OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** Use the Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$ to obtain the first three non-zero terms in the Maclaurin series for $\ln\left(\frac{1+x}{1-x}\right)$. State the range of validity of this series. **[4]**

(ii) Find the value of x for which $\frac{1+x}{1-x} = 3$. Hence find an approximation to $\ln 3$, giving your answer to three decimal places. **[4]**

(b) A curve has polar equation $r = \frac{a}{1+\sin\theta}$ for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant. The points on the curve have cartesian coordinates x and y.

(i) By plotting suitable points, or otherwise, sketch the curve. **[3]**

(ii) Show that, for this curve, $r + y = a$ and hence find the cartesian equation of the curve. **[5]**

2 **(i)** Obtain the characteristic equation for the matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} 3 & 1 & -2 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{pmatrix}.$$

Hence or otherwise obtain the value of $\det(\mathbf{M})$. **[3]**

(ii) Show that -1 is an eigenvalue of $\mathbf{M}$, and show that the other two eigenvalues are not real.

Find an eigenvector corresponding to the eigenvalue -1.

Hence or otherwise write down the solution to the following system of equations. **[9]**

$$\begin{aligned} 3x + y - 2z &= -0.1 \\ -y \quad\quad &= 0.6 \\ 2x \quad\quad + z &= 0.1 \end{aligned}$$

(iii) State the Cayley-Hamilton theorem and use it to show that

$$\mathbf{M}^3 = 3\mathbf{M}^2 - 3\mathbf{M} - 7\mathbf{I}.$$

Obtain an expression for $\mathbf{M}^{-1}$ in terms of $\mathbf{M}^2$, $\mathbf{M}$ and $\mathbf{I}$. **[4]**

(iv) Find the numerical values of the elements of $\mathbf{M}^{-1}$, showing your working. **[3]**

3 **(a)** **(i)** Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. **[1]**

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, justifying the sign of your answer by reference to your sketch. **[4]**

(ii) Find the exact value of the integral $\int_0^1 \frac{1}{\sqrt{2 - x^2}}\,\mathrm{d}x$. **[3]**

(b) The infinite series C and S are defined as follows.

$$C = \cos\theta + \tfrac{1}{3}\cos 3\theta + \tfrac{1}{9}\cos 5\theta + \ldots$$

$$S = \sin\theta + \tfrac{1}{3}\sin 3\theta + \tfrac{1}{9}\sin 5\theta + \ldots$$

By considering $C + \mathrm{j}S$, show that

$$C = \frac{3\cos\theta}{5 - 3\cos 2\theta},$$

and find a similar expression for S. **[11]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Prove, from definitions involving exponentials, that

$$\cosh 2u = 2\cosh^2 u - 1.$$ **[3]**

(ii) Prove that $\operatorname{arsinh} y = \ln\left(y + \sqrt{y^2 + 1}\right)$. **[4]**

(iii) Use the substitution $x = 2\sinh u$ to show that

$$\int \sqrt{x^2 + 4}\,\mathrm{d}x = 2\operatorname{arsinh}\tfrac{1}{2}x + \tfrac{1}{2}x\sqrt{x^2 + 4} + c,$$

where c is an arbitrary constant. **[6]**

(iv) By first expressing $t^2 + 2t + 5$ in completed square form, show that

$$\int_{-1}^{1} \sqrt{t^2 + 2t + 5}\,\mathrm{d}t = 2\left(\ln(1 + \sqrt{2}) + \sqrt{2}\right).$$ **[5]**

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 Fig. 5 shows a circle with centre C $(a, 0)$ and radius a. B is the point $(0, 1)$. The line BC intersects the circle at P and Q; P is above the x-axis and Q is below.

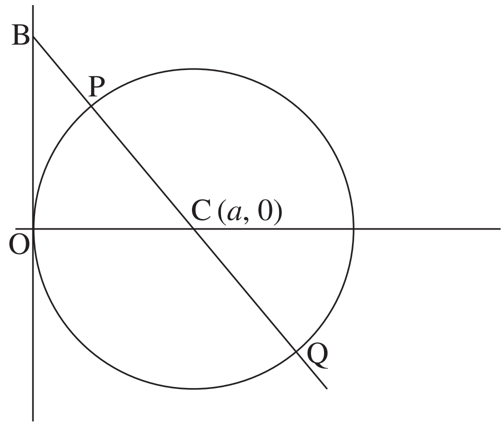

Fig. 5

(i) Show that, in the case $a = 1$, P has coordinates $\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Write down the coordinates of Q. **[3]**

(ii) Show that, for all positive values of a, the coordinates of P are

$$x = a\left(1 - \frac{a}{\sqrt{a^2 + 1}}\right), \qquad y = \frac{a}{\sqrt{a^2 + 1}}. \qquad (*)$$

Write down the coordinates of Q in a similar form. **[4]**

Now let the variable point P be defined by the parametric equations $(*)$ for all values of the parameter a, positive, zero and negative. Let Q be defined for all a by your answer in part **(ii)**.

(iii) Using your calculator, sketch the locus of P as a varies. State what happens to P as $a \to \infty$ and as $a \to -\infty$.

Show algebraically that this locus has an asymptote at $y = -1$.

On the same axes, sketch, as a dotted line, the locus of Q as a varies. **[8]**

(The single curve made up of these two loci and including the point B is called a *right strophoid*.)

(iv) State, with a reason, the size of the angle POQ in Fig. 5. What does this indicate about the angle at which a right strophoid crosses itself? **[3]**

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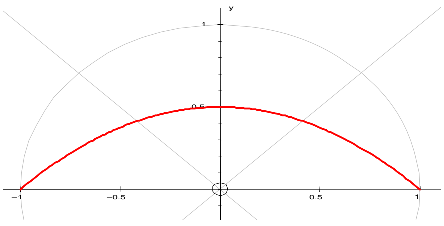
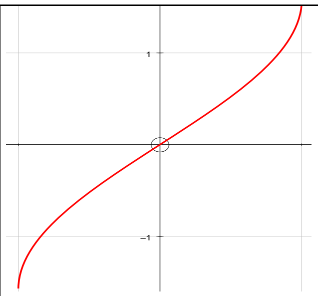
RECOGNISING ACHIEVEMENT

4756 (FP2) Further Methods for Advanced Mathematics

1 (a)(i)	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \ldots$ $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} \ldots$ $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$ $= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} \ldots$ Valid for $-1 < x < 1$	B1 M1 A1 B1 **4**	Series for $\ln(1-x)$ as far as x^5 s.o.i. Seeing series subtracted Inequalities must be strict
(ii)	$\frac{1+x}{1-x} = 3$ $\Rightarrow 1 + x = 3(1-x)$ $\Rightarrow 1 + x = 3 - 3x$ $\Rightarrow 4x = 2$ $\Rightarrow x = \frac{1}{2}$ $\ln 3 \approx 2\times\frac{1}{2} + \frac{2}{3}\times\left(\frac{1}{2}\right)^3 + \frac{2}{5}\times\left(\frac{1}{2}\right)^5$ $= 1 + \frac{1}{12} + \frac{1}{80}$ $= 1.096$ (3 d.p.)	M1 A1 M1 A1 **4**	Correct method of solution B2 for $x = ½$ stated Substituting their x into their series in (a) (i), even if outside range of validity. Series must have at least two terms *SR*: if >3 correct terms seen in (i), allow a better answer to 3 d.p. Must be 3 decimal places
(b)(i)		G1 G1 G1 **3**	$r(0) = a$, $r(\pi/2) = a/2$ indicated Symmetry in $\theta = \pi/2$ Correct basic shape: flat at $\theta = \pi/2$, not vertical or horizontal at ends, no dimple Ignore beyond $0 \le \theta \le \pi$
(ii)	$r + y = r + r\sin\theta$ $= r(1+\sin\theta) = \frac{a}{1+\sin\theta} \times (1+\sin\theta)$ $= a$ $\Rightarrow r = a - y$ $\Rightarrow x^2 + y^2 = (a-y)^2$ $\Rightarrow x^2 + y^2 = a^2 - 2ay + y^2$ $\Rightarrow 2ay = a^2 - x^2$ $\Rightarrow y = \frac{a^2 - x^2}{2a}$	M1 A1 (ag) M1 A1 A1 **5**	Using $y = r\sin\theta$ Using $r^2 = x^2 + y^2$ in $r + y = a$ Unsimplified A correct final answer, not spoiled **16**

2 (i)	$\mathbf{M}-\lambda\mathbf{I}=\begin{pmatrix}3-\lambda & 1 & -2\\ 0 & -1-\lambda & 0\\ 2 & 0 & 1-\lambda\end{pmatrix}$		
	det(**M** – λ**I**) = $(3-\lambda)[(-1-\lambda)(1-\lambda)] + 2[2(-1-\lambda)]$	M1	Attempt at det(**M** – λ**I**) with all elements present. Allow sign errors
	$= (3-\lambda)(\lambda^2-1) + 4(-1-\lambda)$	A1	Unsimplified. Allow signs reversed. Condone omission of = 0
	$\Rightarrow \lambda^3 - 3\lambda^2 + 3\lambda + 7 = 0$		
	det **M** = –7	B1	
		3	
(ii)	$f(\lambda) = \lambda^3 - 3\lambda^2 + 3\lambda + 7$		
	$f(-1) = -1 - 3 - 3 + 7 = 0 \Rightarrow -1$ eigenvalue	B1	Showing –1 satisfies a correct characteristic equation
	$f(\lambda) = (\lambda+1)(\lambda^2 - 4\lambda + 7)$	M1	Obtaining quadratic factor
	$\lambda^2 - 4\lambda + 7 = (\lambda-2)^2 + 3 \geq 3$ so no real roots	A1	www
	$(\mathbf{M}-\lambda\mathbf{I})\mathbf{s} = \mathbf{0}$, $\lambda = -1$		$(\mathbf{M}-\lambda\mathbf{I})\mathbf{s} = (\lambda)\mathbf{s}$ M0 below
	$\Rightarrow \begin{pmatrix}4 & 1 & -2\\ 0 & 0 & 0\\ 2 & 0 & 2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$		
	$\Rightarrow 4x + y - 2z = 0$ $2x + 2z = 0$	M1	Obtaining equations relating x, y and z
	$\Rightarrow x = -z$ $y = 2z - 4x = 2z + 4z = 6z$	M1	Obtaining equations relating two variables to a third. Dep. on first M1
	$\Rightarrow \mathbf{s} = \begin{pmatrix}-1\\ 6\\ 1\end{pmatrix}$	A1	Or any non-zero multiple
	$\begin{pmatrix}3 & 1 & -2\\ 0 & -1 & 0\\ 2 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}-0.1\\ 0.6\\ 0.1\end{pmatrix}$	M1	Solution by any method, e.g. use of multiple of **s**, but M0 if **s** itself quoted without further work
	$\Rightarrow x = 0.1$, $y = -0.6$, $z = -0.1$	A2	Give A1 if any two correct
		9	
(iii)	C-H: a matrix satisfies its own characteristic equation	B1	Idea of $\lambda \leftrightarrow$ **M**
	$\Rightarrow \mathbf{M}^3 - 3\mathbf{M}^2 + 3\mathbf{M} + 7\mathbf{I} = \mathbf{0}$		
	$\Rightarrow \mathbf{M}^3 = 3\mathbf{M}^2 - 3\mathbf{M} - 7\mathbf{I}$	B1 (ag)	Must be derived www. Condone omitted **I**
	$\Rightarrow \mathbf{M}^2 = 3\mathbf{M} - 3\mathbf{I} - 7\mathbf{M}^{-1}$	M1	Multiplying by $\mathbf{M}^{-1}$
	$\Rightarrow \mathbf{M}^{-1} = -\frac{1}{7}\mathbf{M}^2 + \frac{3}{7}\mathbf{M} - \frac{3}{7}\mathbf{I}$	A1	o.e.
		4	
(iv)	$\mathbf{M}^2 = \begin{pmatrix}3 & 1 & -2\\ 0 & -1 & 0\\ 2 & 0 & 1\end{pmatrix}\begin{pmatrix}3 & 1 & -2\\ 0 & -1 & 0\\ 2 & 0 & 1\end{pmatrix} = \begin{pmatrix}5 & 2 & -8\\ 0 & 1 & 0\\ 8 & 2 & -3\end{pmatrix}$	M1	Correct attempt to find $\mathbf{M}^2$
	$-\frac{1}{7}\begin{pmatrix}5 & 2 & -8\\ 0 & 1 & 0\\ 8 & 2 & -3\end{pmatrix} + \frac{3}{7}\begin{pmatrix}3 & 1 & -2\\ 0 & -1 & 0\\ 2 & 0 & 1\end{pmatrix} - \frac{3}{7}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$	M1	Using their (iii)
	$= \begin{pmatrix}\frac{1}{7} & \frac{1}{7} & \frac{2}{7}\\ 0 & -1 & 0\\ -\frac{2}{7} & -\frac{2}{7} & \frac{3}{7}\end{pmatrix}$ or $\frac{1}{7}\begin{pmatrix}1 & 1 & 2\\ 0 & -7 & 0\\ -2 & -2 & 3\end{pmatrix}$	A1	SC1 for answer without working

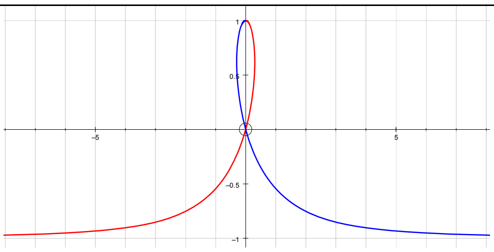
	OR Matrix of cofactors: $\begin{pmatrix} -1 & 0 & 2 \\ -1 & 7 & 2 \\ -2 & 0 & -3 \end{pmatrix}$	M1		Finding at least four cofactors
	Adjugate matrix $\begin{pmatrix} -1 & -1 & -2 \\ 0 & 7 & 0 \\ 2 & 2 & -3 \end{pmatrix}$: det **M** = –7	M1	**3**	Transposing and dividing by determinant. Dep. on M1 above **19**

3(a)(i)		G1	1	Correct basic shape (positive gradient, through (0, 0))
	$y = \arcsin x \Rightarrow \sin y = x$	M1		$\sin y =$ and attempt to diff. both sides
	$\Rightarrow \frac{dx}{dy} = \cos y$	A1		Or $\cos y \frac{dy}{dx} = 1$
	$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$	A1		www. SC1 if quoted without working
	Positive square root because gradient positive	B1	4	Dep. on graph of an increasing function
(ii)	$\int_0^1 \frac{1}{\sqrt{2-x^2}}dx = \left[\arcsin\frac{x}{\sqrt{2}}\right]_0^1$	M1		arcsin function alone, or any sine substitution
		A1		$\frac{x}{\sqrt{2}}$, or $\int 1\,d\theta$ www without limits
	$= \frac{\pi}{4}$	A1	3	Evaluated in terms of π
(b)	$C + jS = e^{j\theta} + \frac{1}{3}e^{3j\theta} + \frac{1}{9}e^{5j\theta} + \ldots$	M1		Forming $C + jS$ as a series of powers
	This is a geometric series	M1		Identifying geometric series and attempting sum to infinity or to n terms
	with first term $a = e^{j\theta}$, common ratio $r = \frac{1}{3}e^{2j\theta}$	A1		Correct a and r
	Sum to infinity $= \frac{a}{1-r} = \frac{e^{j\theta}}{1-\frac{1}{3}e^{2j\theta}}$ (= $\frac{3e^{j\theta}}{3-e^{2j\theta}}$)	A1		Sum to infinity
	$= \frac{3e^{j\theta}}{3-e^{2j\theta}} \times \frac{3-e^{-2j\theta}}{3-e^{-2j\theta}}$	M1*		Multiplying numerator and denominator by $1-\frac{1}{3}e^{-2j\theta}$ o.e. Or writing in terms of trig functions and realising the denominator
	$= \frac{9e^{j\theta} - 3e^{-j\theta}}{9 - 3e^{-2j\theta} - 3e^{2j\theta} + 1}$	M1		Multiplying out numerator and denominator. Dep. on M1*
	$= \frac{9(\cos\theta + j\sin\theta) - 3(\cos\theta - j\sin\theta)}{10 - 3(\cos 2\theta - j\sin 2\theta) - 3(\cos 2\theta + j\sin 2\theta)}$	M1		Valid attempt to express in terms of trig functions. If trig functions used from start, M1 for using the compound angle formulae and Pythagoras Dep. on M1*
	$= \frac{6\cos\theta + 12j\sin\theta}{10 - 6\cos 2\theta}$	A1		
	$\Rightarrow C = \frac{6\cos\theta}{10 - 6\cos 2\theta}$	M1		Equating real and imaginary parts. Dep. on M1*

	$= \dfrac{3\cos\theta}{5-3\cos 2\theta}$	A1 (ag)	
	$S = \dfrac{6\sin\theta}{5-3\cos 2\theta}$	A1	o.e.
		11	**19**

4 (i)	cosh $u = \frac{e^u+e^{-u}}{2}$ $\Rightarrow$ 2 cosh$^2u = \frac{e^{2u}+2+e^{-2u}}{2}$ $\Rightarrow$ 2 cosh^2u – 1 = $\frac{e^{2u}+e^{-2u}}{2}$ = cosh 2u	 B1 B1 B1 (ag) **3**	 $\left(e^u+e^{-u}\right)^2=e^{2u}+2+e^{-2u}$ cosh 2u = $\frac{e^{2u}+e^{-2u}}{2}$ Completion www
(ii)	x = arsinh y $\Rightarrow$ sinh $x = y$ $\Rightarrow$ $y = \frac{e^x-e^{-x}}{2}$ $\Rightarrow$ $e^{2x}-2ye^x-1=0$ $\Rightarrow$ $(e^x-y)^2-y^2-1=0$ $\Rightarrow$ $(e^x-y)^2=y^2+1$ $\Rightarrow$ $e^x-y=\pm\sqrt{y^2+1}$ $\Rightarrow$ $e^x=y\pm\sqrt{y^2+1}$ Take + because $e^x > 0$ $\Rightarrow$ $x=\ln(y+\sqrt{y^2+1})$	 M1 M1 B1 A1 (ag) **4**	 Expressing y in exponential form ($\frac{1}{2}$, – must be correct) Reaching e^x by quadratic formula or completing the square. Condone no ± Or argument of ln must be positive Completion www but independent of B1
(iii)	$x = 2\sinh u \Rightarrow \frac{dx}{du} = 2\cosh u$ $\int\sqrt{x^2+4}\,dx=\int\sqrt{4\sinh^2u+4}\times2\cosh u\,du$ $=\int 4\cosh^2u\,du$ $=\int 2\cosh 2u+2\,du$ = sinh 2u + 2u + c = 2 sinh u cosh u + 2u + c $= x\sqrt{1+\frac{x^2}{4}}$ + 2 arsinh $\frac{x}{2}$ + c $=\frac{1}{2}x\sqrt{4+x^2}$ + 2 arsinh $\frac{x}{2}$ + c	M1 A1 M1 A1 M1 A1 (ag) **6**	$\frac{dx}{du}$ and substituting for all elements Substituting for all elements correctly Simplifying to an integrable form Any form, e.g. $\frac{1}{2}e^{2u}-\frac{1}{2}e^{-2u}+2u$ Condone omission of + c throughout Using double "angle" formula and attempt to express cosh u in terms of x Completion www
(iv)	$t^2+2t+5=(t+1)^2+4$ $\int_{-1}^{1}\sqrt{t^2+2t+5}\,dt=\int_{-1}^{1}\sqrt{(t+1)^2+4}\,dt$ $=\int_0^2\sqrt{x^2+4}\,dx$ $=\left[\frac{1}{2}x\sqrt{4+x^2}+2\text{arsinh}\frac{x}{2}\right]_0^2$	B1 M1 A1	Completing the square Simplifying to an integrable form, by substituting $x = t + 1$ s.o.i. or complete alternative method Correct limits consistent with their method seen anywhere

	$= \sqrt{8} + 2$ arsinh 1 $= 2\sqrt{2} + 2\ln(1 + \sqrt{2})$ $= 2(\ln(1 + \sqrt{2}) + \sqrt{2})$	M1 A1 (ag) **5**	Using (iii) or otherwise reaching the result of integration, and using limits Completion www. Condone $\sqrt{8}$ etc. **18**

5 (i)	If $a = 1$, angle OCP = 45° so P is (1 – cos 45°, sin 45°) $\Rightarrow$ P $\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$	M1 A1 (ag)	Completion www
	OR Circle $(x-1)^2 + y^2 = 1$, line $y = -x + 1$ $(x-1)^2 + (-x+1)^2 = 1$ M1 $\Rightarrow$ $x = 1 \pm \frac{1}{\sqrt{2}}$ and hence P A1		Complete algebraic method to find x
	Q $\left(1 + \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	B1 **3**	
(ii)	cos OCP = $\frac{a}{\sqrt{a^2+1}}$ sin OCP = $\frac{1}{\sqrt{a^2+1}}$ P is ($a - a$ cos OCP, a sin OCP) $\Rightarrow$ P $\left(a - \frac{a^2}{\sqrt{a^2+1}}, \frac{a}{\sqrt{a^2+1}}\right)$	M1 A1 A1 (ag)	Attempt to find cos OCP and sin OCP in terms of a Both correct Completion www
	OR Circle $(x-a)^2 + y^2 = a^2$, line $y = -\frac{1}{a}x + 1$ $(x-a)^2 + \left(-\frac{1}{a}x+1\right)^2 = a^2$ M1 $\Rightarrow$ $x = \dfrac{2a + \frac{2}{a} \pm \sqrt{\left(2a+\frac{2}{a}\right)^2 - 4\left(1+\frac{1}{a^2}\right)}}{2\left(1+\frac{1}{a^2}\right)}$ A1 $\Rightarrow$ $x = a \pm \frac{a^2}{\sqrt{a^2+1}}$ and hence P A1		Complete algebraic method to find x Unsimplified
	Q $\left(a + \frac{a^2}{\sqrt{a^2+1}}, -\frac{a}{\sqrt{a^2+1}}\right)$	B1 **4**	
(iii)	 As $a \to \infty$, P $\to$ (0, 1) As $a \to -\infty$, y co-ordinate of P $\to -1$ $\frac{a}{\sqrt{a^2+1}} \to \frac{a}{-a} = -1$ as $a \to -\infty$	G1 G1 G1 G1ft B1 B1 M1 A1 **8**	Locus of P (1st & 3rd quadrants) through (0, 0) Locus of P terminates at (0, 1) Locus of P: fully correct shape Locus of Q (2nd & 4th quadrants: dotted) reflection of locus of P in y-axis Stated separately Stated Attempt to consider y as $a \to -\infty$ Completion www
(iv)	POQ = 90° Angle in semicircle Loci cross at 90°	B1 B1 B1 **3**	o.e. **18**

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

This paper included some challenging tests for the most able, while weaker candidates were offered plenty of part-questions which should have been familiar and reasonably straightforward, and indeed there were very few very low marks: the vast majority of candidates were able to show that they knew and understood topics from across the specification. Unfortunately weaker (and many stronger) candidates struggled to sustain accuracy through the paper; their “basic” algebra was often suspect, and many believed that
$\sqrt{x^2+y^2}+y=a \Rightarrow x^2+y^2+y^2=a^2$ or even $\sqrt{x^2+y^2}+y=a \Rightarrow x+y+y=a$ (Q1(b)(ii))
or $\sqrt{t^2+2t+5}=\sqrt{(t+1)^2+4}=(t+1)+2=t+3$ (Q4(iv)), which was rather worrying.

The presentation of scripts was usually good, although once again there were some candidates who split up questions and even part-questions, which does not help them or the examiner. Some candidates used three 16-page answer books. It is expected that candidates sketch graphs on the lined paper: there is no need to use separate pieces of graph paper. There was very little evidence of any time trouble.

In question 3, candidates were asked to “find” $\frac{dy}{dx}$ for y = arcsin x. Many candidates just quoted the result from the formula book without deriving it. In future no credit will be given in such cases: if candidates are asked to “find” a result which appears in the formula book, it is clear that a derivation is expected. When candidates did attempt to derive other given results, their answers often lacked the necessary detail: each step should be shown. For example, when the result of a definite integral is given, we would like to see the explicit substitution of limits.

In Section B, the overwhelming majority of candidates chose the hyperbolic functions question: although the Investigations of Curves question did produce some creditable responses this time, the majority of attempts at it were fragmentary.

Comments on Individual Questions

1) (Maclaurin series, polar curves)
The mean mark on this question was about 10 (out of 16).

(a) Most candidates approached part (i) in the way intended, quoting the series for ln(1 + x) from the formula book, replacing x by $-x$, and subtracting. Some gave only two terms. Quite a few wasted time by deriving the series for ln(1 – x) by differentiation, and several even derived the series for ln(1 + x); fortunately only a very few tried to differentiate $\ln\left(\frac{1+x}{1-x}\right)$. The range of validity was badly done, with many quoting non-strict inequalities or forgetting to answer at all.

The equation in (ii) was very often, but not always, solved correctly, with some believing that $4x = 2 \Rightarrow x = 2$. Most then went on to substitute their x into their series from (i), although a few just used their calculators to find ln 3. The instruction to give the answer to three decimal places was usually followed.

(b) Most candidates were able to produce a good sketch; where marks were lost it was usually because they failed to show that $r(0) = a$ and $r(\pi/2) = a/2$ (these sometimes appeared without the a) or drew a graph with a "dimple" in the top. Then most were able to show that $r + y = a$ by using $y = r \sin \theta$, although some just checked the given statement for one or two points. A significant number of candidates found it difficult to obtain a Cartesian equation for the curve: the formula $r^2 = x^2 + y^2$ was often quoted, but not always used. Of candidates who could produce a correct equation, a substantial number spoiled their answer by poor algebra.

2) (Matrices)
This was the best-answered question: the mean mark was about 14 (out of 19).

(i) Most candidates were able to produce a correct characteristic polynomial, although an equation appeared more rarely. Most expanded by the top row, although expansion by the middle row would have produced an answer more easily. The determinant of **M** was not so well done: those who worked it out from scratch were generally more successful than those who used the characteristic equation, because most reversed all their signs to obtain a leading coefficient of 1: thus $\lambda = 0$ gave –det(**M**). 1 was another common incorrect answer.

(ii) This part of the question had three "sub-parts" and some candidates did not answer all of them. Most candidates successfully showed that –1 was an eigenvalue and that the other two eigenvalues were not real. Then a substantial number missed out the part requiring the eigenvector to be found; of those who tried this, most knew the procedure, but there were many algebraic and arithmetical slips. Most candidates correctly solved the simultaneous equations, with the most common method of solution being elimination: a few found and used the inverse matrix here. Not many followed the instruction to "write down".

(iii) Only a minority of candidates stated the Cayley-Hamilton theorem correctly in words: if the intention to replace λ in the characteristic equation by **M** was clear, the mark was awarded. Candidates sometimes just wrote down the result that they were required to show, but most knew how to use this result to introduce $\mathbf{M}^{-1}$, although there were many sign errors.

(iv) Most candidates failed to produce a completely correct inverse. There was a roughly equal split between those who tried to use their expression in (iii) and those who found cofactors, transposed and divided by their determinant. There were a great many arithmetical errors with both methods.

3) (Calculus with inverse trigonometrical functions, complex numbers)
The mean mark for this question was about 11 (out of 19). Part(a) was done much better than part (b).

(a) The sketch in part (i) was mostly correct although there were a few candidates who reflected the whole sine graph in $y = x$, thus not producing the graph of a function. As mentioned above, $\frac{dy}{dx}$ was often quoted without proof. Candidates often omitted the explanation of the sign of their answer. The integral in part (ii) was very well done although some introduced an extra factor of $\frac{1}{\sqrt{2}}$.

(b) Most candidates were able to recognise $C + jS$ as an infinite geometric series and sum it correctly to gain the first 4 marks. It was then necessary to "realise the denominator" by multiplying numerator and denominator by an appropriate expression, and only the better candidates could do this. Those who worked with exponential forms for as long as possible generally made fewer mistakes and more progress than those who attempted to introduce trigonometry at an early stage.

4) (Hyperbolic functions)
Although this was by far the more popular question in Section B, and each part of the question allowed candidates who had not succeeded in the other parts to attempt it, the mean mark was less than 10 (out of 18).

(i) This part was usually answered well, although weaker candidates did not always "prove from definitions involving exponentials".

(ii) This part was also done well, although some candidates failed to get beyond an exponential expression for *y* or confused themselves by poor choice of variables. Only the best candidates could comment correctly on why the minus sign should be rejected.

(iii) Many candidates could make the hyperbolic substitution and perform the integration correctly, although the weakest only substituted for $\sqrt{x^2+4}$ and not for "*dx*", or mixed up $\frac{dx}{du}$ and $\frac{du}{dx}$. The last two marks, for obtaining the printed answer, were rarely awarded: again, "proof by blatant assertion" was commonly employed.

(iv) Completing the square was by no means universally remembered and such things as $t^2+2t+5=\left(t^2+\sqrt{2}t\right)^2$ were seen. Most successful solutions used $x = t + 1$ and the result in (iii), although some candidates went back to the beginning and substituted $t = 2\sinh x - 1$. Often (iii) was not used at all, with a "result" such as $\text{arsinh}\frac{t+1}{2}$ appearing. Lack of necessary detail, such as explicit substitution of limits, in deriving the given answer prevented very many from scoring full marks here.

5) (Investigations of Curves)
Few attempts at this question were seen. Most were fragmentary, but a small number of candidates gained a substantial number of marks.

(i),(ii) These parts could be approached via algebra or trigonometry. The trigonometric approach was far easier, yet there were creditable attempts using both methods.

(iii),(iv) The sketch, where attempted, was done correctly. Those who could produce the sketch went on to gain most of the rest of the marks.