OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

4754A

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Turn over

Section A (36 marks)

1 **(i)** Use the trapezium rule with four strips to estimate $\int_{-2}^{2} \sqrt{1 + e^x}\, dx$, showing your working. **[4]**

Fig. 1 shows a sketch of $y = \sqrt{1 + e^x}$.

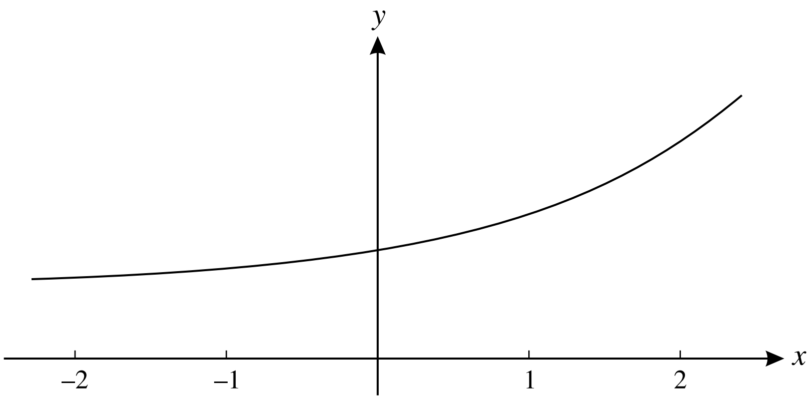

Fig. 1

(ii) Suppose that the trapezium rule is used with more strips than in part **(i)** to estimate $\int_{-2}^{2} \sqrt{1 + e^x}\, dx$. State, with a reason but no further calculation, whether this would give a larger or smaller estimate. **[2]**

2 A curve is defined parametrically by the equations

$$x = \frac{1}{1+t}, \qquad y = \frac{1-t}{1+2t}.$$

Find t in terms of x. Hence find the cartesian equation of the curve, giving your answer as simply as possible. **[5]**

3 Find the first three terms in the binomial expansion of $\frac{1}{(3-2x)^3}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[7]**

4 The points A, B and C have coordinates (2, 0, −1), (4, 3, −6) and (9, 3, −4) respectively.

(i) Show that AB is perpendicular to BC. **[4]**

(ii) Find the area of triangle ABC. **[3]**

5 Show that $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$. **[3]**

6 **(i)** Find the point of intersection of the line $\mathbf{r} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ and the plane $2x - 3y + z = 11$. **[4]**

(ii) Find the acute angle between the line and the normal to the plane. **[4]**

Section B (36 marks)

7 A particle is moving vertically downwards in a liquid. Initially its velocity is zero, and after t seconds it is $v\,\mathrm{m\,s^{-1}}$. Its terminal (long-term) velocity is $5\,\mathrm{m\,s^{-1}}$.

A model of the particle's motion is proposed. In this model, $v = 5(1 - \mathrm{e}^{-2t})$.

(i) Show that this equation is consistent with the initial and terminal velocities. Calculate the velocity after 0.5 seconds as given by this model. **[3]**

(ii) Verify that v satisfies the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 2v$. **[3]**

In a second model, v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 0.4v^2.$$

As before, when $t = 0$, $v = 0$.

(iii) Show that this differential equation may be written as

$$\frac{10}{(5-v)(5+v)}\frac{\mathrm{d}v}{\mathrm{d}t} = 4.$$

Using partial fractions, solve this differential equation to show that

$$t = \tfrac{1}{4}\ln\left(\frac{5+v}{5-v}\right).$$ **[8]**

This can be re-arranged to give $v = \frac{5(1 - \mathrm{e}^{-4t})}{1 + \mathrm{e}^{-4t}}$. [You are **not** required to show this result.]

(iv) Verify that this model also gives a terminal velocity of $5\,\mathrm{m\,s^{-1}}$.

Calculate the velocity after 0.5 seconds as given by this model. **[3]**

The velocity of the particle after 0.5 seconds is measured as $3\,\mathrm{m\,s^{-1}}$.

(v) Which of the two models fits the data better? **[1]**

8 Fig. 8 shows a searchlight, mounted at a point A, 5 metres above level ground. Its beam is in the shape of a cone with axis AC, where C is on the ground. AC is angled at α to the vertical. The beam produces an oval-shaped area of light on the ground, of length DE. The width of the oval at C is GF. Angles DAC, EAC, FAC and GAC are all β.

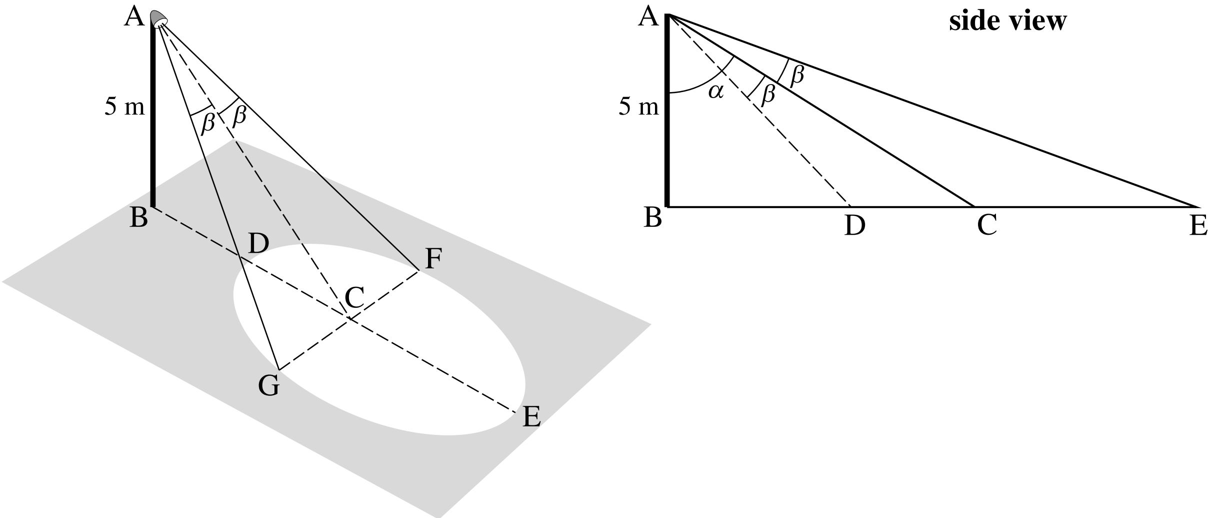

Fig. 8

In the following, all lengths are in metres.

(i) Find AC in terms of α, and hence show that GF = $10\sec\alpha\tan\beta$. **[3]**

(ii) Show that CE = $5\big(\tan(\alpha+\beta) - \tan\alpha\big)$.

Hence show that CE = $\dfrac{5\tan\beta\sec^2\alpha}{1-\tan\alpha\tan\beta}$. **[5]**

Similarly, it can be shown that CD = $\dfrac{5\tan\beta\sec^2\alpha}{1+\tan\alpha\tan\beta}$. [You are **not** required to derive this result.]

You are now given that $\alpha = 45^\circ$ and that $\tan\beta = t$.

(iii) Find CE and CD in terms of t. Hence show that DE = $\dfrac{20t}{1-t^2}$. **[5]**

(iv) Show that GF = $10\sqrt{2}t$. **[2]**

For a certain value of β, DE = 2GF.

(v) Show that $t^2 = 1 - \dfrac{1}{\sqrt{2}}$.

Hence find this value of β. **[3]**

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ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the question paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Friday 14 January 2011
Afternoon

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES
- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- This paper should be attached to the candidate's paper A script before sending to the examiner.

Examiner's Use Only:	
1	
2	
3	
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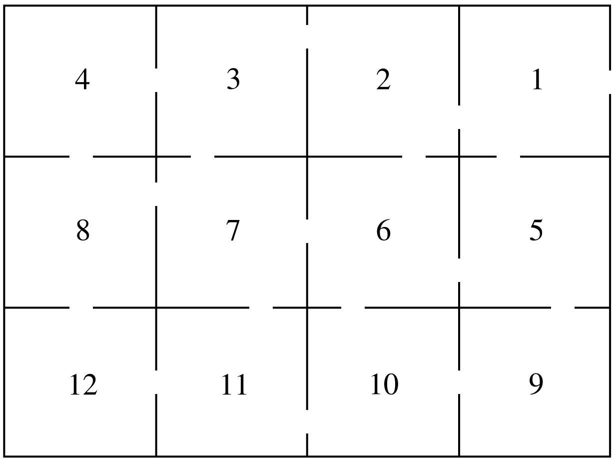
6	
Total	

Turn over

For Examiner's Use

1 The gallery shown below is a 3 by 4 grid of rectangular rooms.



(i) Mark on the diagram the positions of six guards so that the whole gallery can be observed. **[1]**

(ii) Give a counterexample to disprove the following proposition:

For an m by n grid of rectangular rooms, $\left\lfloor \frac{mn}{2} \right\rfloor$ guards are required. **[2]**

For Examiner's Use

2 **(i)** Show that $\frac{(2r+1)-(-1)^r}{4} = \left\lfloor \frac{r+1}{2} \right\rfloor$ in the case where $r = 4$. **[2]**

....

....

....

(ii) The ceiling function, $\lceil x \rceil$, is defined as the smallest integer greater than or equal to x.

Complete the following table. **[2]**

x	1	2	3	4	5
$\left\lceil \frac{x}{2} \right\rceil$					

3 Justify the statement in lines 79 and 80. **[2]**

....

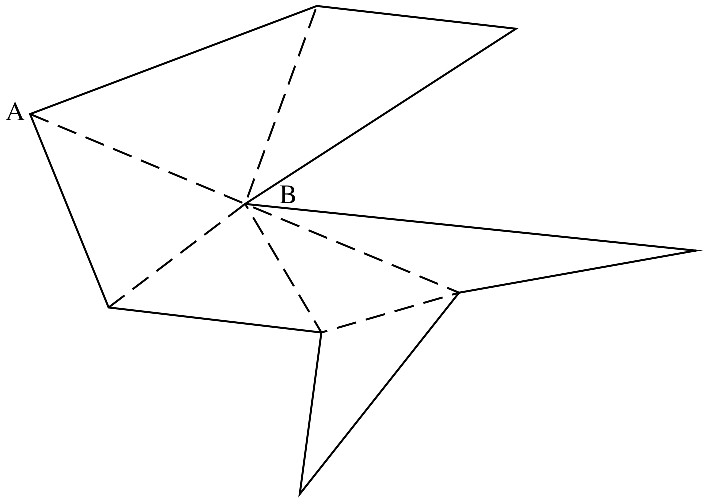
....

....

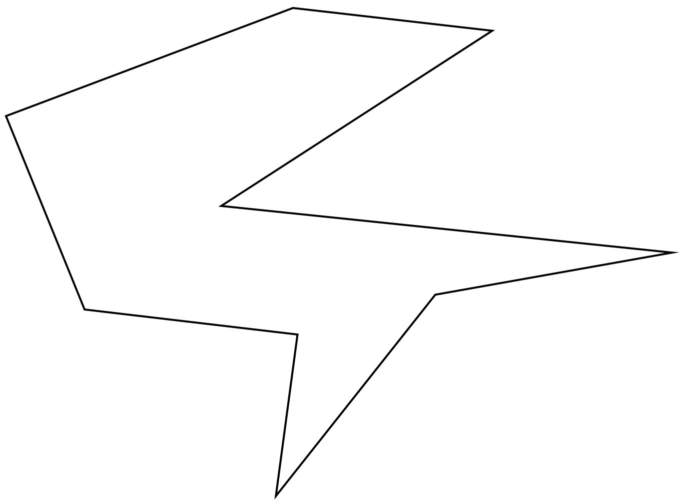
....

For Examiner's Use

4 **(i)** Following the procedure in Fig. 6, complete the labelling of the polygon shown below. **[2]**

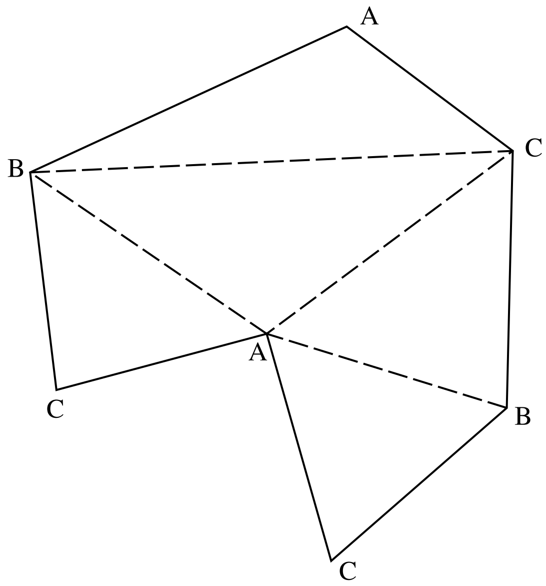


(ii) In order to use the minimum number of cameras, show on the diagram below where your answer to part **(i)** indicates the cameras should be placed. **[1]**



For Examiner's Use

5 With reference to the labelled triangulation shown below, state with a reason whether each of the following statements is true or false.



(i) The triangulation shows that 2 cameras are sufficient. **[2]**

....

Reason: ....

....

(ii) The triangulation shows that 2 cameras are necessary. **[2]**

....

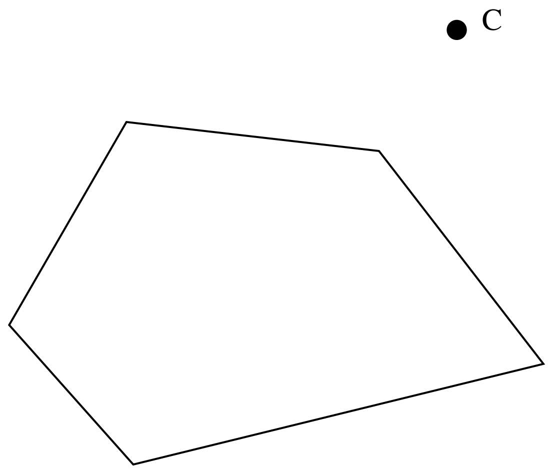
Reason: ....

....

For Examiner's Use

6 On lines 96 and 97 it says "If the cameras did not need to be mounted on the walls, but could be positioned further away from the building, then fewer cameras would usually suffice."

Using the diagram below, which shows a pentagon and one external camera, C, indicate by shading the region in which a second camera must be positioned so that all the walls could be observed by the two cameras. **[2]**



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ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Friday 14 January 2011
Afternoon

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

- This insert contains the text for use with the questions.
- This document consists of **12** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this insert for marking; it should be retained in the centre or destroyed.

The Art Gallery Problem

Introduction

Closed-circuit television (CCTV) is widely used to monitor activities in public areas. Usually a CCTV camera is fixed to a wall, either on the inside or the outside of a building, in such a way that it can rotate to survey a wide region.

Art galleries need to take surveillance very seriously. Many galleries use a combination of guards, who can move between rooms, and fixed cameras.

This article addresses the problem of how to ensure that all points in a gallery can be observed, using the minimum number of guards or cameras. Two typical layouts will be considered: a standard layout consisting of a chain of rectangular rooms with one route through them; and a polygonal, open-plan gallery.

Standard layout

Fig. 1 shows the plan view of an art gallery. It contains six rectangular rooms in a chain.

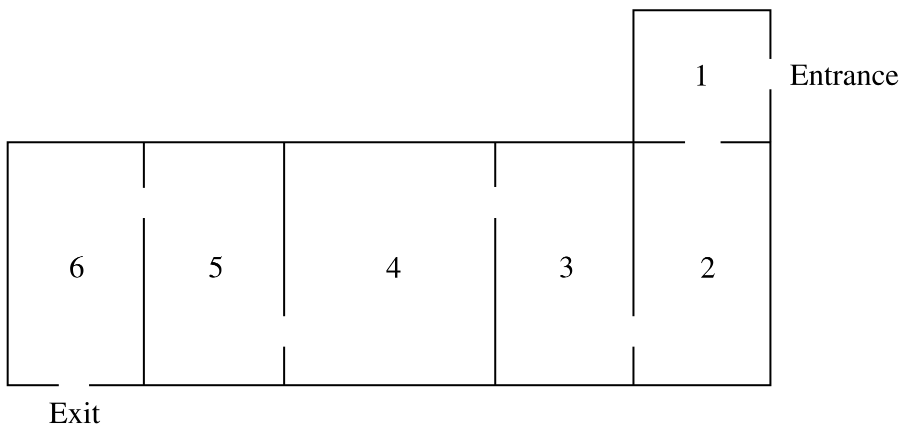


Fig. 1

Imagine you want to employ the minimum number of guards so that every point in the gallery can be observed by at least one of them. How many guards are needed and where would you position them?

It turns out that 3 guards are needed; they should be positioned in the doorways between rooms 1 and 2, rooms 3 and 4 and rooms 5 and 6.

Table 2 shows the number of guards, G, needed for different numbers of rectangular rooms, r, arranged in a chain.

Number of rooms, r	1	2	3	4	5	6	7	8	9	10
Number of guards, G	1	1	2	2	3	3	4	4	5	5

Table 2

One formula which expresses G in terms of r is

$$G = \frac{(2r + 1) - (-1)^r}{4}.$$

This can be expressed more concisely using the ‘floor function’, denoted by $\lfloor x \rfloor$. This is defined as the greatest integer less than or equal to x. For example, $\lfloor 3.9 \rfloor = 3$ and $\lfloor 5 \rfloor = 5$.

Using this notation, the formula which expresses G in terms of r is

$$G = \left\lfloor \frac{r + 1}{2} \right\rfloor.$$

It has been assumed that the thickness of the walls does not obscure the view of a guard positioned in a doorway. In reality the guard would need to take a step into a room to ensure that he or she can see into all corners. In this sense, guards have the advantage over fixed cameras as the thickness of the walls between rooms would result in the view of a camera positioned in a doorway being partially blocked.

The remainder of this article is concerned with positioning fixed cameras in galleries which are not divided into separate rooms.

Open-plan gallery

An open-plan gallery has no interior walls.

Figs 3a, 3b, 3c and 3d show the plan views of four different open-plan galleries. Each one is made up of 12 straight exterior walls with no interior walls.

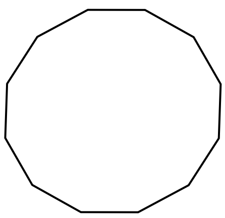


Fig. 3a

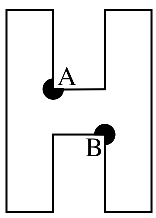


Fig. 3b

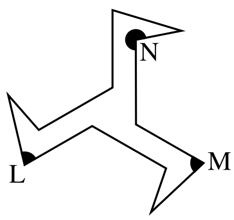


Fig. 3c

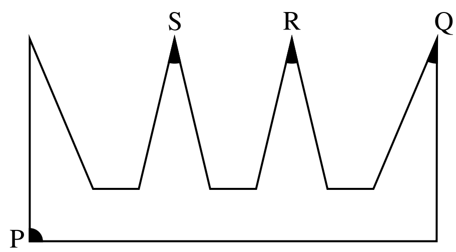


Fig. 3d

Fig. 3a is a gallery in the shape of a convex dodecagon. A single camera mounted at any point on any wall would be able to observe the entire gallery.

In order to ensure that all points can be observed, it turns out that the galleries shown in Figs 3b, 3c and 3d require 2, 3 and 4 cameras respectively. Possible positions for the cameras are shown. The cameras do not have to be positioned in corners but corners are often convenient locations for them.

An interesting question is whether there could be an open-plan gallery with 12 straight walls that requires more than 4 cameras.

A procedure called *triangulation* helps to answer this question and to determine how many cameras may be required for open-plan galleries with any number of walls. This will be illustrated for a gallery with 11 walls.

Triangulation

Fig. 4 shows a polygon with 11 edges.

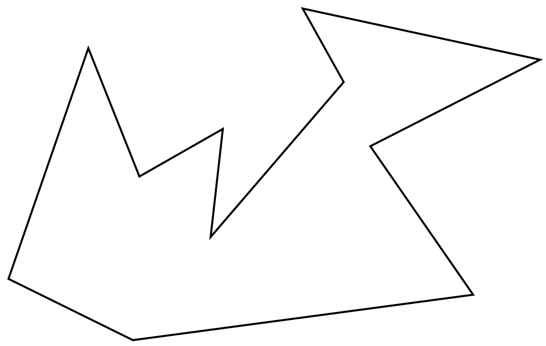


Fig. 4

It can be proved that every polygon can be split into triangles without creating any new vertices; this procedure is called triangulation. Figs 5a and 5b show two ways of triangulating the polygon shown in Fig. 4.

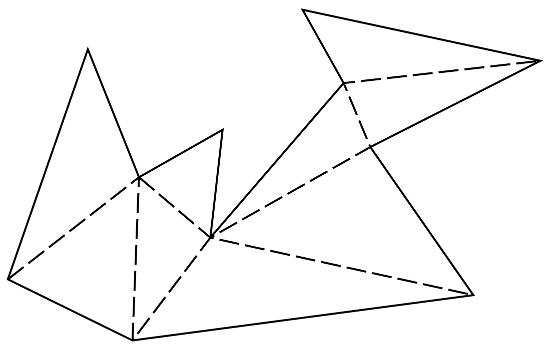


Fig. 5a

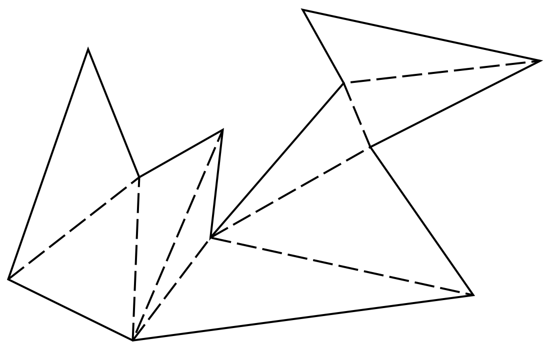


Fig. 5b

There is a two-stage process to decide on possible positions of the cameras in an open-plan gallery. The first stage is to add new internal edges to triangulate the polygon which represents the gallery. Then each vertex is labelled either A, B or C using the procedure shown in Fig. 6.

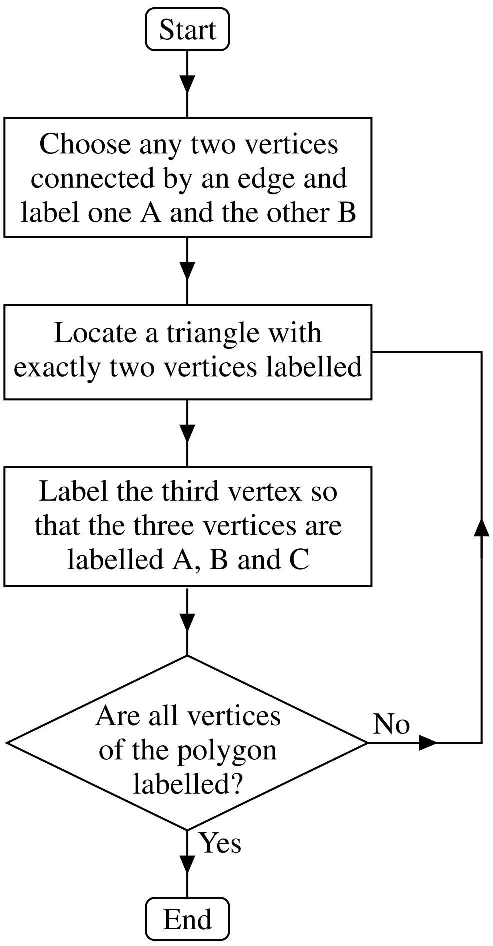


Fig. 6

This procedure is illustrated using the polygon in Fig. 4; for ease of reference the polygon has been reproduced in Fig. 7 with the vertices numbered 1 to 11.

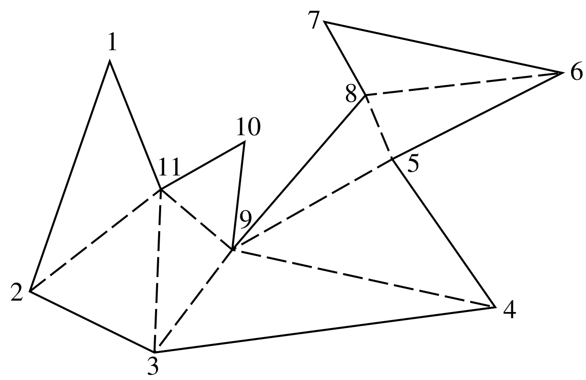


Fig. 7

Fig. 8 shows the result after choosing vertices 1 and 2 and labelling them A and B respectively.

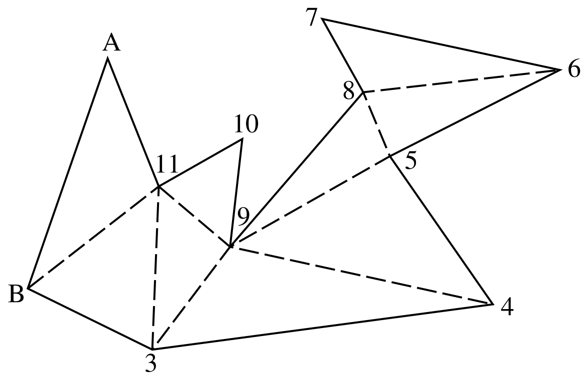


Fig. 8

Vertex 11 is now assigned the label C so that this triangle (shaded in Fig. 9) has vertices labelled A, B and C.

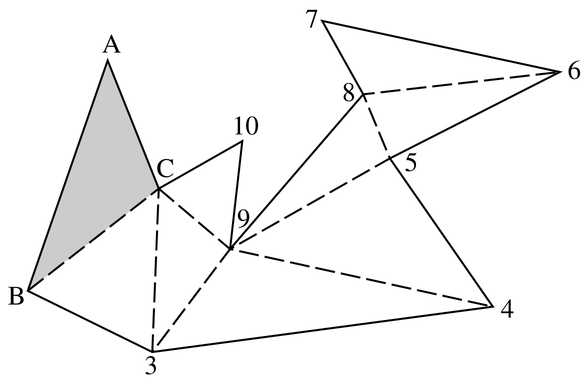


Fig. 9

The next vertex to be labelled is vertex 3; this is assigned the label A. The remaining vertices are labelled in the following order.

Vertex 9 is labelled B;
Vertex 10 is labelled A;
Vertex 4 is labelled C;
Vertex 5 is labelled A;
Vertex 8 is labelled C;
Vertex 6 is labelled B;
Vertex 7 is labelled A.

The resulting labelling is shown in Fig. 10.

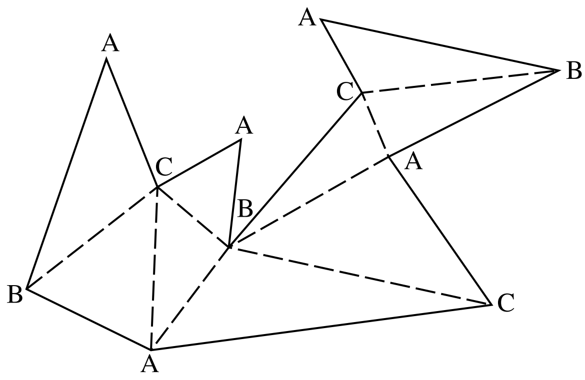


Fig. 10

The 11 vertices have all been assigned a label A, B or C; the numbers of vertices with each label is given in Table 11.

Label	Number of vertices assigned this label
A	5
B	3
C	3

Table 11

Since every triangle contains a vertex labelled A, positioning a camera at each vertex A will ensure that the whole gallery can be observed. This would require 5 cameras.

Alternatively, positioning cameras at the 3 vertices labelled B would ensure that the whole gallery can be observed using only 3 cameras.

Positioning cameras at the 3 vertices labelled C would also be sufficient.

Since $\frac{11}{3} < 4$, in any 11-sided polygon at least one of A, B or C must appear as a label at most 3 times.

A generalisation of this argument demonstrates that an open-plan gallery with n walls can be covered with $\left\lfloor \frac{n}{3} \right\rfloor$ cameras or fewer. Table 12 shows this information.

n	3	4	5	6	7	8	9	10	11	12	13	14	15
$\left\lfloor \frac{n}{3} \right\rfloor$	1	1	1	2	2	2	3	3	3	4	4	4	5

Table 12

It is always possible to design an open-plan gallery with n walls that requires $\left\lfloor \frac{n}{3} \right\rfloor$ cameras.

Triangulation in practice

For a given open-plan gallery, different triangulations may suggest different numbers of cameras.

Figs 13a and 13b show two different triangulations on a particular hexagon. Fig. 13a shows that the whole gallery can be observed using 2 cameras (as in Table 12). However, Fig. 13b shows that only one camera is necessary.

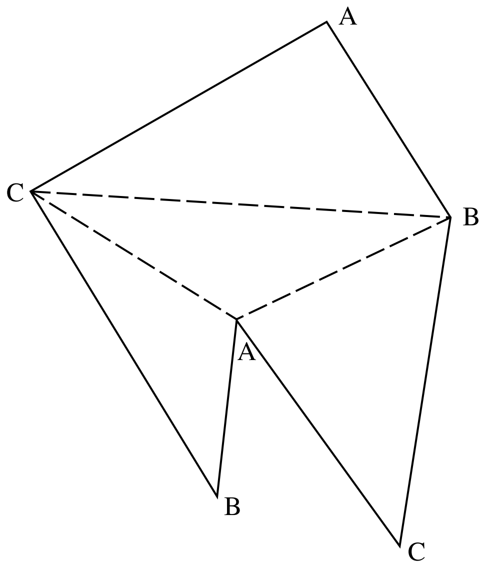


Fig. 13a

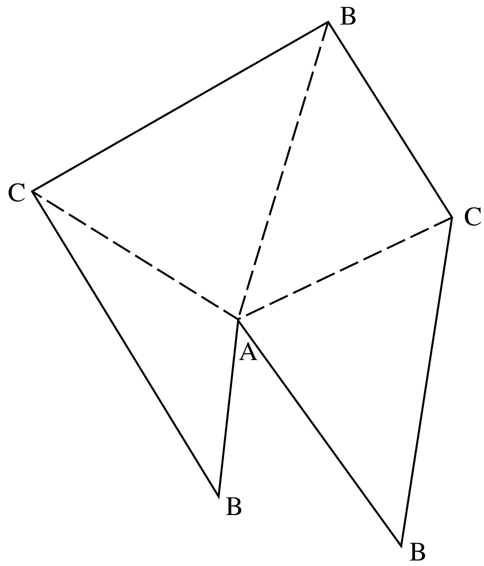


Fig. 13b

Surveillance of the outside of a building

Fig. 14 shows a pentagonal building with corners numbered, in order, from 1 to 5. To observe all of the outside of this building, 3 cameras could be positioned at the odd-numbered corners as shown.

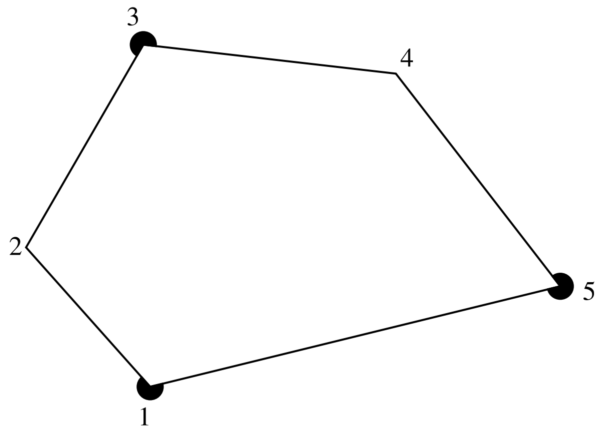


Fig. 14

This method of positioning cameras at odd-numbered corners can be extended to a polygonal building with any number of walls, showing that $\left\lfloor \frac{n+1}{2} \right\rfloor$ cameras are sufficient to observe all the outside of any n-sided polygonal building.

If the cameras did not need to be mounted on the walls, but could be positioned further away from the building, then fewer cameras would usually suffice.

Conclusion

Triangulation provides an elegant proof when analysing the minimum number of cameras needed in open-plan galleries. With more complex layouts in two and three dimensions, such elegant solutions have not been discovered although some necessary and some sufficient conditions have been found. In general, optimal solutions are found by applying computer algorithms to mathematical models of galleries.

Section A

1(i)

x	-2	-1	0	1	2
y	1.0655	1.1696	1.4142	1.9283	2.8964

(table above)	B2,1,0	table values
$A \approx ½×1\{1.0655+2.8964+2(1.1696+1.4142+1.9283)\}$	M1	formula
$= 6.493$	A1	6.5 or better www
	[4]	
(ii) Smaller, as the trapezium rule is an over-estimate in this case and the error is less with more strips	B1	
	B1	
	[2]	
2 $x = \frac{1}{1+t} \Rightarrow 1+t = \frac{1}{x}$	M1	attempt to solve for t
$\Rightarrow t = \frac{1}{x} - 1$	A1	oe
$y = \frac{1-t}{1+2t} = \frac{1-\frac{1}{x}+1}{1+\frac{2}{x}-2}$	M1	substituting for t in terms of x
$= \frac{2-\frac{1}{x}}{\frac{2}{x}-1} = \frac{2x-1}{2-x}$	M1	clearing subsidiary fractions
	A1	
	[5]	
3 $(3-2x)^{-3} = 3^{-3}(1-\frac{2}{3}x)^{-3}$	M1	dealing with the '3'
$= \frac{1}{27}(1+(-3)(-\frac{2}{3}x)+\frac{(-3)(-4)}{2}(-\frac{2}{3}x)^2+...)$	B1	correct binomial coeffs
$= \frac{1}{27}(1+2x+\frac{8}{3}x^2+...)$	B2,1,0	1, 2, 8/3 oe
$= \frac{1}{27}+\frac{2}{27}x+\frac{8}{81}x^2+...$	A1	cao
Valid for $-1 < -\frac{2}{3}x < 1$	M1	
$\Rightarrow -\frac{3}{2} < x < \frac{3}{2}$	A1	
	[7]	

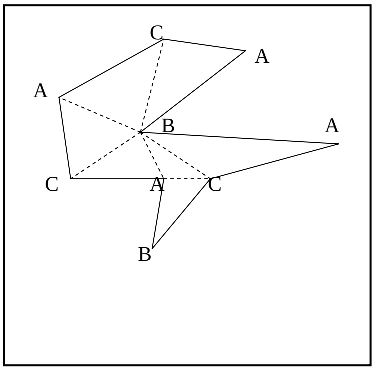
4(i) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ $\overrightarrow{AB}.\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}.\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 2\times5+3\times0+(-5)\times2=0$ $\Rightarrow$ AB is perpendicular to BC.	B1 B1 M1E1 [4]	
(ii) AB = $\sqrt{(2^2 + 3^2 + (-5)^2)} = \sqrt{38}$ BC = $\sqrt{(5^2 + 0^2 + 2^2)} = \sqrt{29}$ Area = ½ ×√38 × √29 = ½ √1102 or 16.6 units^2	M1 B1 A1 [3]	complete method ft lengths of both AB, BC oe www
5 LHS = $\frac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ $= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$ $= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$	M1 M1 E1 [3]	one correct double angle formula used cancelling cos θ's
6(i) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8-3\lambda \\ -2 \\ 6+\lambda \end{pmatrix}$ Substituting into plane equation: 2(–8 –3λ) –3(–2) + 6 + λ = 11 $\Rightarrow$ –16 – 6 λ + 6 + 6 + λ = 11 $\Rightarrow$ 5 λ = –15, λ = –3 So point of intersection is (1, –2, 3)	B1 M1 A1 A1ft [4]	
(ii) Angle between $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ $\cos\theta = \frac{2\times(-3)+(-3)\times0+1\times1}{\sqrt{14}\sqrt{10}}$ = (–)0.423 $\Rightarrow$ acute angle = 65°	B1 M1 A1 A1 [4]	allow M1 for a complete method only for any vectors

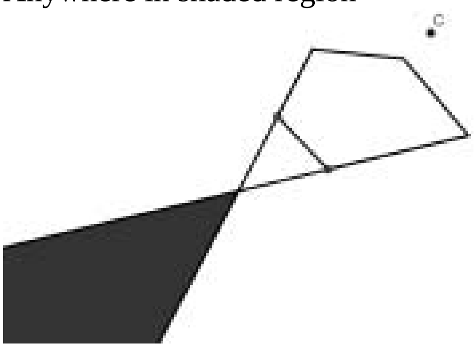
Section B

7(i) When $t = 0$, $v = 5(1 - e^0) = 0$ As $t \to \infty$, $e^{-2t} \to 0, \Rightarrow v \to 5$ When $t = 0.5$, $v = 3.16$ m s^{-1}	E1 E1 B1 [3]	
(ii) $\frac{dv}{dt} = 5 \times (-2)e^{-2t} = 10e^{-2t}$ $10 - 2v = 10 - 10(1 - e^{-2t}) = 10e^{-2t}$ $\Rightarrow \frac{dv}{dt} = 10 - 2v$	B1 M1 E1 [3]	
(iii) $\frac{dv}{dt} = 10 - 0.4v^2$ $\Rightarrow \frac{10}{100-4v^2}\frac{dv}{dt} = 1$ $\Rightarrow \frac{10}{25-v^2}\frac{dv}{dt} = 4$ $\Rightarrow \frac{10}{(5-v)(5+v)}\frac{dv}{dt} = 4$ * $\frac{10}{(5-v)(5+v)} = \frac{A}{5-v} + \frac{B}{5+v}$ $\Rightarrow 10 = A(5+v) + B(5-v)$ $v = 5 \Rightarrow 10 = 10A \Rightarrow A = 1$ $v = -5 \Rightarrow 10 = 10B \Rightarrow B = 1$ $\Rightarrow \frac{10}{(5-v)(5+v)} = \frac{1}{5-v} + \frac{1}{5+v}$ $\Rightarrow \int (\frac{1}{5-v} + \frac{1}{5+v})\,dv = 4\int dt$ $\Rightarrow \ln(5+v) - \ln(5-v) = 4t + c$ when $t = 0$, $v = 0$, $\Rightarrow 0 = 4\times0 + c \Rightarrow c = 0$ $\Rightarrow \ln\left(\frac{5+v}{5-v}\right) = 4t$ $\Rightarrow t = \frac{1}{4}\ln\left(\frac{5+v}{5-v}\right)$ *	 M1 E1 M1 A1 M1 A1 A1 E1 [8]	 for both A=1,B=1 separating variables correctly and indicating integration ft their $A,B,$ condone absence of c ft finding c from an expression of correct form
(iv) When $t \to \infty$, $e^{-4t} \to 0, \Rightarrow v \to 5/1 = 5$ when $t = 0.5$, $t = \frac{5(1-e^{-2})}{1+e^{-2}} = 3.8\,\text{m}\,\text{s}^{-1}$	E1 M1A1 [3]	
(v) The first model	E1 [1]	www

8(i) AC = 5sec α $\Rightarrow$ CF = AC tan β = 5sec α tan β $\Rightarrow$ GF = 2CF = 10sec α tan β *	B1 M1 E1 [3]	oe ACtanβ
(ii) CE = BE – BC = 5 tan($\alpha + \beta$) – 5 tan α = 5(tan($\alpha + \beta$) – tan α) $= 5\left(\frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} - \tan\alpha\right)$ $= 5\left(\frac{\tan\alpha + \tan\beta - \tan\alpha + \tan^2\alpha\tan\beta}{1-\tan\alpha\tan\beta}\right)$ $= \frac{5(1+\tan^2\alpha)\tan\beta}{1-\tan\alpha\tan\beta}$ $= \frac{5\tan\beta\sec^2\alpha}{1-\tan\alpha\tan\beta}$ *	 E1 M1 M1 DM1 E1 [5]	 compound angle formula combining fractions $\sec^2 = 1 + \tan^2$
(iii) $\sec^2 45° = 2$, tan 45° = 1 $\Rightarrow$ $CE = \frac{5t \times 2}{1-t} = \frac{10t}{1-t}$ $CD = \frac{10t}{1+t}$ $\Rightarrow$ $DE = \frac{10t}{1-t} + \frac{10t}{1+t} = 10t(\frac{1}{1-t} + \frac{1}{1+t})$ $= 10t\left(\frac{1+t+1-t}{(1-t)(1+t)}\right) = \frac{20t}{1-t^2}$ *	B1 M1 A1 M1 E1 [5]	used substitution for both in CE or CD oe for both adding their CE and CD
(iv) cos 45° = 1/√2 $\Rightarrow$ sec α = √2 $\Rightarrow$ GF = 10√2 tan β = 10√2 t	M1 E1 [2]	
(v) DE = 2GF $\Rightarrow$ $\frac{20t}{1-t^2} = 20\sqrt{2}t$ $\Rightarrow$ $1 - t^2 = 1/\sqrt{2}$ $\Rightarrow$ $t^2 = 1 - 1/\sqrt{2}$ * $\Rightarrow$ t = 0.541 $\Rightarrow$ β = 28.4°	 E1 M1 A1 [3]	 invtan t

<table>
<tr><td>Qn</td><td>Answer</td><td>Marks</td></tr>
<tr><td>1(i)</td><td>6 correct marks</td><td>B1</td></tr>
<tr><td>1(ii)</td><td>Either state both m and n odd or give a diagram (doorways between rooms not necessary)
justification</td><td>B1

B1ft</td></tr>
<tr><td>2(i)</td><td>$\frac{9-1}{4}=2=\left\lfloor\frac{4+1}{2}\right\rfloor$</td><td>B2
(B1 for LHS correct)</td></tr>
<tr><td>2(ii)</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$\left\lceil\frac{x}{2}\right\rceil$</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr></table></td><td>B2,1,0</td></tr>
<tr><td>3.</td><td>If each of A, B and C appeared at least four times then the total number of vertices would have to be at least 3×4=12</td><td>E2</td></tr>
<tr><td>4(i)</td><td>
</td><td>M1
allow if one error

A1</td></tr>
<tr><td>4(ii)</td><td>Two points labelled B above clearly marked (or f.t. from (i))</td><td>A1</td></tr>
<tr><td>5(i)</td><td>True.
Two cameras at the vertices labelled A or at the vertices labelled B would cover the entire gallery</td><td>A1
M1 for either</td></tr>
<tr><td>5(ii)</td><td>False.
One camera at either vertex labelled A would be sufficient (or C on RHS)</td><td>A1
M1</td></tr>
<tr><td>6</td><td>Anywhere in shaded region


correct construction
correct shading</td><td>M1
A1</td></tr>
</table>

4754 Applications of Advanced Mathematics

General Comments

This paper proved to be of a similar standard to that set in recent years. The questions were accessible to all and a wide range of marks-from full marks to single figures-was seen. As usual in the January paper most candidates achieved good or high marks. The most disappointing feature was the poor algebra which caused an unnecessary loss of marks for some candidates. The failure to use brackets was part of the problem as could be seen in question 2 and others. The comprehension was understood well and most candidates achieved good scores in that part.

Some candidates did not complete question 8 in Paper A or appeared rushed at this stage. In some cases this was due to using inefficient methods earlier in the paper, but the paper may have been longer than usual.

Comments on Individual Questions

Paper A

Section A

1) Most candidates successfully evaluated the integral using the trapezium rule. Those that failed commonly either failed to evaluate the terms correctly, failed to use the term at x=0, or misquoted (or misused) the formula.
In the second part candidates needed to refer both to the fact that the trapezium rule gave an overestimate in this case and that increasing the number of strips improves accuracy. Often one of these was omitted (or perhaps assumed).

2) Almost all candidates found t in terms of x correctly and substituted this in the equation for y. A great deal of poor algebra, both omission of brackets and incorrect signs followed. For example, on the numerator

$1-(1/x -1) = \pm 1/x$ was common.

Others failed to clear the subsidiary fractions, either by not trying, inverting fractions term by term or by making multiple errors.

3) Most candidates attempted a binomial expansion with power -3 and usually found the correct coefficients. Many could not deal correctly with factorising out the 1/27. Common errors were 3^3, 3 and 3^{-1}. For those who chose the correct factorisation it was disappointing to notice that so many failed to reintroduce it after completing the expansion. In some cases the term in the bracket stayed as $2x$ even after the factorisation. A more common error, however, was to use $2x/3$ instead of $-2x/3$. There were other algebraic errors after failing to use brackets or signs correctly.

The validity was often correct. Errors included having the signs pointing in the opposite directions, using ≥ instead of > and expressions such as $|x| < -3/2$.

4) This question was well answered. Most candidates scored full marks in the first part. The second part was also generally answered well but some candidates tried to use the position vectors of the vertices rather than the direction vectors of the sides. Some failed to see the relevance of part (i) to the area in (ii).

5) Almost all candidates correctly stated that sin $2\theta=2\sin\theta\cos\theta$. Some used the incorrect double angle formula on the denominator, usually $\cos2\theta=1-\cos^2\theta$ or $=2\sin^2\theta-1$. The majority, however, scored all three marks.

6) This question was well answered. Most knew the method to find where the line intersected the plane.
The majority used the correct vectors in the second part although some did not find the acute angle.

Section B

7) (i) The most common error here was to substitute a value when trying to show that as $t\to\infty$, $e^{-2t}\to0$, $v\to5$.

(ii) Only good candidates realised what was required here. Those who realised what was required usually scored well. A few solved the equation rather than verifying as requested. If it was fully correct they could obtain the marks this way, but it was more difficult and time consuming and few were successful. Candidates should be encouraged to verify when requested.

(iii) The first part was often omitted although there were many candidates who showed correctly that the two differential equations were equivalent. This was attempted both backwards and forwards with success.
The method of partial fractions was, as usual, well known. A few, however, fiddled their answers to obtain say A=1 and B= -1in anticipation of the given answer.
The majority separated their variables correctly. It was disappointing, however, to see some candidates only working with one side and failing to have a t in their working until reaching the final given answer. There were two common errors here. The first was to integrate $1/(5-v)$ as $+\ln(5-v)$ and the second, which was very, very common was to omit the constant of integration. In the latter case candidates are then precluded from obtaining the marks for evaluating c and establishing the given final result.

(iv) As in (i), as $t\to\infty$ or equivalent was not always seen or used.

(v) This was almost always correct if the correct values had been obtained in (i) and (iv).

8) (i) This was usually answered correctly.

(ii) Some failed to show that CE=BE –BC. Most substituted the compound angle formula. The subsequent adding of fractions, cancelling, factorising and the use of $\sec^2\theta=1+\tan^2\theta$ was not often seen. The best candidates found this straightforward but others who attempted this made sign errors and fiddling of results was seen.

(iii) There was some poor and unclear work in this part. Failing to use $\sec^2 45=2$ and using tan t instead of t or $\tan\beta$ were common errors. The addition of CD and DE to obtain the final given answer was often poor.

(iv) This question required candidates to 'show' the given result. Stating the result was therefore not enough. Both equations, GF = $10\sec\alpha\tan\beta$ and GF=$\sqrt{2}t$ were given in the question and so the substitution of α=45 needed to be seen.

(v) Poor algebra was often seen when showing that $t^2=1-1/\sqrt{2}$. However, the result was usually used correctly to find the value β.

Paper B
The Comprehension

1) **(i)** Almost all candidates showed the positions of the 6 guards correctly.

(ii) Most candidates realised that the problem arose when *m* and *n* were both odd. Their justifications, however, were not always correct. Answers such as '3x5/2= 7.5, there cannot be half a guard' were seen instead of stating that the floor function gave 7 guards instead of the required 8.

2) **(i)** 2.5=2.5 was often seen.

(ii) Often correct, but some incorrectly interpreted the symbol, or left the symbol in the answers.

3) This was the least successful question in the comprehension. Those giving a counter-example were the most successful.

4) Both parts here were almost always correct.

5) This was well answered. Some candidates did not understand what necessary and sufficient meant in these cases. Others did not state which points in the diagram they were using. In part (ii), some said three Cs were necessary or changed the original triangulation but did not refer to it.

6) There were many good solutions here with clear constructions and shading but there were also many who did not understand the question.

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