

ADVANCED GCE
MATHEMATICS (MEI)
Mechanics 2

4762

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

- 1 (a) Two small objects, P of mass m kg and Q of mass km kg, slide on a smooth horizontal plane. Initially, P and Q are moving in the same straight line towards one another, each with speed u m s⁻¹.

After a direct collision with P, the direction of motion of Q is reversed and it now has a speed of $\frac{1}{3}u$ m s⁻¹. The velocity of P is now v m s⁻¹, where the positive direction is the original direction of motion of P.

- (i) Draw a diagram showing the velocities of P and Q before and after the impact. [1]
- (ii) By considering the linear momentum of the objects before and after the collision, show that $v = (1 - \frac{4}{3}k)u$. [3]
- (iii) Hence find the condition on k for the direction of motion of P to be reversed. [2]

The coefficient of restitution in the collision is 0.5.

- (iv) Show that $v = -\frac{2}{3}u$ and calculate the value of k . [5]

- (b) Particle A has a mass of 5 kg and velocity $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ m s⁻¹. Particle B has mass 3 kg and is initially at rest. A force $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ N acts for 9 seconds on B and subsequently (in the absence of the force), A and B collide and stick together to form an object C that moves off with a velocity \mathbf{V} m s⁻¹.

- (i) Show that $\mathbf{V} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. [4]

The object C now collides with a smooth barrier which lies in the direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The coefficient of restitution in the collision is 0.5.

- (ii) Calculate the velocity of C after the impact. [3]

- 2 (a) A small block of mass 25 kg is on a long, horizontal table. Each side of the block is connected to a small sphere by means of a light inextensible string passing over a smooth pulley. Fig. 2 shows this situation. Sphere A has mass 5 kg and sphere B has mass 20 kg. Each of the spheres hangs freely.



Fig. 2

Initially the block moves on a smooth part of the table. With the block at a point O, the system is released from rest with both strings taut.

- (i) (A) Is mechanical energy conserved in the subsequent motion? Give a brief reason for your answer. [1]
 (B) Why is no work done by the block against the reaction of the table on it? [1]

The block reaches a speed of 1.5 m s^{-1} at point P.

- (ii) Use an energy method to calculate the distance OP. [5]

The block continues moving beyond P, at which point the table becomes rough. After travelling two metres beyond P, the block passes through point Q. The block does 180 J of work against resistances to its motion from P to Q.

- (iii) Use an energy method to calculate the speed of the block at Q. [5]

- (b) A tree trunk of mass 450 kg is being pulled up a slope inclined at 20° to the horizontal.

Calculate the power required to pull the trunk at a steady speed of 2.5 m s^{-1} against a frictional force of 2000 N. [5]

- 3 A non-uniform beam AB has weight 85 N. The length of the beam is 5 m and its centre of mass is 3 m from A. In this question all the forces act in the same vertical plane.

Fig. 3.1 shows the beam in horizontal equilibrium, supported at its ends.

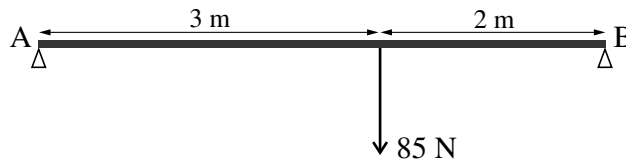


Fig. 3.1

- (i) Calculate the reactions of the supports on the beam. [4]

Using a smooth hinge, the beam is now attached at A to a vertical wall. The beam is held in equilibrium at an angle α to the horizontal by means of a horizontal force of magnitude 27.2 N acting at B. This situation is shown in Fig. 3.2.

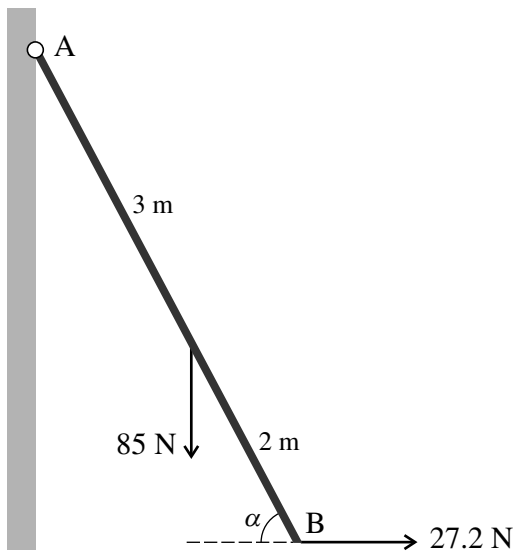


Fig. 3.2

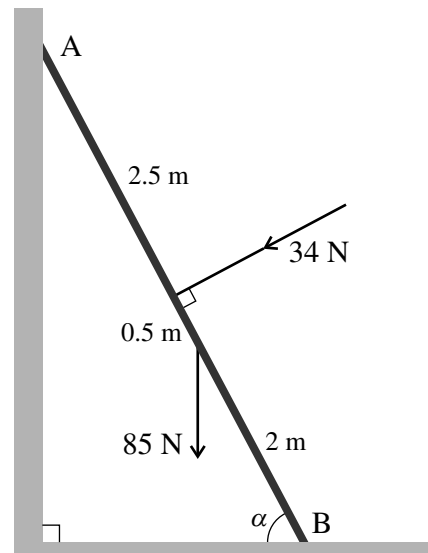


Fig. 3.3

- (ii) Show that $\tan \alpha = \frac{15}{8}$. [4]

The hinge and 27.2 N force are removed. The beam now rests with B on a rough horizontal floor and A on a smooth vertical wall, as shown in Fig. 3.3. It is at the same angle α to the horizontal. There is now a force of 34 N acting at right angles to the beam at its centre in the direction shown. The beam is in equilibrium and on the point of slipping.

- (iii) Draw a diagram showing the forces acting on the beam.

Show that the frictional force acting on the beam is 7.4 N.

Calculate the value of the coefficient of friction between the beam and the floor. [10]

- 4 In this question you may use the following facts: as illustrated in Fig. 4.1, the centre of mass, G, of a uniform thin open hemispherical shell is at the mid-point of OA on its axis of symmetry; the surface area of this shell is $2\pi r^2$, where r is the distance OA.

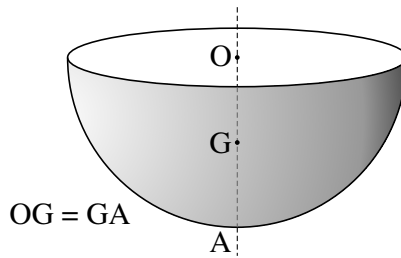
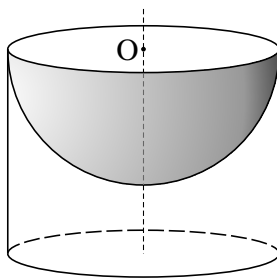
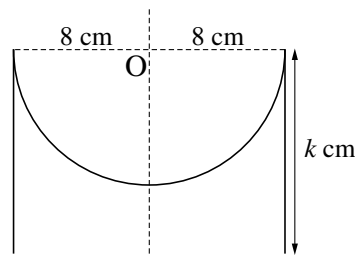


Fig. 4.1

A perspective view and a cross-section of a dog bowl are shown in Fig. 4.2. The bowl is made throughout from thin uniform material. An open hemispherical shell of radius 8 cm is fitted inside an open circular cylinder of radius 8 cm so that they have a common axis of symmetry and the rim of the hemisphere is at one end of the cylinder. The height of the cylinder is k cm. The point O is on the axis of symmetry and at the end of the cylinder.

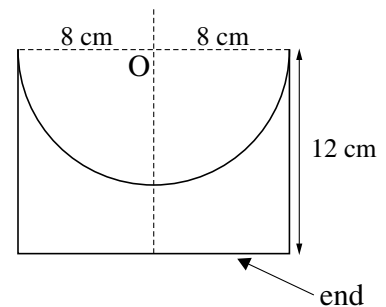


perspective view



cross-section

Fig. 4.2



cross-section

Fig. 4.3

- (i) Show that the centre of mass of the bowl is a distance $\frac{64 + k^2}{16 + 2k}$ cm from O. [6]

A version of the bowl for the 'senior dog' has $k = 12$ and an end to the cylinder, as shown in Fig. 4.3. The end is made from the same material as the original bowl.

- (ii) Show that the centre of mass of this bowl is a distance $6\frac{1}{3}$ cm from O. [5]

This bowl is placed on a rough slope inclined at θ to the horizontal.

- (iii) Assume that the bowl is prevented from sliding and is on the point of toppling.

Draw a diagram indicating the position of the centre of mass of the bowl with relevant lengths marked.

Calculate the value of θ . [5]

- (iv) If the bowl is not prevented from sliding, determine whether it will slide when placed on the slope when there is a coefficient of friction between the bowl and the slope of 1.5. [3]

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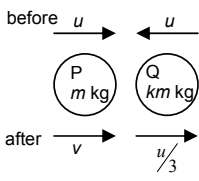
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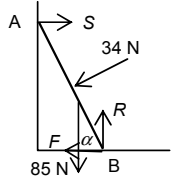
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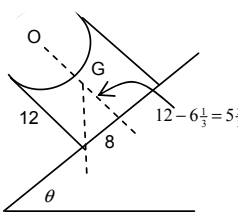
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Q 1	mark	comment	sub
(a) (i) 	B1		1
(ii) $mu - kmu = mv + km\frac{u}{3}$ $v = \left(1 - \frac{4k}{3}\right)u$	M1 A1 E1	PCLM applied Either side correct (or equiv) Must at least show terms grouped	3
(iii) Need $v < 0$ so $k > \frac{3}{4}$	E1 B1	Accept $\frac{4k}{3} > 1$ without reason [SC1: $v = 0$ used and inequality stated without reason]	2
(iv) $\frac{\frac{u}{3} - v}{-u - u} = -\frac{1}{2}$ so $v = -\frac{2u}{3}$ $-\frac{2u}{3} = u\left(1 - \frac{4k}{3}\right)$ so $k = 1.25$	M1 A1 E1 M1 A1	Use of NEL cao	5
(b) (i) $9\begin{pmatrix} 1 \\ -2 \end{pmatrix} + 5\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 8\mathbf{v}$ $\mathbf{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$	M1 B1 M1 E1	Use of PCLM Use of mass 8 in coalescence Use of $\mathbf{I} = \mathbf{F}t$	4
(ii) i cpt $3 \rightarrow -3 \times \frac{1}{2}$	M1	Allow wrong sign	

j cpt unchanged	B1	May be implied	
new velocity $\begin{pmatrix} -1.5 \\ -1 \end{pmatrix} \text{ m s}^{-1}$	A1	cao [Award 2/3 if barrier taken as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$]	
			3
	18		

Q 2	mark	comment	sub
(a)			
(i) (A) Yes. Only WD is against conservative forces.	E1	Accept only WD is against gravity or no work done against friction.	
(B) Block has no displacement in that direction	E1		2
(ii)			
$0.5 \times 50 \times 1.5^2 = 20gx - 5gx$	M1	Use of WE with KE. Allow $m = 25$.	
	B1	Use of 50	
	M1	At least 1 GPE term	
	A1	GPE terms correct signs	
$x = 0.38265\dots$ so 0.383 m (3 s. f.)	A1	cao	5
(iii)			
$0.5 \times 50 \times V^2 - 0.5 \times 50 \times 1.5^2$	M1	WE equation with WD term. Allow GPE terms missing	
	B1	Both KE terms. Accept use of 25.	
$= 2 \times 20g - 2 \times 5g - 180$	B1	Either GPE term	
	B1	180 with correct sign	
$V = 2.6095\dots$ so 2.61 m s ⁻¹	A1	cao	5
(b)			
Force down the slope is			
$2000 + 450g \sin 20$	M1	Both terms. Allow mass not weight	
	B1	Weight term correct	
Using $P = Fv$	M1		
$P = (2000 + 450g \sin 20) \times 2.5$	F1	FT their weight term	
$P = 8770.77\dots$ so 8770 W (3 s. f.)	A1	cao	5
	17		

Q 3	mark	comment	sub
(i)			
c.w. moments about A	M1	Moments equation.	
$5R_B - 3 \times 85 = 0$ so $R_B = 51$ giving 51 N \uparrow	A1	Accept no direction given	
Either a.c. moments about B or resolve \uparrow	M1		
$R_A = 34$ so 34 N \uparrow	F1	Accept no direction given	
			4
(ii)			
c.w. moments about A	M1	Moments with attempt to resolve at least one force. Allow $s \leftrightarrow c$.	
$85 \times 3 \cos \alpha - 27.2 \times 5 \sin \alpha = 0$	B1	Weight term	
	B1	horiz force term	
so $\tan \alpha = \frac{3 \times 85}{27.2 \times 5} = \frac{15}{8}$	E1	Must see some arrangement of terms or equiv	
			4
(iii)			
	B1	All forces present and labelled	
a.c. moments about B	M1	Moments with attempt to resolve forces and all relevant forces present	
$85 \times 2 \times \cos \alpha + 34 \times 2.5 - 5S \times \sin \alpha = 0$	B1	34×2.5	
	A1	All other terms correct. Allow sign errors.	
$S = 37.4$	A1	All correct	
Resolving horizontally and vertically	M1	Either attempted	
$\rightarrow S - F - 34 \sin \alpha = 0$ so $F = 7.4$	E1		
$\uparrow R - 85 - 34 \cos \alpha = 0$	A1	$R = 101$ need not be evaluated here [Allow A1 for the two expressions if correct other than $s \leftrightarrow c$]	
Using $F = \mu R$	M1		
$\mu = \frac{7.4}{101} = 0.07326\dots$ so 0.0733 (3 s. f.)	A1	cao	
			10
	18		

Q 4	mark	comment	sub
<p>(i)</p> <p>Taking a y-axis vert downwards from O</p> $2\pi\sigma \times 8^2 \times 4 + 2\pi\sigma \times 8 \times k \times \frac{k}{2}$ $= (2\pi\sigma \times 8^2 + 2\pi\sigma \times 8k) \bar{y}$ <p>so $\bar{y} = \frac{64+k^2}{16+2k}$</p>	<p>M1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>E1</p>	<p>Allow areas used as masses</p> <p>Method for c.m.</p> <p>'4' used</p> <p>$16\pi k$</p> <p>$k/2$ used</p> <p>Masses correct</p> <p>Must see some evidence of simplification</p> <p>Need no reference to axis of symmetry</p>	6
<p>(ii)</p> <p>$k = 12$ gives OG as 5.2 and mass as $320\pi\sigma$</p> $320\pi\sigma \times 5.2 + \pi\sigma \times 8^2 \times 12$ $= (320\pi\sigma + 64\pi\sigma) \bar{y}$ $\bar{y} = 6\frac{1}{3}$	<p>B1</p> <p>M1</p> <p>B1</p> <p>B1</p> <p>E1</p>	<p>Allow for either. Allow $\sigma = 1$</p> <p>Method for c.m. combining with (i) or starting again</p> <p>One term correct</p> <p>Second term correct</p> <p>Some simplification shown</p>	5
<p>(iii)</p>  <p>$\tan \theta = \frac{8}{5\frac{2}{3}}$</p> <p>$\theta = 54.6887\dots$ so 54.7° (3 s. f.)</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>G above edge of base</p> <p>$12 - 6\frac{1}{3} = 5\frac{2}{3}$ seen here or below</p> <p>8 seen here or below</p> <p>Accept $\frac{5\frac{2}{3}}{8}$ or attempts based on $6\frac{1}{3}$ and 8.</p> <p>cao</p>	5
<p>(iv)</p> <p>Slips when $\mu = \tan \theta$</p> $\frac{8}{5\frac{2}{3}} = 1.4117\dots$ <p>< 1.5 so does not slip</p>	<p>M1</p> <p>B1</p> <p>A1</p>	<p>Or</p> <p>There must be a reason</p>	3
	19		

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General Comments

Many good responses to this paper were seen and the majority of candidates could attempt at least some part of every question and gain credit for their efforts. The standard of presentation was variable and some candidates did not appreciate that poor notation and failure to state the principles or processes being employed could lead to avoidable errors and to loss of marks. As in previous sessions, those parts of the questions that required a candidate to explain or show a given answer were the least well done. Many candidates did not give enough detail in either case. As has also happened in previous sessions, those candidates that attempted to work back from a given answer usually obtained less credit than those who had attempted to employ the principles required to solve the problem.

There was evidence to suggest that some candidates found the paper long and this was taken into account at the Award. Some candidates ran out of time because they used inefficient methods of solution, particularly in Q1 and Q4.

Comments on Individual Questions

- 1) This was the highest scoring question on the paper with the vast majority of candidates able to produce work worthy of significant credit.
 - (a)(i) The majority of candidates obtained the mark for this part but some omitted labels or failed to indicate direction of the velocities.
 - (ii) Almost all of the candidates showed understanding of the principle of conservation of momentum and obtained full marks for this part. However, a small number failed to show sufficient evidence of how they obtained the given answer.
 - (iii) Many obtained full marks for this part but a significant minority failed to explain the reasoning behind their answer. Others interpreted the fact that the direction of motion of P had to be reversed by replacing v by $-v$ instead of taking $v < 0$
 - (iv) It was encouraging to see that the majority of candidates employed Newton's experimental law correctly and obtained the right answer; failure to do this was usually due to a sign error.
- (b)(i) Many fully correct solutions were seen to this part.
- (ii) This part caused problems for a large number of candidates. Many failed to realise that the velocity parallel to the barrier would be unchanged and applied Newton's experimental law in both directions. Others took the barrier to be at right angles to the direction specified.

- 2) Candidates seemed to understand the principles to be employed in this question but did not always appreciate that all of the masses needed to be considered.
- (a)(i) Only a minority of candidates could give a clear reason as to why energy would be conserved in this situation. Many stated, incorrectly, that no external force was acting. Others seemed to have little idea regarding cause and effect and offered the answer that energy was conserved because potential energy was equal to kinetic energy.
- A
- There were few correct answers seen to this part. Some candidates seemed to understand that the reaction force was perpendicular to the table but could not explain the relevance of this to work done. Others said that it was perpendicular to it, without any indication as to what 'it' was. It was common to see the arguments advanced in part A repeated.
- B
- (ii) Most candidates obeyed the instruction in the question to employ an energy method and gained some marks. A majority, though, used $m = 25$ (instead of 50) when calculating the kinetic energy terms i.e. ignored the masses of the spheres. Despite the instructions in the question, some candidates attempted to use the constant acceleration formulae and Newton's second law without any consideration of energy at all.
- (iii) As in the previous part, the majority of candidates considered $m = 25$ and many went on to omit the gravitational potential energy terms as well.
- (b) A large number of candidates obtained all 5 marks for this part of the question. Those that did not, usually failed because they had omitted 'g' from the weight term or had the cosine component rather than the sine component.
- 3) Many excellent answers were seen to this question but a significant minority struggled with the trigonometry and the manipulation of equations.
- (i) Almost all of the candidates obtained full marks for this part.
- (ii) Most of the candidates understood that they needed to take moments for this part but could not manipulate their moments equation to show the given answer. Many thought that $\tan\alpha = \sin\alpha \times \cos\alpha$ and some 'fudging' of the algebra was seen.
- (iii) The quality of the diagrams offered was, in many cases, poor. Forces at A and/or B were missing or shown at an acute angle to the beam. Additional spurious forces such as a reaction perpendicular to the beam at the point where the weight acts were included by some candidates. Those who drew a clearly labelled and correct diagram were usually able to make more progress in the following working than those who did not. Candidates gained some credit for appreciating the need to take moments and later to resolve but fully correct equations were not always obtained. Confusion with the use of sine and cosine was common as were equations where moment and force terms were mixed. The majority of candidates recovered by using the given answer to calculate the value of the coefficient of friction.

- 4) Some candidates were obviously pressed for time with this question and solutions appeared rushed. However, most made some progress worthy of credit, appearing to understand the method to be employed.
- (i) The majority of the candidates gained the marks for this part. The main errors made by those who did not were lack of knowledge of the formula for the surface area of a cylinder (a GCSE topic) and, even though the formula for the surface area of the shell was given, many chose not to use it. False cancelling abounded in attempts to obtain the given answer.
 - (ii) Those candidates that were successful in part (i) were usually as successful in this part but some did not use what they had already derived and started from scratch, causing themselves time problems.
 - (iii) Many fully correct solutions were seen. Even those candidates who did not do well on the previous parts of this question obtained some credit for this part. Diagrams in many cases were good. Those that were not were usually too small to be helpful and did not clearly show the centre of mass above the edge of the base.
 - (iv) This part of the question was found difficult by the majority of the candidates. Few set out a complete statement of how they intended to test if sliding would occur and then offered random calculations without any statement as to their relevance. Those who clearly stated the criteria they were going to use to test almost always obtained full credit.