OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Tuesday **17 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8\ \text{m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 Two identical light springs have natural length 1 m and stiffness 400 N m^{-1}. One is suspended vertically with its upper end fixed to a ceiling and a particle of mass 2 kg hanging in equilibrium from its lower end.

(i) Calculate the extension of the spring. [2]

The second spring is then attached to the particle and its other end is attached to the floor. The system is in equilibrium with the two springs in the same vertical line. The distance between the floor and ceiling is 2.5 m and the extension of the upper spring is e metres. This situation is shown in Fig. 1.

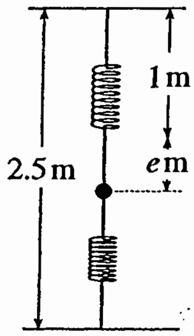

Fig. 1

(ii) Write down the extension of the lower spring in terms of e. [1]

(iii) Write down the equilibrium equation for the particle and hence calculate e. [4]

(iv) Calculate the tensions in the springs. [2]

The particle is now pulled vertically downwards a short distance and released.

(v) Show that the particle performs simple harmonic motion with period $\frac{1}{10}\pi$ seconds. [6]

2 Newton's Law of Gravitation states that the gravitational force, F, between two bodies of masses m_1 and m_2 a distance r apart is given by

$$F = \frac{Gm_1m_2}{r^2},$$

where G is the universal gravitational constant.

(i) Use this equation to show that the dimensions of G are $M^{-1}L^3T^{-2}$. [3]

A ray of light passing close to a star is bent by the gravitational field of the star. It is suggested that the bending angle, θ, can be expressed as

$$\theta = kG^{\alpha}m^{\beta}r^{\gamma}c^{\delta},$$

where m is the mass of the star, r is the distance between the ray of light and the star, c is the speed of light, G is the universal gravitational constant and k is a dimensionless constant.

(ii) Show that an angle in radians is dimensionless. [1]

(iii) Assuming that the above relationship is correct,

(A) explain why the values of α, β, γ and δ cannot be determined by dimensional analysis, [2]

(B) use dimensional analysis to find β, γ and δ in terms of α and hence show that

$$\theta = k\left(\frac{Gm}{rc^2}\right)^{\alpha}.$$ [6]

Two observations are made of light passing a particular star.

Observation	Distance, r	Angle, θ
1	2×10^9 m	3×10^{-5} rad
2	1.5×10^9 m	4×10^{-5} rad

(iv) Use these figures to determine α. [2]

(v) Given also that the mass of the star is 2.015×10^{31} kg, $G = 6.7 \times 10^{-11}$ m^3 s^{-2} kg^{-1} and $c = 3.0 \times 10^8$ m s^{-1}, calculate the value of k correct to 3 significant figures. [1]

3 Part of the track of a theme park ride consists of a vertical drop, AB, of 62.5 m and the arc, BC, of a circle with centre O and radius 25 m. BO is horizontal. Fig. 3.1 shows the situation with the car in its initial position. Fig. 3.2 shows the car in a later position when it has turned through an angle θ about O.

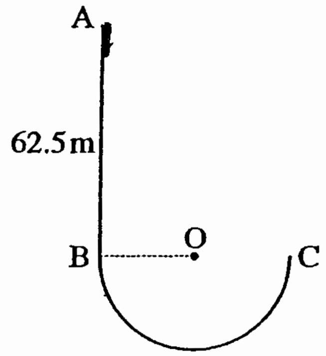

Fig. 3.1

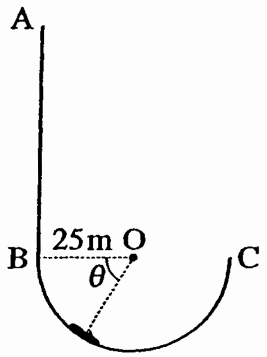

Fig. 3.2

The car is modelled as a particle of mass m kg dropped from rest at A. All resistance forces are neglected.

(i) Show that the speed of the car at B is $35\,\mathrm{m\,s^{-1}}$. [1]

When the car has turned through an angle θ, its speed is $v\,\mathrm{m\,s^{-1}}$.

(ii) Find an expression for v^2 in terms of θ and g and hence show that the force, R N, of the track on the car is given by $R = mg(3\sin\theta + 5)$. [8]

(iii) Find expressions for the radial and tangential components of the acceleration in terms of θ and g. Hence show that the magnitude of the acceleration, in $\mathrm{m\,s^{-2}}$, is

$$g\sqrt{26 + 20\sin\theta + 3\sin^2\theta}.$$ [6]

4 A uniform solid is made by rotating the region in the first quadrant between the curve $y = \sqrt{1-x}$ and the coordinate axes through one revolution about the y-axis, as shown in Fig. 4.1.

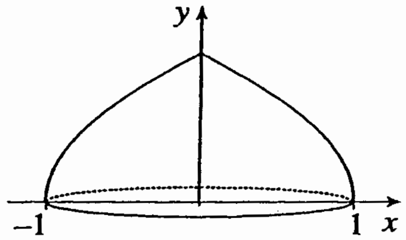

Fig. 4.1

(i) Show that the centre of mass of the solid is at $(0, \frac{5}{16})$. [7]

Fig. 4.2 shows a buoy made by attaching a uniform solid of the above shape and of mass m to a uniform solid hemisphere of radius 1 and mass λm. The buoy has its axis of symmetry vertical.

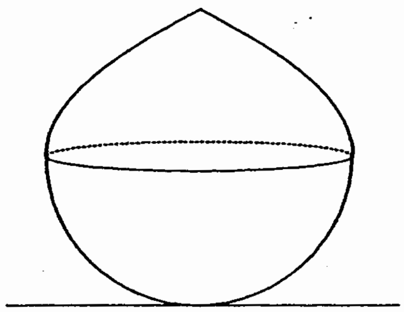

Fig. 4.2

(ii) Show that the distance of the centre of mass of the buoy above its lowest point is

$$\frac{10\lambda + 21}{16(\lambda + 1)}.$$ [4]

[You may assume the standard result for the centre of mass of a hemisphere.]

(iii) The buoy is placed with a point of the hemisphere on horizontal ground. The axis of symmetry is not vertical. Find the range of values of λ for which the buoy will tend to right itself. [4]

Mark Scheme

1(i)	$2g = 400x$	M1	use of Hooke's law and equilibrium	
	$x = 0.049$	A1		2
(ii)	$0.5 - e$	B1	accept unsimplified	1
(iii)	$T_1 = T_2 + mg$	M1	equilibrium equation	
	$400e = 400(0.5 - e) + 2g$	A1	allow their (ii) instead of $(0.5 - e)$	
	$800e = 219.6$	M1	solving an equation with more than one term in e	
	$e = 0.2745$	A1	cao	4
(iv)	$T_1 = 400e = 109.8$	F1	400 times their e	
	$T_2 = 400(0.5 - e) = 90.2$	F1	or $T_1 - 19.6$	2
(v)	(let y be displacement below equilibrium position)			
	$m\ddot{y} = T_2 + mg - T_1$	M1	N2L in general position	
		B1	weight term (ignore sign)	
	$= 400(0.5 - e - y) + mg - 400(e + y)$	M1	two distinct tensions in terms of y (or equivalent)	
		A1	all correct	
	$\Rightarrow \ddot{y} = -400y$, i.e. SHM	E1	must be completely correct and conclude SHM	
	period $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{400}} = \frac{1}{10}\pi$	E1	dependent on both M1 marks	6

2(i)	$[G] = \dfrac{MLT^{-2} \cdot L^2}{M \cdot M}$	M1	substitute dimensions	
		A1	correct dimensions	
	$\Rightarrow [G] = M^{-1}L^3T^{-2}$	E1		3
(ii)	e.g. $s = r\theta \Rightarrow [\theta] = \frac{L}{L} = 1$	E1	any valid reason	1
(iii)	(*A*) dimensional analysis gives only 3 equations	B1	for indication that method provides less information than needed	
	but 4 unknowns so cannot solve	B1	for full explanation	2
	(*B*) $1 = (M^{-1}L^3T^{-2})^\alpha M^\beta L^\gamma (LT^{-1})^\delta$	M1	substitute dimensions	
	M: $-\alpha + \beta = 0$	M1	one equation	
	L: $3\alpha + \gamma + \delta = 0$	M1	good attempt at all three equations	
	T: $-2\alpha - \delta = 0$			
	$\beta = \alpha, \delta = -2\alpha, \gamma = -\alpha$	A1	one correct	
		A1	all correct. Evidence of working is needed as can be deduced from given answer. Use of given answer (verification) earns at most 3 marks: M1 M1 M0 A1 A0 E0.	
	$\theta = kG^\alpha m^\alpha r^{-\alpha} c^{-2\alpha} = k\left(\frac{Gm}{rc^2}\right)^\alpha$	E1	must be shown, not just stated	6
(iv)	$\frac{3 \cdot 10^{-5}}{4 \cdot 10^{-5}} = \left(\frac{1.5 \cdot 10^9}{2 \cdot 10^9}\right)^\alpha$	M1	eliminate constants	
	$0.75 = 0.75^\alpha \Rightarrow \alpha = 1$	A1	must have evidence of working	2
(v)	$k = \frac{3\times10^{-5}\times2\times10^9\times(3.0\times10^8)^2}{6.7\times10^{-11}\ 2.15\times10^{31}} = 4.00$	B1	cao (must be to 3sf)	1

3(i)	$v^2 = 2\times 9.8\times 62.5 \Rightarrow v = 35\text{ m s}^{-1}$	E1	using energy or constant acceleration formula(e)	1
(ii)	$\frac{1}{2}mv^2 = mg(62.5 + 25\sin\theta)$	M1	using energy	
	or $\frac{1}{2}mv^2 - mg\cdot 25\sin\theta = \frac{1}{2}m\cdot 35^2$	A1	$25mg\sin\theta$ (or equivalent) seen	
		A1	correct equation	
	$\Rightarrow v^2 = 125g + 50g\sin\theta$	A1	or equivalent	
	$R - mg\sin\theta = \frac{mv^2}{25}$	M1	N2L with mv^2/r	
		A1		
	$R = \frac{m}{25}(125g + 50g\sin\theta) + mg\sin\theta$	M1	substitute their v^2 (but must be in terms of θ)	
	$R = mg(3\sin\theta + 5)$	E1		8
(iii)	radial $= \frac{v^2}{25} = 5g + 2g\sin\theta$	B1	follow their v^2 (but must be in terms of θ)	
	tangential $m\dot{v} = mg\cos\theta$	M1	N2L with cpt. of weight and no other force	
	$\dot{v} = g\cos\theta$	A1	ignore sign	
	$\lvert acc.\rvert = \sqrt{(5g + 2g\sin\theta)^2 + (g\cos\theta)^2}$	M1	finding magnitude	
	$= g\sqrt{25 + 20\sin\theta + 4\sin^2\theta + 1 - \sin^2\theta}$	M1	using $\cos^2\theta = 1 - \sin^2\theta$	
	$= g\sqrt{26 + 20\sin\theta + 3\sin^2\theta}$	E1		6

4(i)	$\bar{x} = 0$ (symmetry)	B1	need justification	
	$\bar{y} = \dfrac{\int x^2 y\,dy}{\int x^2 dy}$	B1	formula	
	$= \dfrac{\int_0^1 (y - 2y^3 + y^5)dy}{\int_0^1 (1 - 2y^2 + y^4)dy}$	M1	substitute & expand (both) – must have more than two terms for each	
	$= \dfrac{\left[\frac{1}{2}y^2 - \frac{1}{2}y^4 + \frac{1}{6}y^6\right]_0^1}{\left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5\right]_0^1}$	M1	integrate (either) – must have more than one term	
		A1	numerator	
		A1	denominator	
	$= \dfrac{1/6}{8/15} = \frac{5}{16}$	E1		7
(ii)	$\dfrac{\lambda m\cdot\frac{5}{8} + m\cdot\frac{21}{16}}{\lambda m + m}$	M1	$\Sigma mx/\Sigma m$ or moments	
		A1	numerator	
		A1	denominator	
	$= \dfrac{10\lambda + 21}{16(\lambda + 1)}$	E1		4
(iii)	need centre of mass in hemisphere	B1	may be implied	
	$\dfrac{10\lambda + 21}{16(\lambda + 1)} < 1$	B1	inequality	
		M1	solving (accept equation)	
	$\lambda > \frac{5}{6}$	A1	cao (if from equation, > must be justified)	4

Examiner's Report

2609 Mechanics 3

General Comments

Although there were many good scripts showing a sound understanding of mechanics, it was evident that this paper was found difficult in parts, in particular the last part of question 1 and most of question 3. In these parts and others many candidates would have benefited by using clear diagrams and 'signposting' their work.

Comments on Individual Questions

Q.1 The first four parts of this question were often well done, although some of the candidates omitted the weight in the equilibrium equation in part (iii). The last part was rarely done well. Many of the candidates made no attempt to use Newton's second law which was necessary to make any progress. Of the remainder, many omitted the weight, some only used one tension and others used identical tensions in the two springs.

(i) 0.049 m; (ii) $(0.5 - e)$ m; (iii) 0.2745; (iv) 109.8 N, 90.2 N.

Q.2 Many of the candidates showed a good working knowledge of dimensions. However the explanation parts were generally found more difficult and most candidates were unable to perform the calculations accurately. Although many of the candidates struggled to give an adequate justification that an angle is dimensionless, many gave a valid reason, often using the formula for arc length of a circle. In part (iii)(*A*), although many did realise that the problem was fewer equations than unknowns, many other candidates argued that the problem was with the dimensionless angle. Candidates who adopted this approach often stated that dimensional analysis could not be used at all, yet immediately went on to use it in the next part of the question. Most of the candidates made a good attempt at finding the indices in terms of α, although some clearly used the given result rather than dimensional analysis. The calculations were often poorly done, with very few of the candidates efficiently using ratios to find α and only a minority able to find it by setting up two equations and eliminating the constants. Even for candidates who found α correctly, the calculations of k were disappointing, as was the inability of many candidates accurately to correct to three significant figures.

(iii)(*B*) $\beta = \alpha, \delta = -2\alpha, \gamma = -\alpha$; (iv) $\alpha = 1$; (v) 4.00.

Q.3 Almost all of the candidates were able to calculate the speed of the car at B. The calculation of v^2 in the general position was often done well, although some candidates made a mistake with the position of least potential energy and hence errors crept in. Disappointingly, some candidates used constant acceleration formulae and so received no credit. Most candidates who attempted to find R realised that Newton's second law was required, but many attempts were flawed and quite a few candidates made no attempt at all. Omitting the component of weight, resolving the reaction force and a wrong sign for the acceleration were all common errors. Finding the tangential component of acceleration often caused problems, with few candidates using Newton's second law. Finding the radial component was usually more successful, although some added spurious terms to $\frac{v^2}{r}$. When finding the magnitude, candidates often started well but too many manipulated their expression into the given answer with insufficient or plainly incorrect working.

(ii) $v^2 = 125g + 50g\sin\theta$; (iii) radial $5g + 2g\sin\theta$, tangential $g\cos\theta$.

Q.4 Many of the candidates calculated the centre of mass correctly, but many others were unable to quote the correct formula. Candidates should be able accurately to look up and use the relevant formulae for centres of mass. Other common errors were the use of wrong limits and poor expansion of $(1 - y^2)^2$. In part (ii), many correct solutions were seen but also many errors, in particular, candidates using volumes rather than the given masses. Some candidates tried to take the simple mean of the positions of the two centres of mass. In the last part, some candidates started with a correct inequality and then produced good clear solutions. However, many started with an equation and did not justify their final inequality.

(iii) $\lambda > \frac{5}{6}$.