OCR
RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2608/1**

Mechanics 2

Monday **14 JANUARY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 6 printed pages and 2 blank pages.

1 A tractor and its plough have a combined mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of $2.5\,\mathrm{m\,s^{-1}}$ along a horizontal field.

(i) Calculate the resistance to the motion. [2]

The tractor comes to a patch of wet ground where the resistance to motion is different. The power developed by the tractor during the next 10 seconds has an average value of 8 kW over this time. During this time, the tractor accelerates uniformly from $2.5\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$.

(ii) Show that the work done against the resistance to motion during the 10 seconds is 71 750 J.

Assuming that the resistance to the motion is constant, calculate its value. [7]

The tractor now comes to a slope at $\sin^{-1}(\frac{1}{20})$ to the horizontal. The non-gravitational resistance to motion on this slope is 2000 N. The tractor accelerates uniformly from $3\,\mathrm{m\,s^{-1}}$ to $3.25\,\mathrm{m\,s^{-1}}$ over a distance of 100 m while climbing the slope.

(iii) Calculate the time taken to travel this distance of 100 m and the average power required over this time period. [6]

[Total 15]

2

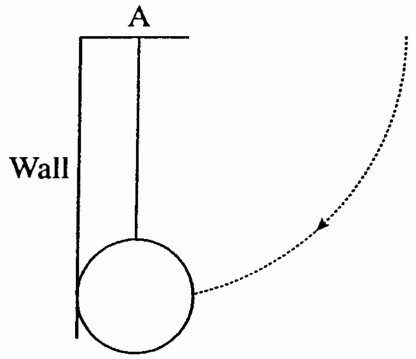


Fig. 2

Fig. 2 shows a uniform sphere of mass 10 kg suspended from a point A by a light string. The sphere just touches a vertical wall when hanging freely. The sphere swings in a vertical plane and makes direct impact with the wall. The sphere is held with the string taut and is released from rest, hitting the wall with a speed of $7\,\mathrm{m\,s^{-1}}$. The coefficient of restitution between the sphere and the wall is 0.6.

The wall does not move when hit by the sphere.

(i) Calculate the impulse on the sphere in the impact. [3]

(ii) Show that after the third impact the sphere comes instantaneously to rest about 0.117 m higher than its position when it hits the wall. [4]

The sphere is released again in the same way so that it hits the wall with a speed of $7\,\mathrm{m\,s^{-1}}$. This time a brick of mass 4 kg is dislodged. You may assume that linear momentum is conserved in this impact. The coefficient of restitution between the sphere and the brick is 0.4.

(iii) Calculate the velocity of the sphere immediately after the impact. [7]

[Total 14]

3

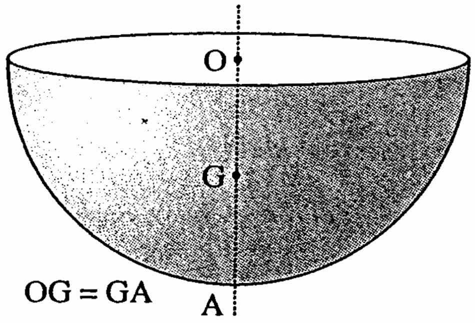
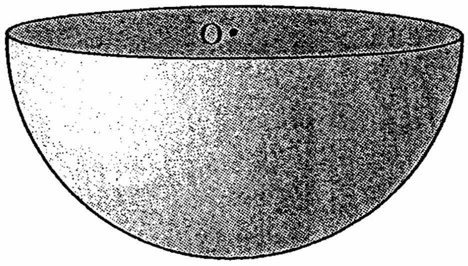


Fig. 3.1



the hollow enclosed hemisphere H

Fig. 3.2

In this question you may use the following fact, illustrated in Fig. 3.1: the centre of mass, G, of a thin open hemispherical shell is at the mid-point of OA, its axis of symmetry.

A thin circular disc of radius r metres is attached to a thin, open hemispherical shell of radius r metres to form a hollow enclosed hemisphere, H, as shown in Fig. 3.2. The disc and hemispherical shell are made from the same uniform material.

(i) Show that the centre of mass of H is on the axis of symmetry a distance $\frac{1}{3}r$ metres from O, the centre of the disc. [The surface area of a sphere of radius r is $4\pi r^2$.] [4]

The mass of H is $12m$ kg. A thin, uniform rod of length y metres with mass m kg per unit length is attached to H at O and lies in its axis of symmetry, as shown in Fig. 3.3.

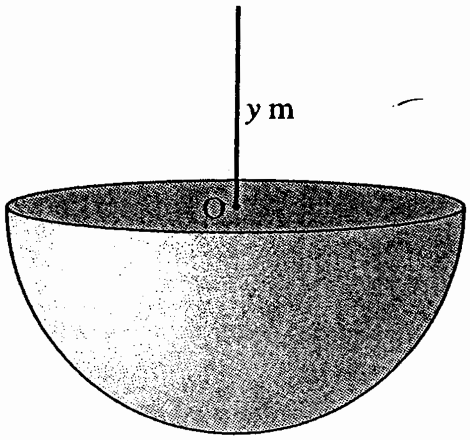


Fig. 3.3

(ii) Show that the centre of mass of this new object is a distance $\dfrac{y^2 - 8r}{24 + 2y}$ metres from O on the axis of symmetry in the direction away from the hemisphere. [5]

Question 3 continues on the next page

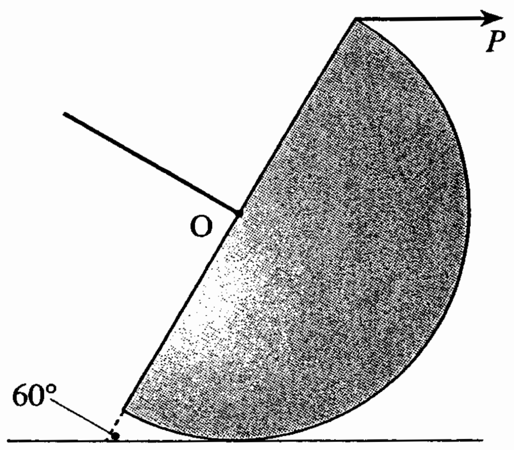


Fig. 3.4

This object is placed on a rough, horizontal table and is held with the disc at 60° to the horizontal by means of a horizontal force, P newtons, acting at a point of the circumference of the disc, as shown in Fig. 3.4. The normal reaction of the table on the object acts through the point O.

(iii) In the case when $m = 0.5$, $y = 3$ and $r = 0.5$, find the value of P. Find also the value of the coefficient of friction between the object and the table if the object is on the point of slipping. [7]

[Total 16]

4

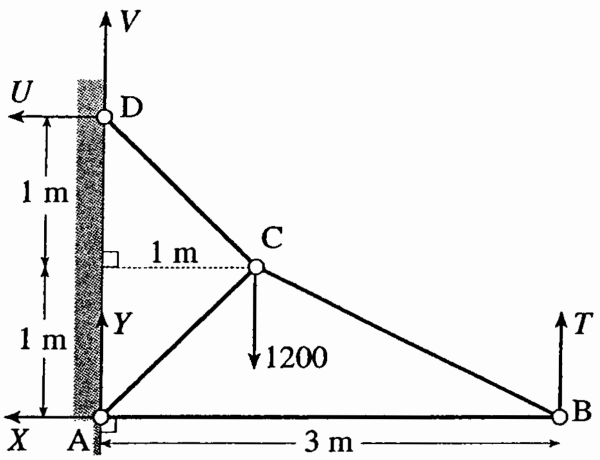


Fig. 4

Fig. 4 shows a light framework ABCD freely pin-jointed together at A, B and C and freely attached to a vertical wall at A and D. There is a load of 1200 N at C and a vertical force of T N acts at B. The other external forces U, V, X and Y N and essential geometrical information are marked in the diagram. The framework is in equilibrium.

(i) Show that $X = -U$ and that $U = \frac{1}{2}(1200 - 3T)$. [3]

(ii) By considering the equilibrium at D, show that $U = V$. [2]

(iii) Show that $Y = \frac{1}{2}(1200 + T)$ and find expressions in terms of T for the internal forces in each of the rods AB, BC, AC and CD. Show that just one of the rods will change from being in tension to being in thrust as T increases. For what value of T is there no internal force in this rod? [10]

[Total 15]

Mark Scheme

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Solutions and mark scheme

Q1		Mark		Subtotal
(i)	$5000 = F \times 2.5$	M1	Use of $P = Fv$	
	$F = 2000$ so 2000 N	A1		2
(ii)	$\frac{1}{2} \times 6000 \times 3^2 - \frac{1}{2} \times 6000 \times 2.5^2$	M1	Work-energy equation with all terms	
		B1	One KE term correct	
	$= 8000 \times 10 - W$	B1	WD by driving force correct	
	$W = 71750$ so 71750 J	E1	Clearly shown	
	Distance travelled is $\frac{2.5+3}{2} \times 10 = 27.5$ m	B1		
	Hence $F \times 27.5 = 71750$	M1	WD is Fd	
	$F = 2609.09...$ so 2610 N (3 s.f.)	A1		7
	or			
	$3 = 2.5 + 10a$ so $a = 0.05$	M1	Appropriate *uvast*	
		A1	For a	
	$3^2 = 2.5^2 + 2 \times 0.05 \times s$ so $s = 27.5$	B1		
	$\frac{8000}{2.75} - R = 6000 \times 0.05$ so $R = 2609$	M1	N2L and use of P/v	
		A1		
	WD = $2609.09.. \times 27.5 = 71750$ J	M1	WD is Fd	
		A1		
(iii)	Time taken is given by			
	$100 = \frac{3+3.25}{2} \times t$ so $t = 32$	B1		
	WD is $\frac{1}{2} \times 6000(3.25^2 - 3^2)$	M1	Work-energy. All terms present	
	$+ 6000 \times 9.8 \times 100 \times \frac{1}{20}$	M1	Attempt at GPE change	
		A1	GPE change correct	
	$+2000 \times 100$			
	$= 498687.5$	A1		
	$P = \frac{498687.5}{32} = 15583.98$ so 15.6 kW (3 s.f.)	F1	FT from **their** WD	6
	or			
	Using *uvast*	M1	Appropriate *uvast* + N2L	
	$a = 0.0078125$	A1		
	$t = 32$	B1		
	$F - 2000 - \frac{6000g}{20} = 6000a$	M1	N2L all terms present	
	so $F = 4986.875$	A1		
	Power is $\frac{4986.875 \times 100}{32} = 15.6$ kW (3 s.f.)	F1		Total 15

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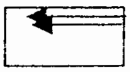
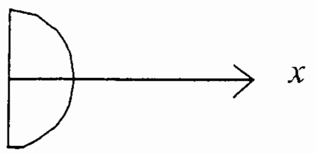
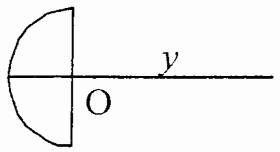
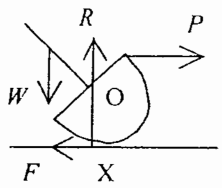
Solutions and mark scheme

Q 2		Mark		Subtotal
(i)	$I = 10(7 \times 0.6 - (-7))$ 	M1	$\mathbf{I} = m(\mathbf{v} - \mathbf{u})$	
		M1	For 7×0.6	
	$= 112$ Ns	A1		3
(ii)	After 3rd impact speed is 7×0.6^3	B1		
	so $10 \times g \times h = \frac{1}{2} \times 10 \times (7 \times 0.6^3)^2$	M1	Equating KE to GPE change	
		B1	Either term correct	
	$h = 0.11664...$ so 0.117 (3 s.f.)	E1		4
(iii)	before: 0 (4 kg), 7 (10 kg); after: v_W (4 kg), v_S (10 kg)	B1	Award if final numerical answer correct with no diagram	
	PCLM $70 = 4v_W + 10v_S$	M1	Use of PCLM	
		A1	Any form	
	NEL $\frac{v_W - v_S}{0 - 7} = -0.4$	M1	Use of NEL	
	so $v_W - v_S = 2.8$	A1	Any form	
	Solving for v_S			
	$v_S = 4.2$ so 4.2 m s^{-1}	A1		
	in its original direction	A1	Must be specified or implied by a diagram	7
				Total 14

Solutions and mark scheme

Q 3		Mark		Subtotal
(i)		M1	Method for c.m.	
	$(\pi r^2 + 2\pi r^2)\sigma\bar{x} = 0 + 2\pi r^2\sigma \times \frac{r}{2}$	B1 B1	LHS correct RHS correct	
	$\Rightarrow \bar{x} = \frac{r}{3}$	E1	Clearly shown [If $4\pi r^2$ used instead of $2\pi r^2$, allow M1 B1 for their equation]	4
(ii)		M1	Method correct	
	$(12m + my)\bar{y} = -12m \times \frac{r}{3} + my \times \frac{y}{2}$	B1 B1 B1	B1 for each term	
	$\bar{y} = \dfrac{\frac{y^2}{2} - 4r}{12 + y} = \dfrac{y^2 - 8r}{24 + 2y}$	E1	Clearly shown	5
(iii)				
	$\bar{y} = \frac{5}{30} = \frac{1}{6}$ Total mass is $15m = 7.5$	B1	Both	
	$\overset{\frown}{X}$ $15mg \times \frac{1}{6}\sin 60 = P(r + r\sin 60)$	M1 B1 B1	Attempt to take moments LHS RHS	
	Hence $P = 11.3704...$ so 11.4 N (3 s.f.)	A1	cao	
	$\mu \geq \frac{11.3704}{15 \times 0.5 \times g} = 0.1547...$	M1	Use of $F = \mu R$	
	so $\mu = 0.155$ (3 s.f.)	F1		7
				Total 16

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Solutions and mark scheme

Q 4		Mark		Subtotal
(i)	$\rightarrow$ $U + X = 0 \Rightarrow X = -U$	E1		
	$\hat{A}$ $2U + 3T = 1200$	M1	Moments about A or D	
	so $-X = U = \frac{1200-3T}{2}$	E1		3
(ii)	$\uparrow$ $V = T_{CD}\cos 45$ $\rightarrow$ $U = T_{CD}\cos 45$ so $U = V$	M1	Resolving in each direction	
		E1	Clearly shown	2
(iii)	$\uparrow$ For the whole system $V + Y + T = 1200$	B1		
	so $Y = 1200 - T - \frac{(1200-3T)}{2} = \frac{1200+T}{2}$	E1	Must be clearly derived	
	Consider all the struts in tension and consider the equilibria at pin-joints	M1 M1	Considering equilibrium at a pin-joint At least two equilibrium equations attempted	
	at D $\rightarrow$ $T_{CD}\cos 45 = U$ so $T_{CD} = \frac{1200-3T}{\sqrt{2}}$	A1		
	at A $\uparrow$ $Y + T_{AC}\cos 45 = 0$ so $T_{AC} = -\frac{(1200+T)}{\sqrt{2}}$	A1		
	$\rightarrow$ $X = T_{AC}\cos 45 + T_{AB}$ so $T_{AB} = -\frac{(1200-3T)}{2} + \frac{(1200+T)}{2} = 2T$	F1		
	at B $\uparrow$ $T_{CB} \times \frac{1}{\sqrt{5}} + T = 0$ so $T_{CB} = -\sqrt{5}T$	M1 A1	Attempt to find angle. [For forces in struts, FT according to order they are determined]	
	Only CD can change sign for $T > 0$. There is zero force in CD when $T = 400$.	E1	Clearly argued	10
				Total 15

Examiner’s Report

Mechanics 2 (2608)

General Comments

The paper appeared to be of an appropriate length with little evidence to suggest that candidates were short of time even when they had adopted inefficient methods. Most of the candidates made reasonable attempts at all of the questions and many perfect solutions were seen to every question. However, poor choice of method and the attempt to use more than one method within a part of a question, especially in the question involving work-energy, caused some candidates to use a lot of time and effort with little return.

Many of the candidates would have benefited from a fuller use of diagrams to clarify their working or establish a sign convention. The general standard of presentation of the scripts was high but too many candidates either tried to 'fudge' the algebra in Question 3 or produced poorly presented arguments.

Comments on Individual Questions

Q1 (Work, energy and power applied to a tractor)

Many candidates obtained their lowest mark on this question.

Most candidates could answer part (i) accurately but struggled to produce complete solutions to the other parts. The main problem seemed to be an inability to formulate and execute a coherent plan to solve this fairly standard problem. Those candidates who used a work-energy approach usually did better than those who used Newton's second law and the *uvast* formulae; many candidates started parts (ii) and (iii) by using one of these methods and then re-started using the other, making little further progress by doing so.

Most of the candidates successfully dealt with the resolution required in part (iii).

[(i) 2000 N; (ii) 2610 N (3 s. f.); (iii) 32 s, 15.6 kW (3 s. f.)]

Q2 (Impulse and momentum applied to a swinging sphere)

Parts (i) and (iii) were generally answered well, especially by those candidates who drew clear diagrams; those who did not produce adequate diagrams often obtained equations with inconsistent signs in part (iii).

Part (ii) was not answered so well. Many candidates did not recognise that they had first to find the speed of the sphere after the 3rd bounce by arguments based on Newton's experimental law and then the vertical height reached by considering the conservation of energy.

[(i) 112 Ns; (iii) 4.2 m s^{-1} in its direction before the impact]

Q3 (Centre of mass of a thin, closed hemispherical shell and frictional force)

Most of the candidates showed a good grasp of the standard methods and scored well on the first two parts of the question but much good work was spoiled by poor algebra; there were many errors in the manipulation and the omission of brackets was common. There were many obvious attempts to 'fudge' the given answer to part (ii).

Part (iii) was completed successfully only by the most able candidates. Many candidates took moments about O but failed to resolve in order to obtain a second expression involving P and the frictional force and so could make no further progress. Most candidates scored marks for correctly trying to use $F = \mu R$.

[(iii) 11.4 N (3 s. f.), 0.155 (3 s. f.)]

Q4 (Pin-jointed light framework)

This question was often answered well with many candidates scoring high marks in each part.

In order to avoid candidates wasting time using their own wrong expressions, many of the vital intermediate results were given. Many candidates failed to work with sufficient rigour to obtain all the marks available for establishing these results. For instance, in part (i) it is necessary to state that *because the framework is in equilibrium, resolving horizontally gives* $U + X = 0$ and so $X = -U$; it is not enough simply to write down $U + X = 0$ when $X = -U$ is *given* in the question.

In most cases, the candidates clearly understood that they had to consider the equilibrium at different pin-joints but there were many errors, especially with signs; most of these errors would have been avoided had a suitable clear diagram been produced.

[(iii) $T_{AB} = 2T$, $T_{BC} = -\sqrt{5}T$, $T_{AC} = -\dfrac{\sqrt{2}(1200+T)}{2}$, $T_{CD} = \dfrac{\sqrt{2}(1200-3T)}{2}$, CD will change from tension to thrust when $T = 400$]