OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2608/1

Mechanics 2

Friday **16 JANUARY 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take g = 9.8 m s^{-2} unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 Spheres A, B and C of masses 2 kg, 3 kg and 5 kg, respectively, are at rest on a smooth, horizontal table, as shown in Fig. 1. The spheres move only in the same straight line and all impacts on the table are direct.

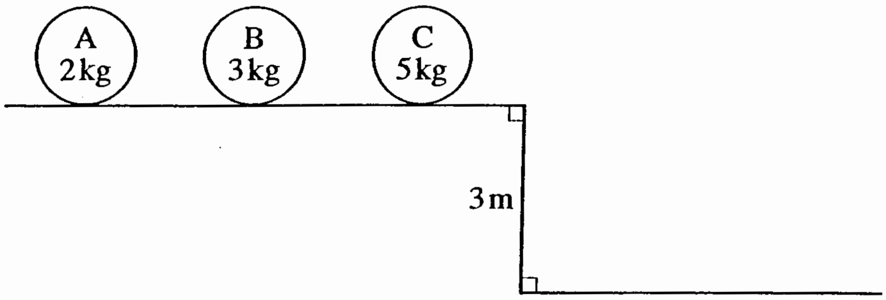


Fig. 1

An impulse of 10 N s is applied to sphere A in the direction A to B.

(i) Find the speed with which the sphere moves off. [1]

Sphere A now collides and coalesces with sphere B to form the object AB.

(ii) Show that the speed of the object AB after the collision is $2\ \mathrm{m\,s^{-1}}$. [2]

Object AB now collides with sphere C. The coefficient of restitution in this collision is 0.6.

(iii) Show that the speed of sphere C after the collision is $1.6\ \mathrm{m\,s^{-1}}$. [5]

Sphere C leaves the table and should now be regarded as a projectile subject to negligible air resistance. Sphere C bounces off the smooth horizontal floor that is 3 m below the table top, as shown in Fig. 1. The coefficient of restitution in this collision is also 0.6.

(iv) Show that sphere C bounces off the floor with a vertical component of velocity of $\sqrt{2.16g}\ \mathrm{m\,s^{-1}}$.

Calculate the angle to the horizontal with which sphere C leaves the floor. [6]

[Total 14]

2

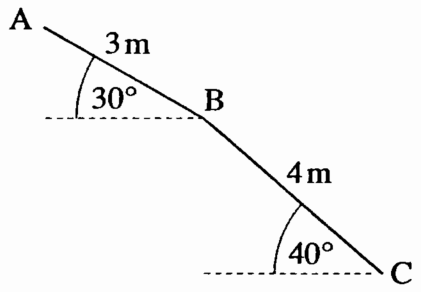


Fig. 2

A small tile of mass 0.5 kg slides down a roof. It slides 3 m from A to B at 30° to the horizontal and, without losing contact, slides 4 m from B to C at 40° to the horizontal. The coefficient of friction, μ, between the tile and the roof is the same for both parts of the motion.

(i) Calculate the gravitational potential energy lost by the tile in travelling from A to C. [3]

(ii) Write down expressions in terms of μ for the work done against friction by the tile as it travels from A to B and from B to C. [3]

The tile has the same speed at C as at A.

(iii) Show that the value of μ is about 0.72. [3]

The speed of the tile at A is 3.5 m s^{-1}.

(iv) Calculate the speed of the tile at B. [5]

[Total 14]

3 In this question, the units of the axes shown in the diagrams are centimetres and all coordinates are referred to these axes.

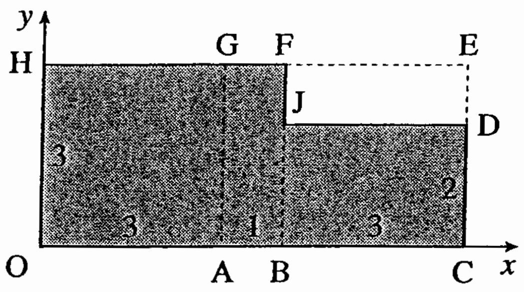


Fig. 3.1

A uniform lamina OCDJFH is in the shape of the rectangle OCEH with the rectangle DEFJ removed, as shown in Fig. 3.1.

(i) Calculate the coordinates of the centre of mass of OCDJFH. [5]

The lamina is now folded along AG and BJ to form the shape shown in Fig. 3.2. Angles OAB and ABC are 90°.

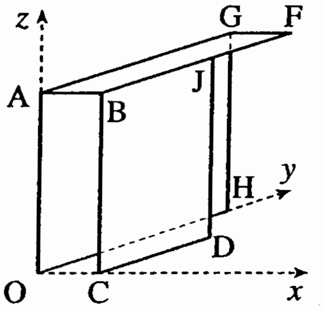


Fig. 3.2

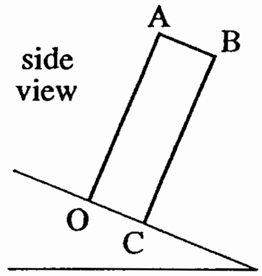


Fig. 3.3

(ii) Calculate the coordinates of the centre of mass of the folded lamina. [5]

The folded lamina is placed on an inclined plane with OC along a line of greatest slope, as shown in Fig. 3.3. The plane is tilted slowly until the lamina is about to turn about the edge CD. At this stage, the lamina is also about to slip down the plane and OC is inclined at an angle λ to the horizontal. The coefficient of friction between the folded lamina and the plane is μ.

(iii) Show that $\tan\lambda = \mu$.

Draw a diagram showing the line of action of the weight of the lamina.

Calculate the value of μ. [5]

[Total 15]

4 (a)

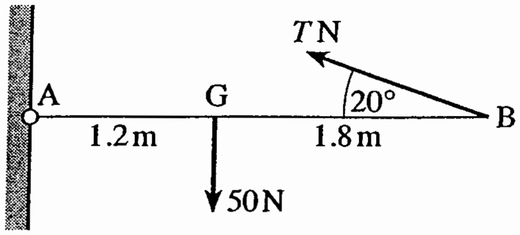


Fig. 4.1

A heavy beam AB of length 3 m is freely hinged at A and its weight of 50 N acts through its centre of mass G, where AG is 1.2 m. It is held horizontally in equilibrium by a force of magnitude T N acting at 20° to the horizontal, as shown in Fig. 4.1.

Calculate the value of T. [3]

(b)

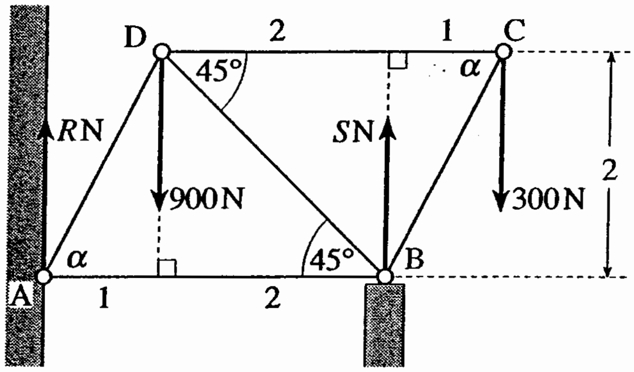


Fig. 4.2

Fig. 4.2 shows a framework of light rods AB, BC, CD, DA and DB, freely pin-jointed together at A, B, C and D and to a wall at A. The framework rests on a smooth, horizontal support at B with AB and CD horizontal. The dimensions of the framework are shown in metres.

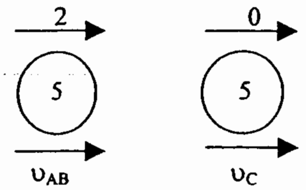
Angle DAB = angle DCB = α, where $\sin\alpha = \frac{2}{\sqrt{5}}$, and $\cos\alpha = \frac{1}{\sqrt{5}}$. Vertical loads of 300 N and 900 N act at C and D, respectively. The vertical support forces at A and B are R N and S N respectively.

(i) Calculate the value of S and show that $R = 500$. [4]

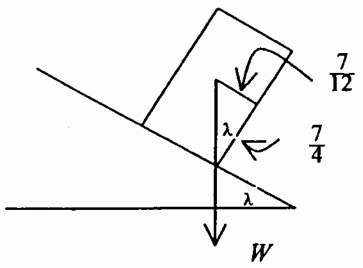
(ii) Draw a diagram showing all of the forces acting on the pin-joints, including the internal forces in the rods. Calculate the internal force in each rod, indicating whether it is a tension or a thrust (compression). Answers may be left in surd form. [10]

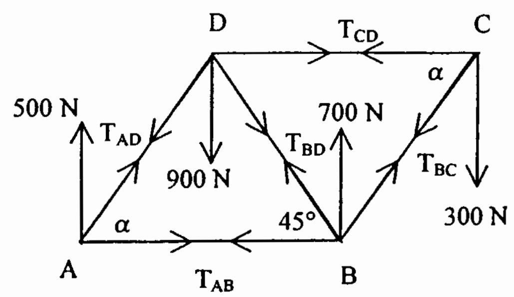
[Total 17]

Mark Scheme

Q 1		mark		sub
(i)	$10 = 2v$ so $v = 5$ so 5 m s^{-1}	B1		1
(ii)	$10 = (2+3)V$	M1	PCLM. Must deal with coalescence.	
	so $V = 2$ so 2 m s^{-1}	E1		2
(iii)	before  after			
	$10 = 5v_C + 5v_{AB}$	M1 A1	PCLM Any form	
	$\dfrac{v_C - v_{AB}}{0 - 2} = -0.6$	M1	NEL	
		A1	Any form	
	Solving $v_C = 1.6$ so 1.6 m s^{-1}	E1	Must be clearly shown. (Either solved or v_C subst in one equation to get v_{AB} and both values checked in the other)	5
(iv)	NEL applied perpendicular to the floor	M1	May be implied	
	Vert cpt ↓ $\sqrt{6g}$	B1	May be implied	
	After bounce ↑ $\sqrt{6g} \times 0.6 = \sqrt{2.16g}$	E1	If decimal equivalence used need at least 1 d.p.	
	Linear momentum conserved parallel to floor	M1	May be implied by using 1.6 after the bounce	
	Angle with the floor is $\arctan \dfrac{\sqrt{6g} \times 0.6}{1.6}$	M1	Use of arctan or equiv with numerator **their** $\sqrt{2.16g}$	
	$= 70.8243\ldots$ so 70.8° (3 s. f.)	A1	cao	6
			total	14

Q 2		mark		sub
(i)	$(3\sin 30 + 4\sin 40)\times 0.5\times 9.8$	M1	Use of *mgh*	
		A1	Either GPE term correct	
	19.94867… so 19.9 J (3 s. f.)	A1		3
(ii)	$3\times 0.5g\cos 30\mu$ $(12.73057\ldots\mu)$	M1	Use of $F = \mu R$. Accept g not used.	
		M1	Use of WD = Fd	
	$4\times 0.5g\cos 40\mu$ $(15.01447\ldots\mu)$	A1	Both. Any form.	3
(iii)	$(3\times 0.5g\cos 30 + 4\times 0.5g\cos 40)\mu$	M1	Use of work-energy equation	
	= 19.94867…	A1		
	so $\mu = 0.71899\ldots$, so 0.72 (2 s. f.)	E1	Accept any reasonable accuracy	
	or			
	N2L to find a and *uvast*	M1	Used on both sections. Accept use of 3.5 for v_A	
	Eliminate velocities	A1	Statement correct in any form	
	so $\mu = 0.71899\ldots$, so 0.72 (2 s. f.)	E1	Accept any reasonable accuracy	3
(iv)	$0.5\times 0.5\times 3.5^2 + 0.5\times 9.8\times 3\sin 30$ $- 3\times 0.5\times 9.8\cos 30\mu = 0.5\times 0.5v^2$	M1	work-energy equation with at least two of KE, GPE, WD against friction	
		B1	Use of $0.5mv^2$ for KE and Δ KE	
		B1	GPE term correct	
		A1	All correct	
	$v = 2.2443\ldots$ so 2.24 m s^{-1} (3 s. f.)	A1	cao. Accept 2.23.	
	or			
	N2L on AB	M1	All forces present	
		A1	Expression or $a = 1.21..$ or $-1.21\ldots$	
	Use of appropriate *uvast*	M1	Dep on 1st M1. Signs consistent	
		F1	FT **their** a	
		A1	cao Accept 2.23.	5
			total	14

Q 3		mark		sub
(i)	$18\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 21\begin{pmatrix}3.5\\1.5\end{pmatrix} - 3\begin{pmatrix}5.5\\2.5\end{pmatrix} = \begin{pmatrix}57\\24\end{pmatrix}$	M1	Correct method	
		B1	Masses correct	
		B1	At least one c.m. correct (if x, y done separately, at least two c. m. cpts correct)	
	$\bar{x} = \frac{19}{6}$, $\bar{y} = \frac{4}{3}$	A1 A1		5
(ii)	$\bar{y} = \frac{4}{3}$. No change.	B1	Accept WW. Or work out below.	
	$18\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 9\begin{pmatrix}0\\1.5\\1.5\end{pmatrix} + 3\begin{pmatrix}0.5\\1.5\\3\end{pmatrix} + 6\begin{pmatrix}1\\1\\1.5\end{pmatrix}$	M1	Correct method dealing with fold	
		B1	At least one c.m. correct (if x, y, z done separately, at least two c. m. cpts correct)	
	$\bar{x} = \frac{5}{12}$, $\bar{z} = \frac{7}{4}$	A1 A1		5
(iii)	Since about to slip, $F = \mu R$ $F = mg\sin\lambda$ and $R = mg\cos\lambda$	M1	Use of $F = \mu R$ and attempts to find F and R	
	Hence $\tan\lambda = \mu$	A1	Clearly shown	
	Line of action of weight acts through CD	B1		
		M1	Identify both relevant lengths and correct angle (accept the use of $\frac{5}{12}$ in place of $\frac{7}{12}$ but not $\frac{4}{3}$ instead of $\frac{7}{4}$)	
	$\mu = \tan\lambda = \frac{7}{12} \div \frac{7}{4} = \frac{1}{3}$	A1	cao	5
			total	15

Q 4		mark		sub
(a) (i)	$\hat{A}$ $1.2 \times 50 = 3T \sin 20$ $T = 58.4760\ldots$ so 58.5 N (3 s. f.)	M1 B1 A1	Moments about A, or equivalent $3T \sin 20$	 3
(b) (i)	$\hat{A}$ $900 \times 1 + 300 \times 4 = 3S$ so $S = 700$ $R + S = 1200$ so $R = 500$	M1 A1 M1 E1	Moments about A or … Resolve or take moments again	 4
(ii)		B1	All labelling correct. All forces present. Accept no angles marked.	
		M1 A1 A1	Equilibrium considered at pin-joint(s) One equilibrium equation correct Second equilibrium equation correct	
	A↑ $T_{AD} \sin\alpha + 500 = 0$ $T_{AD} = -250\sqrt{5}$ so $250\sqrt{5}$ N (C)	A1	This and the following marks are A1 and F1 as appropriate according to the order in which the equilibria are considered.	
	A → $T_{AD} \cos\alpha + T_{AB} = 0$ $T_{AB} = 250$ so 250 N (T)	F1		
	C ↓ $T_{BC} \sin\alpha + 300 = 0$ $T_{BC} = -150\sqrt{5}$ so $150\sqrt{5}$ N (C)	F1		
	C → $T_{BC} \cos\alpha + T_{CD} = 0$ $T_{CD} = 150$ so 150 N (T)	F1		
	B↑ $T_{BC} \sin\alpha + T_{DB} \cos 45 + 700 = 0$ $T_{DB} = -400\sqrt{2}$ so $400\sqrt{2}$ N (C)	F1 F1	 Tensions/compressions all consistent [SC Repeated error: signs in working inconsistent with the diagram. Condone and FT working up to maximum of 8/10]	 10
			total	17

Examiner's Report

2608 Mechanics 2

General Comments

The majority of the candidates made progress with every question and many complete answers were seen to every part of every question. Some candidates produced rather inefficient solutions, especially to part (iv) of Question 1, to parts (ii) and (iii) of Question 2 and to part (b) (ii) of Question 4. Some candidates found the paper a little too long but most seemed to have done all they could in the time.

Some candidates penalised themselves by their poor presentation; there were many examples of 'unforced errors' consequent upon poor organisation and marks lost because of poor or even no indication of the method used to obtain a wrong answer or a given answer. As always, many marks were lost because of poor or no diagrams being used, a common consequence being that a force was omitted or there was inconsistency with signs. Premature approximation caused some candidates to have problems with accuracy. There was also a further increase in the number of candidates who, when given an answer, waste their time trying to 'fudge' their working instead of using it more profitably to try to decide what principles might apply.

Comments on Individual Questions

Question 1 (Impulse, linear momentum and the impact of two objects; oblique impact with a smooth plane)

Many candidates understood the principles involved and scored very well on this question.

Almost every candidate obtained full marks for parts (i) and (ii) and most did so in part (iii) as well. Problems with part (iii) were usually caused by the lack of a diagram establishing a sign convention so that the candidate's equations had inconsistent signs. Candidates who try to produce a single initial expression involving both the conservation principle and the use of Newton's experimental law are also prone to error.

There were many correct solutions to part (iv) from candidates who understood that they should apply Newton's experimental law to the component of velocity normal to the plane and conserve linear momentum parallel to the plane. However, some candidates were clearly not familiar with these principles and made little progress. Quite a few candidates seemed unable to find the vertical component of velocity with which the sphere hit the plane and others used a spurious method based on energy; some candidates deliberately found the total speed of the sphere and applied Newton's experimental law to this.

(i).5 m s^{-1}; (iv) 70.8° (3 s. f.)

Question 2 (Friction, work and energy applied to a tile sliding down slopes of different angle and common coefficient of friction)

The standard of the answers to this question was higher than seen to similar questions in previous sessions, with almost all of the candidates employing work-energy methods rather than directly applying Newton's second law and then using the *uvast* results.

The majority of the candidates obtained the correct answer to part (i) but some used cosine in place of sine.

Most, but not as many, knew what to do in part (ii). Apart from errors with sine and cosine (not always consistent with those in part (i)) a few candidates thought they should multiply the sum of the frictional forces by the sum of the distances.

A minority found it difficult to relate their answers to parts (i) and (ii) to part (iii). Some candidates omitted the part. Others found the values of μ for the tile to slide at constant speed down each part of the slope and averaged them (this gives a wrong answer); others calculated μ for the tile to slide at constant speed down the average slope (although this gives the correct answer no credit was given without justification for this method of solution).

Most candidates used a correct work-energy method for part (iv) but very many omitted one of the terms (usually the work done against friction). A diagram showing all of the forces could help candidates avoid such an elementary error. A small number of candidates used rounded figures from earlier in the question and so did not obtain the correct result. In this part of the question, Newton's second law may be applied directly and the solution is (unusually) just as efficient as that based on work-energy.

(i) 19.9 J (3 s. f.); (ii) 12.7μ, 15.0μ (3 s. f.); (iv) 2.24 m s^{-1}

Question 3 (The centre of mass of a folded lamina and the condition for sliding and toppling on an inclined plane)

Most candidates scored well on this question. Apart from arithmetic errors, parts (i) and (ii) caused few problems. A small number of candidates could not deal properly with the fold in part (ii); a common error was to omit one of the dimensions. However, a little surprisingly, there were more slips seen in part (i), usually because of giving the centre of mass of a part of the lamina relative to a vertex of that part instead of relative to the origin. Some candidates who did not express their answers as fractions in parts (i) and/or (ii) rounded to 1 decimal place with a consequent loss of accuracy in later parts.

Most candidates knew how to establish $\mu = \tan\lambda$ in part (iii) but this was poorly done in a small number of cases, often because g was omitted. The diagram required to show the line of action of the normal reaction force was often not sufficiently clear. Most candidates located the correct angle required but many did not obtain the correct lengths, using $\frac{5}{12}$ instead of $1-\frac{5}{12}=\frac{7}{12}$.

(i) $\left(\frac{19}{6}, \frac{4}{3}\right)$; (ii) $\left(\frac{5}{12}, \frac{4}{3}, \frac{7}{4}\right)$; (iii) $\mu=\frac{1}{3}$

Question 4 (The use of moments and the internal forces in a pin-jointed light framework)

There were many excellent answers to the whole of this question but there were also some poor attempts at it. Generally speaking, candidates who were well organised and had a system made few errors while those whose solutions were less clear made many more.

The majority of the candidates correctly obtained the solutions to parts (a)(i), and (b)(i). Almost everybody appreciated that moments were required but some made errors either in calculation or in resolution.

Finding the internal forces in the framework caused problems for a significant minority, with most of the difficulties stemming from a poor diagram or, indeed, a lack of one. Many diagrams lacked arrows designating whether the rod was in tension or compression, others lacked labels for the forces. When resolving at the pin-joints, errors were made with signs both within the equations and in the solutions to them. Unfortunately, some candidates who drew an acceptable diagram then went on to ignore it, producing equations at pin-joints that were inconsistent with the diagram and in some cases, inconsistent with each other. Lack of consistency was also seen in the interpretation of tension and compression; it is generally found that candidates who start by assuming all the internal forces are tensions make fewer mistakes when interpreting their results.

(a)(i) 58.5 N (3 s. f.); (b)(i) $S = 700$; (ii) $T_{AD} = 250\sqrt{5}$ N (C), $T_{AB} = 250$ N (T), $T_{BC} = 150\sqrt{5}$ N (C), $T_{CD} = 150$ N (T), $T_{BD} = 400\sqrt{2}$ N (C)