02 4.72 5.14 6.36 6.50 6.58 6.68 6.78

Test at the 5% level of significance whether a median of 4.42 is consistent with these data. **[10]**

3 Cholesterol is a lipid (fat) which is manufactured by the liver from the fatty foods that we eat. It plays a vital part in allowing the body to function normally. However, when high levels of cholesterol are present in the blood there is a risk of arterial disease. Among the factors believed to assist with achieving and maintaining low cholesterol levels are weight loss and exercise.

A doctor wishes to test the effectiveness of exercise in lowering cholesterol levels. For a random sample of 12 of her patients, she measures their cholesterol levels before and after they have followed a programme of exercise. The measurements obtained are as follows.

Patient	A	B	C	D	E	F	G	H	I	J	K	L
Before	5.7	5.7	4.0	6.8	7.4	5.5	6.7	6.4	7.2	7.2	7.1	4.4
After	5.8	4.0	5.2	5.7	6.0	5.0	5.8	4.2	7.3	5.2	6.4	4.1

(i) A *t* test is to be used in order to see if, on average, the exercise programme seems to be effective in lowering cholesterol levels. State the distributional assumption necessary for the test, and carry out the test using a 1% significance level. **[11]**

(ii) A second random sample of 12 patients gives a 95% confidence interval of (−0.5380, 1.4046) for the true mean reduction (before – after) in cholesterol level. Find the mean and standard deviation for this sample. How might the doctor interpret this interval in relation to the exercise programme? **[7]**

4 The weights of a particular variety (A) of tomato are known to be Normally distributed with mean 80 grams and standard deviation 11 grams.

(i) Find the probability that a randomly chosen tomato of variety A weighs less than 90 grams. **[3]**

The weights of another variety (B) of tomato are known to be Normally distributed with mean 70 grams. These tomatoes are packed in sixes using packaging that weighs 15 grams.

(ii) The probability that a randomly chosen pack of 6 tomatoes of variety B, including packaging, weighs less than 450 grams is 0.8463. Show that the standard deviation of the weight of single tomatoes of variety B is 6 grams, to the nearest gram. **[5]**

(iii) Tomatoes of variety A are packed in fives using packaging that weighs 25 grams. Find the probability that the total weight of a randomly chosen pack of variety A is greater than the total weight of a randomly chosen pack of variety B. **[5]**

(iv) A new variety (C) of tomato is introduced. The weights, c grams, of a random sample of 60 of these tomatoes are measured giving the following results.

$$\Sigma c = 3126.0 \qquad \Sigma c^2 = 164\,223.96$$

Find a 95% confidence interval for the true mean weight of these tomatoes. **[5]**

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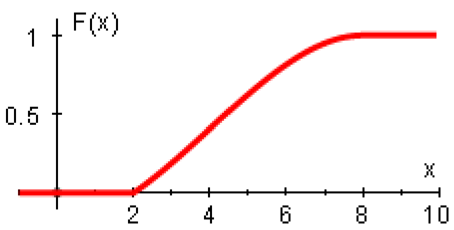
1 (i)	H_0: The number of eggs hatched can be modelled by B(3, ½) H_1: The number of eggs hatched cannot be modelled by B(3, ½)	B1 B1		

With p = ½

Probability	0.125	0.375	0.375	0.125
Exp'd frequency	10	30	30	10
Obs'd frequency	7	23	29	21

		M1 A1	Probs × 80 for expected frequencies. All correct.	
	X^2 = 0.9 + 1.6333 + 0.0333 + 12.1 = 14.666(7)	M1 A1	Calculation of X^2. c.a.o.	
	Refer to χ_3^2.	M1	Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. P(X^2 > 14.667) = 0.00212.	
	Upper 5% point is 7.815. Significant. Suggests it is reasonable to suppose model with p = ½ does not apply.	A1 A1 A1	No ft from here if wrong. ft only c's test statistic. ft only c's test statistic.	**[10]**
(ii)	$\bar{x} = \frac{144}{80} = 1.8$ $\therefore \hat{p} = \frac{1.8}{3} = 0.6$	B1 B1	C.a.o. Use of E(X) = np. ft c's mean, provided $0 < \hat{p} < 1$.	**[2]**
(iii)	Refer to χ_2^2.	M1	Allow df 1 less than in part (i). No ft if wrong.	
	Upper 5% point is 5.991.	A1	No ft if wrong.	
	Suggests it is reasonable to suppose model with estimated p does apply.	A1	ft provided previous A mark awarded.	**[3]**
(iv)	For example: Estimating p leads to an improved fit … … at the expense of the loss of 1 degree of freedom. The model in (i) fails due to a large underestimate for X = 3.	E2	Reward any two sensible points for E1 each.	**[2]**
			Total	**[17]**

2 (a)	$\mathrm{f}(x)=\frac{1}{72}\left(8x-x^2\right)\ ,\ 2\le x\le 8$			
(i)	$\mathrm{F}(x)=\int_2^x \frac{1}{72}\left(8t-t^2\right)\mathrm{d}t$ $=\frac{1}{72}\left[4t^2-\frac{t^3}{3}\right]_2^x$ $=\frac{1}{72}\left(4x^2-\frac{x^3}{3}-16+\frac{8}{3}\right)=\frac{12x^2-x^3-40}{216}$	M1 A1 A1	Correct integral with limits (which may be implied subsequently). Correctly integrated Limits used. Accept unsimplified form.	**[3]**
(ii)		G1 G1 G1	Correct shape; nothing below $y=0$; non-negative gradient. Labels at (2, 0) and (8, 1). Curve (horizontal lines) shown for $x<2$ and $x>8$.	**[3]**
(iii)	$\mathrm{F}(m)=½\quad \therefore \frac{12m^2-m^3-40}{216}=\frac{1}{2}$ $\therefore 12m^2-m^3-40=108$ $\therefore m^3-12m^2+148=0$ Either F(4.42) = 0.5003(977) ≈ 0.5 Or $4.42^3-12\times 4.42^2+148=-0.0859(12)\approx 0$ $\therefore m\approx 4.42$	M1 A1 E1	Use of definition of median. Allow use of c's F(x). Convincingly rearranged. Beware: answer given. Convincingly shown, e.g. 4.418 or better seen.	**[3]**

2 (b)	H_0: $m = 4.42$ H_1: $m \neq 4.42$ where m is the population median	B1 B1	Both. Accept hypotheses in words. Adequate definition of m to include "population".	

Weights	− 4.42	Rank of |diff|
3.16	−1.26	7
3.62	−0.80	6
3.80	−0.62	4
3.90	−0.52	3
4.02	−0.40	2
4.72	0.30	1
5.14	0.72	5
6.36	1.94	8
6.50	2.08	9
6.58	2.16	10
6.68	2.26	11
6.78	2.36	12

M1 for subtracting 4.42.

M1 for ranks.
A1 ft if ranks wrong.

	$W_- = 2 + 3 + 4 + 6 + 7 = 22$	B1	($W_+ = 1 + 5 + 8 + 9 + 10 + 11 + 12 = 56$)	
	Refer to Wilcoxon single sample tables for $n = 12$.	M1	No ft from here if wrong.	
	Lower 2½% point is 13 (or upper is 65 if 56 used).	A1	i.e. a 2-tail test. No ft from here if wrong.	
	Result is not significant.	A1	ft only c's test statistic.	
	Evidence suggests that a median of 4.42 is consistent with these data.	A1	ft only c's test statistic.	**[10]**
			Total	**[19]**

3 (i)	Must assume • Normality of population … • … of differences. H_0: $\mu_D = 0$ H_1: $\mu_D > 0$	 B1 B1 B1	 Both. Accept alternatives e.g. $\mu_D <$ 0 for H_1, or $\mu_B - \mu_A$ etc provided adequately defined. Hypotheses in words only must include "population". Do NOT allow "$\bar{X} = ...$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a population mean.	
	Where μ_D is the (population) mean reduction/difference in cholesterol level.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used.	
	MUST be PAIRED COMPARISON t test. Differences (reductions) (before – after) are:		 Allow "after – before" if consistent with alternatives above.	
	–0.1 1.7 –1.2 1.1 1.4 0.5 0.9 2.2 –0.1 2.0 0.7 0.3			
	$\bar{x} = 0.7833$ $s_{n-1} = 0.9833(46)$ ($s_{n-1}^2 = 0.966969$)	B1	Do not allow $s_n = 0.9415$ ($s_n^2 = 0.8864$)	
	Test statistic is $\dfrac{0.7833 - 0}{\frac{0.9833}{\sqrt{12}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0 + (c's 2.718) × $\frac{0.9833}{\sqrt{12}}$ (= 0.7715) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 2.718) × $\frac{0.9833}{\sqrt{12}}$ (= 0.0118) for comparison with 0.)	
	= 2.7595.	A1	c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{x}$ scores M1A0, but ft.	
	Refer to t_{11}.	M1	No ft from here if wrong. P(t > 2.7595) = 0.009286.	
	Single-tailed 1% point is 2.718.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean cholesterol level has fallen.	A1	ft only c's test statistic.	**[11]**
(ii)	CI is $\bar{x} \pm$	M1	Overall structure, seen or implied.	
	2.201	B1	From t_{11}, seen or implied.	
	$\times \dfrac{s}{\sqrt{12}} = (-0.5380, 1.4046)$	A1	Fully correct pair of equations using the given interval, seen or implied.	
	$\bar{x} = ½(1.4046 - 0.5380) = 0.4333$	B1		
	$s = (1.4046 - 0.4333) \times \dfrac{\sqrt{12}}{2.201} = 1.5287$	M1 A1	Substitute $\bar{x}$ and rearrange to find s. c.a.o.	
	Using this interval the doctor might conclude that the mean cholesterol level did not seem to have been reduced.	E1	Accept any sensible comment or interpretation of this interval.	**[7]**
			Total	**[18]**

4	$A \sim N(80,\ \sigma = 11)$ $B \sim N(70,\ \sigma = v)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$P(A < 90) = P\left(Z < \frac{90-80}{11} = 0.9091\right)$ $= 0.8182$	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	[3]
(ii)	$W_B = B_1 + B_2 + \ldots + B_6 + 15 \sim N(435,$ $\sigma^2 = v^2 + v^2 + \ldots + v^2 = 6v^2)$ $P(\text{this} < 450) = P\left(Z < \frac{450-435}{v\sqrt{6}}\right) = 0.8463$ $\therefore \frac{450-435}{v\sqrt{6}} = \Phi^{-1}(0.8463) = 1.021$ $\therefore v = \frac{15}{1.021 \times \sqrt{6}} = 5.9977 = 6$ grams (nearest gram)	B1 B1 M1 B1 A1	Mean. Expression for variance. Formulation of the problem. Inverse Normal. Convincingly shown, beware A.G.	[5]
(iii)	$W_A = A_1 + A_2 + \ldots + A_5 + 25 \sim N(425,$ $\sigma^2 = 11^2 + 11^2 + \ldots + 11^2 = 605\ )$ $D = W_A - W_B \sim N(-10,$ $605 + 216 = 821)$ Want $P(W_A > W_B) = P(W_A - W_B > 0)$ $= P\left(Z > \frac{0-(-10)}{\sqrt{821}} = 0.3490\right) = 1 - 0.6365 = 0.3635$	B1 M1 A1 M1 A1	Mean. Accept "$B - A$". Variance. Accept sd (= 28.65). c.a.o.	[5]
(iv)	$\bar{x} = \frac{3126.0}{60} = 52.1$, $s = \sqrt{\frac{164223.96 - 60 \times 52.1^2}{59}} = 4.8$ CI is given by 52.1 ± 1.96 $\times \frac{4.8}{\sqrt{60}}$ = 52.1 ± 1.2146= (50.885(4), 53.314(6))	B1 M1 B1 M1 A1	Both correct. c.a.o. Must be expressed as an interval.	[5]
			Total	**[18]**

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General Comments

There were 280 candidates from 41 centres (January 2009: 291 from 40) for this sitting of the paper. Overall the general standard of many of the scripts seen compared favourably with those seen in recent series: there were many examples of good, thorough and well-organised work. However, at the same time, some candidates showed considerable carelessness, particularly in the quality of their comments, interpretations and explanations. For example, some candidates stated their hypotheses badly or neglected to state them at all, and the final conclusions were sometimes badly expressed. It should be noted that, when a test produces a significant result, it is not correct to say "there is no evidence to suggest that H_0 is true"; but rather "there is evidence to suggest that H_0 may be false." Furthermore, candidates need to be aware that, when asked to show a result that is given in the question, the working leading to that result needs to be absolutely explicit in order to convince the examiner that the candidate has genuinely carried out every step of the necessary work.

All four questions were attempted. Marks for Question 4 were found to be a little higher on average than Questions 1, 2 and 3, which were equally well answered. There was no evidence to suggest that candidates were unable to complete the paper through a shortage of time.

Comments on Individual Questions

1 Chi-squared test of goodness of fit of a binomial model; numbers of eggs per nest that hatch.

(i) Sometimes the hypotheses for this test were missing completely while on other occasions they were badly expressed: for example, "H_0: $p = ½$" etc was seen on many occasions.
In contrast, the calculation of the expected frequencies and the test statistic were usually done correctly. The last part, the critical value and the conclusion, was generally correct, although there were many, as in the past, whose choice of language was too assertive.

(ii) The mean was usually correct, but occasionally there were errors in finding the estimate of p.

(iii) The majority of candidates, but by no means all, realised that an adjustment to the number of degrees of freedom and hence to the critical value was appropriate, but, as in part (i), some candidates were not as careful about their conclusion as they should have been.

(iv) Marks were not awarded for simply repeating what had already been established in parts (i) and (iii); candidates were expected to consider the different outcomes in a little more depth, for example by attributing the improved fit of the model to the fact that p had been estimated from the data.

2 Continuous random variables: the cdf and the median; Wilcoxon single sample test: birth weights of lambs.

(a) (i) There were many failed attempts to set up the necessary integral correctly. Limits were either incorrect or omitted altogether. Sometimes candidates were seen attempting to backtrack, probably as a result of realising in part (iii) that something was wrong.

(ii) Many responses did not show the basic characteristics of a cumulative probability curve. For full marks the sketches were expected to show the horizontal portions for $x < 2$ and for $x > 8$.

(iii) When the answer to part (i) was correct then it was not difficult for candidates to derive the required cubic equation for m. However, for this and for the verification of the value of m, the working needed to be convincing.

(b) The Wilcoxon test was almost always carried out with little difficulty. There were just two minor common shortcomings that incurred a loss of marks. In the hypotheses it was necessary to indicate that it is the population median that is being tested. (Also, it is not a good idea to use μ for the median.) As in other tests, the conclusion must be expressed using non-assertive language.

3 Paired *t* test for the population mean reduction in cholesterol levels; confidence interval for the true population mean.

(i) Both the distributional assumption and the hypotheses were sometimes carelessly expressed. There were occasional errors in calculating the differences in the data, thus leading to an incorrect test statistic, but broadly speaking candidates knew what they were required to do. The test itself was usually correct apart from the conclusion, which was almost always deficient in that either it was too assertive and/or (more usually) it contained no recognition that on average cholesterol levels appeared to be reduced.

(ii) By working backwards from the confidence interval, the mean and standard deviation of the second sample were frequently worked out correctly and, in some cases, very efficiently indeed. However, there were also many instances of candidates who could set up the correct simultaneous equations but who could not then solve them correctly.
The final part of this question asked candidates to interpret this particular interval. The standard response of "95% of intervals …" was seen frequently but did not impress the examiners. Some candidates tried to link it to the mean of the sample in part (i). Many tried to interpret it in terms of individuals. Hardly any candidates discussed it in terms of the population mean, let alone concluding that, since 0 is in the interval, it probably meant no improvement on the whole.

4 **Combinations of Normal distributions; confidence interval for the true population mean; packets of different varieties of tomatoes.**

On the whole, this question was answered very well by very many candidates.

(i) Intended as an easy introduction to the question this part was almost always answered correctly. However there was a noticeable number of candidates who looked up $\Phi(0.99)$ in the Normal distribution tables instead of $\Phi(0.909)$.

(ii) The majority of candidates made good progress with this part, and many scored full marks. Difficulties arose in respect of the expression needed for the variance of the weight of tomatoes in the pack. This often resulted in some adjustment at a later stage. Sometimes the candidate was able to recover the situation. On other occasions it meant that spurious calculations were used to arrive at the given value. Once again, the working leading to a printed answer needed to be totally convincing.

(iii) As in part (ii) considerable progress was made in this part, with full marks often being awarded. Again the main problem was in the calculation of the variance, which was not always shown clearly enough to allow a judgement about the validity of the method used for it. This time a further error was sometimes seen when candidates neglected to include the weights of the different packagings.

(iv) This part was very well answered, and it was quite common for candidates to score full marks for the confidence interval.