OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2607**

Mechanics 1

Friday **16 JANUARY 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(a)** Cars A and B are travelling in the same direction along a straight road. The time t is in seconds.

At $t = 0$, car A is at rest. It accelerates at $3\,\mathrm{m\,s^{-2}}$ for $0 \leqslant t \leqslant 10$ and then travels at a constant speed.

Car B travels at $15\,\mathrm{m\,s^{-1}}$ for $0 \leqslant t \leqslant 30$ and then accelerates at $1\,\mathrm{m\,s^{-2}}$ until it reaches a speed of $25\,\mathrm{m\,s^{-1}}$, after which it continues at this constant speed.

(i) Draw v-t diagrams for the motion of car A and of car B, where v is the speed in $\mathrm{m\,s^{-1}}$ and $0 \leqslant t \leqslant 80$. [4]

(ii) Show that, in the first 40 seconds, car A travels 400 m further than car B. [4]

(iii) Given that car A is 500 m behind car B at $t = 0$, at what value of t does car A catch up with car B? [2]

(b) The velocity, v, of a particle is given as

$$v = 2t^2 - 3t - \tfrac{1}{3}t^3.$$

(i) Show that the acceleration of the particle is zero when $t = 1$ and when $t = 3$. [3]

(ii) Calculate the displacement of the particle from its position when $t = 1$ to its position when $t = 2$. [4]

[Total 17]

2

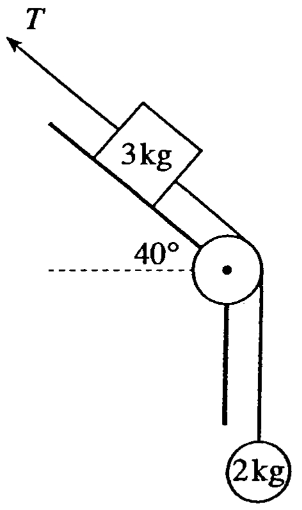

Fig. 2.1

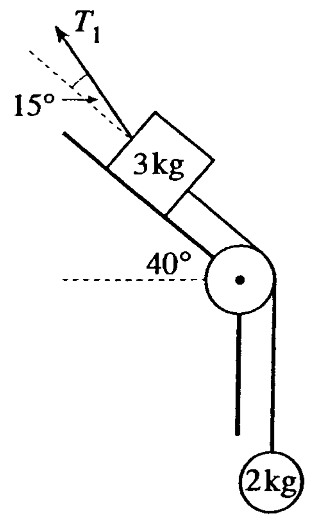

Fig. 2.2

Fig. 2.1 shows a block of mass 3 kg on a smooth plane inclined at 40° to the horizontal. A light string is attached to the lower end of the block. The string passes over a smooth pulley and is attached to a sphere of mass 2 kg that hangs freely. The part of the string between the pulley and the block is parallel to the plane. There is a tension of T N in another light string parallel to the plane and attached to the upper end of the block. The system is in equilibrium.

(i) Why is the tension in the string joining the block to the sphere 19.6 N throughout its length? [2]

(ii) Calculate the value of T. [4]

The string connected to the upper end of the block is now inclined at 15° to the plane, as shown in Fig. 2.2. The tension in this string is now T_1 N and the system remains in equilibrium.

(iii) Calculate the value of T_1 and also the value of the normal reaction of the plane on the block. [5]

(iv) Calculate the magnitude of the force acting on the pulley due to the tension in the string connecting the block and the hanging sphere. [3]

[Total 14]

3 (a)

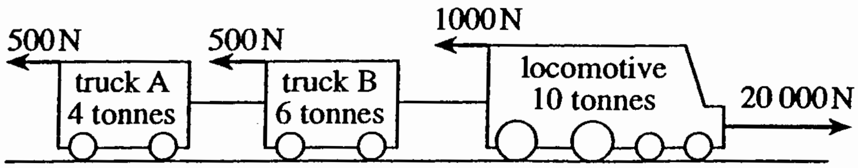

Fig. 3

A locomotive and two trucks are travelling along a straight, level track. Their masses, in tonnes, resistances to motion and the driving force of 20 000 N on the locomotive are shown in Fig. 3. The couplings between the locomotive and truck B and between trucks A and B are light and horizontal.

(i) Show that the acceleration of the locomotive and trucks is $0.9\ \mathrm{m\,s^{-2}}$. [3]

(ii) Calculate the force in the coupling between the trucks. [3]

The driving force changes but the resistances remain as before. The tension in the coupling between the trucks is now 5500 N.

(iii) Calculate the new values of the acceleration and of the driving force. [5]

(b) Ann and Beryl are both pushing a piano. Ann pushes in the direction $4\mathbf{i} + 3\mathbf{j}$ and Beryl in the direction $12\mathbf{i} + 5\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors. Together they produce a force of $(68\mathbf{i} + 33\mathbf{j})$ N so that

$$p(4\mathbf{i} + 3\mathbf{j}) + q(12\mathbf{i} + 5\mathbf{j}) = 68\mathbf{i} + 33\mathbf{j}, \text{ where } p \text{ and } q \text{ are constants.}$$

Calculate the force with which Beryl pushes. [4]

[Total 15]

4 In this question you should assume that air resistance is negligible.

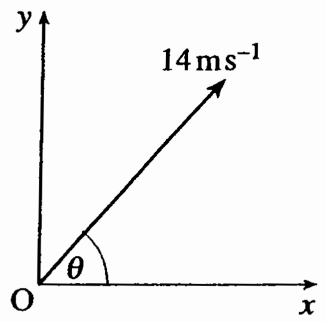

Fig. 4

A small ball is thrown from a point O with speed 14 m s^{-1} at an angle of θ to the horizontal. The x- and y-axes have origin O, as shown in Fig. 4, and the units of the axes are metres.

At time t seconds after projection, the ball is at the point (x, y).

(i) Find expressions, in terms of θ and t, for x and for y. [3]

(ii) In the case where $\theta = 30°$, find

(*A*) the maximum height of the ball above the level of projection,

(*B*) the horizontal range. [6]

(iii) Using your results in part (i), eliminate t to show that

$$y = x\tan\theta - 0.025x^2(1 + \tan^2\theta).$$ [3]

(iv) You are given that, if the ball is to pass through the point (a, b), then $\tan\theta$ satisfies the quadratic equation

$$\tan^2\theta - \frac{40}{a}\tan\theta + 1 + \frac{40b}{a^2} = 0.$$

Note that you are **not** expected to establish this result.

What can you conclude about possible trajectories if there are

(*A*) two solutions for $\tan\theta$,

(*B*) no solutions for $\tan\theta$? [2]

[Total 14]

Mark Scheme

Q 1		mark		sub
(a) (i)	Car A (0, 0) , (10, 30), (80, 30) Car B (0, 15), (30, 15), (40, 25), (80, 25)	B1 B1 B1 B1	Labelled axes; at least 4 sets co-ords (not O) marked or inferred One acceleration phase correct Car A correct; condone $t > 80$ Car B correct; condone $t > 80$ [SC1 in place of last B1 B1 if both graphs correct but terminate $40 < t < 80$] [SC1 if both graphs correct shape but no scales]	4
(ii)	For A $0.5 \times 10 \times 30 + 900 = 1050$ m For B $450 + 0.5 \times 10 \times 40 = 650$ m so extra is $1050 - 650 = 400$ m	M1 A1 M1 E1	Attempt at any distance as area under curve or by *uvast* Correct. FT from (i) only if two phases $0 \le t \le 40$ Attempt at second distance. Both cars must have 2 phases of motion $0 \le t \le 40$ Subtraction and clearly shown	4
(iii)	A has extra 500 - 400 = 100 m to catch up with difference in speeds of $30 - 25 = 5$ m s^{-1} so extra time is $\frac{100}{5} = 20$ s and $t = 60$	M1 A1	Both separation of 100 m and relative speed (or equivalent). FT only **their** $v = 30$. Allow 20s if clear, otherwise cao. Do not FT **their** 400 SC2 for 60 s seen. SC1 for $s_A = 1650$ or $s_B = 1150$ seen	2
(b) (i)	$a = 4t - 3 - t^2$ $t = 1,\ a = 4 - 3 - 1 = 0$ $t = 3,\ a = 12 - 3 - 9 = 0$	M1 A1 E1	Differentiating Both shown. Complete working required for $t = 3$ only.	3
(ii)	$\int_1^2 \left(2t^2 - 3t - \frac{t^3}{3}\right) dt$ $= \left[\frac{2t^3}{3} - \frac{3t^2}{2} - \frac{t^4}{12}\right]_1^2$ **either** $= \left(\frac{16}{3} - 6 - \frac{4}{3}\right) - \left(\frac{2}{3} - \frac{3}{2} - \frac{1}{12}\right)$ $= -\frac{13}{12} = -1\frac{1}{12}$ m **or** $s = \frac{2t^3}{3} - \frac{3t^2}{2} - \frac{t^4}{12} + C$ $s = 0$ when $t = 1$ gives $C = \frac{11}{12}$ Put $t = 2$, giving $s = -\frac{13}{12}$	M1 A1 M1 A1 M1 A1	Integrate; neglect limits and arb const Evidence of integration correct in at least one term Limits properly substituted and difference attempted Accept $+\frac{13}{12}$ m and $\pm 1.083...$ m Sub $t = 1$, $s = 0$ to give arb const and subsequently sub $t = 2$ Accept $+\frac{13}{12}$ m and $\pm 1.083...$ m	4
			total	17

Q 2		mark		sub
(i)	T is weight of 2 kg = $2g$ = 19.6 N	E1	Allow 2×9.8 seen. Allow 'T is the weight'	
	Pulley is smooth	E1		2
(ii)	$T - 3g\sin 40 = 19.6$	M1	Equating other forces on block to 19.6. Condone sign errors, condone $3g$ used. Allow $\sin \leftrightarrow \cos$. Condone T resolved. No extra forces.	
		M1	Weight term $3g$ sin40 seen in any equation. Allow $\sin \leftrightarrow \cos$. Allow $T = 3g\sin 40$.	
		A1	Correct equation	
	so T = 38.4979... so 38.5 N (3 s. f.)	A1		4
(iii)	$T_1\cos 15 = 38.4979...$	B1	$T_1\cos 15$ or $T_1\sin 15$ seen in an equation.	
	so $T_1 = 39.8560...$ so 39.9 N (3 s. f.)	B1	FT **their** T from (ii) / $\cos 15$.	
	$R + T_1\sin 15 = 3g\cos 40$	M1	Resolution attempted on any terms that should be resolved. Allow $\sin \leftrightarrow \cos$ and wrong angles. All forces present. No extra forces.	
		B1	Any resolved term correct (allow sign error) seen in an equation. FT **their** T_1.	
	so R = 12.20... so 12.2 N (3 s. f.)	A1	cao	5
(iv)	**either** $2\times 19.6\cos\left(\frac{90+40}{2}\right)$	M1	Attempt to resolve $2g$ along line of symmetry. Allow $\sin \leftrightarrow \cos$.	
		B1	$\cos\left(\frac{90+40}{2}\right)$	
	= 16.5666... so 16.6 N (3 s. f.)	A1		3
	or Resolve into cpts	M1	Use $2g$. Both cpts attempted. No missing terms. Allow $\sin \leftrightarrow \cos$.	
	Use Pythagoras	M1	Pythag used with 2 unequal cpts	
	16.6 N	A1		
	or Draw triangle of forces	M1	Arrows correct and triangle used for finding force	
		B1	Angles correct	
	16.5666... so 16.6 N (3 s. f.)	A1		
			total	14

Q 3		mark		sub
(a) (i)	$20000 - 2000 = 20\,000a$ $a = 0.9$ so 0.9 m s^{-2}	M1 B1 E1	Use of N2L. Condone use of $F = mga$ and sign errors and missing force. Use of 2000 N and 20 000 kg	3
(ii)	500 N ← [4000 kg] → T_1 ⟶⟶ 0.9 m s^{-2} $T_1 - 500 = 4000 \times 0.9$ $T_1 = 4100$ so 4100 N	 M1 A1 A1	 N2L and use of appropriate mass(es) with $a = 0.9$. Condone $F = mga$, sign errors and extra forces. All correct forces present. Correct equation cao	3
(iii)	$5500 - 500 = 4000a$ so $a = 1.25$ $D - 2000 = 20\,000 \times 1.25$ $D = 27\,000$ so 27 000 N	M1 A1 M1 F1 A1	Use of N2L with $F = ma$ and all terms present. Condone sign errors. No extra forces. Use of whole train (or Loco + B) with **their** new accn. All forces present. Condone sign errors. No extra forces. All correct FT **their** new accn. cao	5
			[If error with tonnes, penalise a maximum of 2 (including E) and FT]	
(b)	$4p + 12q = 68$ $3p + 5q = 33$ solving gives $q = 4.5$ Beryl pushes with force (54 **i** + 22.5 **j**) N	M1 M1 A1 A1	Attempt to obtain two scalar equations Dep on 1st M1. Solving simultaneously Obtaining $q = 4.5$ (FT) [Award SC3 for $q = 4.5$ WW] cao. Any form. Accept magnitude (58.5 N)	4
			total	15

Q 4		mark		sub
(i)	$x = 14\cos\theta\ t$	B1		
	$y = 14\sin\theta\ t - 4.9t^2$	M1	Appropriate *uvast* used with $a = \pm g$ etc. Accept omission of sin θ. Allow $\sin \leftrightarrow \cos$.	
		A1	cao. Accept $0.5gt^2$ in place of $4.9t^2$	3
(ii) (A)	**either**			
	Using $v^2 = u^2 + 2as$ we have	M1	Must have $v = 0$ and use $\pm g$, ±9.8, etc . Allow $\sin \leftrightarrow \cos$. Accept omission of sin 30.	
	$0 = (14\sin 30)^2 - 2\times 9.8\times s$	A1		
	giving $s = 2.5$ so 2.5 m	A1	cao [SC1 if $u = 0$ and direction not clear]	
	or			
	Need $14\sin 30 - 9.8t = 0$	M1	Must have $v = 0$ and use $\pm g$, ±9.8, etc . Accept omission of sin 30. Allow $\sin \leftrightarrow \cos$.	
	so $t = \frac{7}{9.8}$	A1		
	$y = 7\times\frac{7}{9.8} - 4.9\times\frac{7^2}{9.8^2} = 2.5$ so 2.5 m	A1	cao	
(B)	**either**			
	Need $y = 0$ so $t = \frac{7}{4.9}$	M1	Either use (i) or $2\times$ **their** t from (A) (FT)	
	Subst in expression for x so $x = 14\cos 30\times\frac{7}{4.9}$	M1	Sub in **their** x.	
	= 17.32… so 17.3 m (3 s. f.)	A1	cao. Accept $10\sqrt{3}$	
	or			
	Need $v_y = 0$ to find ½ range so $t = \frac{7}{9.8}$	M1	Distance found must be doubled. Either use $0 = u + at$ (with $u = 7,\ 7\sqrt{3}$ or 14), $a = \pm g$ etc or t from (A),	
	$5\sqrt{3}$	M1	Sub in **their** x	
	×2 to give 17.32… so 17.3 m (3 s. f.)	A1	cao. Accept $10\sqrt{3}$ [If work in (i) repeated, use same criteria for acceptable expressions]	6
(iii)	$y = 14\sin\theta\times\frac{x}{14\cos\theta} - 4.9\times\left(\frac{x}{14\cos\theta}\right)^2$	M1	Substitute for t. FT from (i) only if x is linear in t and y is at least a 2 term quadratic in t.	
	$= x\tan\theta - 0.025x^2(1+\tan^2\theta)$	E1	$\frac{4.9}{14^2}$ or $\frac{4.9}{196}$ seen or stating that $\frac{1}{\cos^2\theta} = 1+\tan^2\theta$	
		E1	All clearly shown	3
(iv) (A)	Two different trajectories	E1	Accept 'paths' etc.	
(B)	Cannot be reached	E1		2
			total	14

Examiner’s Report

2607 Mechanics 1

General Comments

Most of the paper was answered well by many of the candidates and few failed to make some progress with each question. It seemed that the content examined in Question 3(b) and in Question 4(iv) was not familiar to many of the candidates but these parts attracted excellent answers from some centres. On the whole candidates fared better with Question 1 and Question 4 than with Question 2 and Question 3. Question 1 was done extremely well by many candidates who scored at least 15 marks (losing marks only on part (iii)). Overall, the resolution of forces in Question 2 was poor, with wrong components and missing forces prevalent. Question 3(a) was either done extremely well or extremely poorly.

Comments on Individual Questions

Question 1 (Kinematics applied to two cars; kinematics involving calculus)

(a) Parts (i) and (ii) were usually fully correct. Errors in part (i) included failure to indicate scales, failure to draw for the whole domain, an error in the calculation of the time for which car B accelerates and some cases of curious number lines instead of *v-t* diagrams. In part (ii) there were slips with the arithmetic, especially from candidates whose working was poorly organised, usually followed by a false claim that the given answer had been confirmed.

Part (iii) presented a far greater challenge than expected and was done poorly overall. Many candidates omitted the part altogether and many considered Car A having to gain 500m whilst forgetting that Car B was moving as well. However, some candidates recognised that they had one car 100m behind travelling 5 ms^{-1} faster than the other and wrote the answer down (as intended).

(b) A minority of the candidates (and rather fewer than in recent sessions) used the *uvast* results throughout part *(b)* instead of calculus and the general standard of the answers was high.

In part (i), the majority of the candidates differentiated accurately and then demonstrated the given result. Some reversed the sign of the acceleration *before* equating to zero.

In part (ii), many candidates integrated correctly although some were unable to deal with $\frac{1}{3} \div 4$ correctly in the final term. Most of the candidates substituted the limits correctly but there were a number of errors with signs, especially at the stage of the final $-2-\left(-\frac{11}{12}\right)$.

(a)(iii) 60; (b)(ii) $-\frac{13}{12}$m

Question 2 (Static equilibrium of a block on a plane)

There were many poor solutions to this question, mostly due to inadequate diagrams, the omission of forces and the inability to resolve correctly.

In part (i), many candidates were able to establish the 19.6 N as $2g$, the weight of the hanging sphere, and most mentioned the smooth pulley, but quite a few failed to give both parts of the answer. Quite a number of candidates thought that the reason for the tension being the same throughout its length was because it was the same piece of string.

Part (ii) was poorly done by many candidates. Common errors were to omit a force, fail to resolve the weight of the 3 kg block at all, resolve incorrectly or apply the wrong signs.

In part (iii), many candidates were able to follow through to obtain their T_1 correctly. The normal reaction was not done particularly well, even by candidates who in other areas showed themselves to be strong. Many correctly found the component of the weight of the block normal to the plane but forgot that there was also a component of T_1 in this direction.

In part (iv), quite a few candidates used the symmetry of the problem and wrote down the answer. Others wrongly found the length of the base of the isosceles triangle with equal sides of $2g$. Those who attempted to find the components of the force on the pulley usually missed out a term when considering the vertical component. Many candidates omitted the part or wrote down expressions not clearly based on any relevant considerations.

(ii) 38.5 (3 s. f.); (iii) 39.9 (3 s. f.), 12.2 N (3 s. f.); (iv) 16.6 N (3 s. f.)

Question 3 (Applications of Newton's second law to train couplings; one vector as the sum of multiples of two other vectors)

(a) Although many candidates worked efficiently and accurately through this part, others struggled to apply the principals appropriately and quite a few seemed not to know how to deal with connected particles.

Part (i) was answered correctly by most of the candidates.

In part (ii), some candidates didn't know to which body/bodies they should apply Newton's second law and consequently their work was extremely muddled. It was quite common to see forces missing (usually the resistance to motion on one or more bodies) or the wrong masses used. A common incorrect method was to calculate the tension in the coupling as the difference between the values of *ma* for the bodies at either end of the coupling.

In part (iii), many candidates thought that the driving force could be found first by considering only one equation. Those who correctly attempted to find the acceleration first tended to make similar errors to those made in part (ii) but were able to gain follow through marks if they substituted their acceleration into a correct equation for the motion of the system (or part of it). However, many other candidates had no problems and gained five marks with little apparent difficulty.

(b) Very few complete solutions were seen to this part and they were invariably well presented by candidates who knew exactly what to do. Many candidates made no progress at all. Some candidates found the magnitudes and directions of the vectors, presumably in the hope that these might be relevant; others invented techniques such as "vector division" or a "method" involving the ratios of components.

(a)(ii) 4100 N; (iii) 1.25 ms^{-2}, 27 000 N; (b) (54 **i** + 22.5 **j**) N

Question 4 (Projectile motion)

Part (i) was done correctly by most of the candidates but some left their expressions with u un-substituted. Others obtained x correctly but thought y followed the same pattern (i.e. zero acceleration) or thought that the expression for x had an acceleration term. A surprisingly large number of candidates stumbled during the establishment of these basic equations. It was also quite common to see errors in this part by the same candidates handling the equations perfectly correctly in part (ii)!

In part (ii), there were many efficient and correct solutions. Common errors were to use the wrong component of the initial velocity (or even the speed) or to use the time of the whole flight in (*A*) and/or the time of half the flight in (*B*).

In part (iii), most candidates followed the instructions to use their equations from part (i). Some, however, instead substituted into the general Cartesian equation that they had (sometimes correctly) remembered, so they could not obtain marks for the 'show'. The majority of candidates who had suitable expressions for x and y eliminated t but many then failed to score further as their working was not sufficiently clear. It is imperative that all steps are shown when an answer is given; in this case the identity used had to be made explicit.

In part (iv), few candidates clearly indicated that in *(A)* there were two distinct trajectories. Some candidates clearly did not know what the significance was while others may have done but could not be awarded a mark for their poor descriptions.

In *(B),* many candidates appreciated that the ball couldn't pass through (a, b).

Both parts attracted many exotic suggestions. A large number thought that in *(B)* the trajectory would be vertical or horizontal and there were many references in both parts to bouncing on the x-axis.

(ii)*(A)* 2.5 m, *(B)* 17.3 m (3 s. f.)